Generation of Coherent Light by a Moving Medium

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Generation of coherent light by a moving medium

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We show that steady nonuniform motion of a medium through an optical resonator can yield light amplification at the resonator frequency. High gain can be achieved if at the generated frequency the medium refractive index is close to zero or medium has very strong frequency dispersion. We also discuss an analogy between light amplification by a moving medium and generation of sound waves when gas flows along a tube with acoustically closed-open boundaries.

It is know for more than a century that laminar motion of air through pipes can result in generation of sound waves. Rijke tube is an example of thermoacoustic oscillations that dates back to 1859 [1]. Rijke tube is a straight tube with the heated gauze placed inside. Air flows through the tube and heats up when passing the gauze. Depending on the position of the gauze and its temperature, pressure oscillations with amplitude of several hundred Pa at one of the tube's natural frequency can be generated [2]. There are other instabilities which yield generation of waves in acoustics and hydrodynamics. An example is the Kelvin–Helmholtz instability occurring when there is a velocity difference across the interface between two fluids. The instability manifests, e.g., in waves on the water surface when wind is blowing over water.

Motivated by the effect of sound wave generation in a steady flow of gas or liquid through pipes, we here explore a possibility of generation of coherent light by a dielectric medium moving through an optical cavity. Measurement of the light velocity in a moving dielectric medium by Fizeau in 1851, as well as the Michelson-Morley experiment, was an important step for the later development of special relativity [3]. In the Fizeau experiment two light rays were investigated traveling in the direction of water flow and opposite to it. The original intention of the experiment was to measure the absolute motion of the earth. The result, as is well known, was not in support of the original ether idea but instead with Einstein's addition formula,

$$u = \frac{c+nV}{n+V/c},$$

where u is the light velocity in the laboratory system, c is the speed of light in vacuum, n is the water refractive index in its proper frame and V is the water velocity. The Fizeau experiment has later been repeated with the use of ring lasers [4, 5].

If a uniform medium moves with velocity $\mathbf{V}$ then in the laboratory frame evolution of the electric field is described by the following equation [6]

$$n^2\frac{\partial^2 E}{\partial t^2} - c^2\nabla^2 E + 2(n^2-1)\mathbf{V}\cdot\nabla\left(\frac{\partial E}{\partial t}\right) = 0, \qquad (1)$$

where n is the refractive index. Assuming that $E(t,\mathbf{r}) = e^{-i\omega t}E(\mathbf{r})$ Eq. (1) yields the following equation for $E(\mathbf{r})$

$$n^2\omega^2 E + c^2\nabla^2 E + 2i(n^2-1)\omega\mathbf{V}\cdot\nabla E = 0. \qquad (2)$$

If $V \ll c$ the last term in Eq. (2) can be considered as a small perturbation. Multiplying both sides of Eq. (2) by E^* and integrating over volume of the system we obtain

$$\omega^2 = -c^2\frac{\int d\mathbf{r}E^*\nabla^2 E}{\int d\mathbf{r}n^2|E|^2} + 2i\omega\frac{\int d\mathbf{r}(1-n^2)E^*\mathbf{V}\cdot\nabla E}{\int d\mathbf{r}n^2|E|^2}. \qquad (3)$$

Writing $\omega = \omega_m + \delta\omega_m$, where ω_m are normal mode frequencies for non moving medium and neglecting $(\delta\omega_m)^2$, we find the following expression for the small correction to the frequency produced by motion of the medium

$$\delta\omega_m = i\frac{\int d\mathbf{r}(1-n^2)E^*\mathbf{V}\cdot\nabla E}{\int d\mathbf{r}n^2|E|^2}. \qquad (4)$$

In Eq. (4) E is the electric field of the normal mode ω_m for non moving medium. If $\delta\omega_m$ has an imaginary part the system is unstable towards the generation of electromagnetic waves in a similar way as the flow of gas through pipes can be unstable towards generation of sound waves.

Here we consider a system shown in Fig. 1. Namely, in the present setup a liquid or a gas medium flows in a tube that occupies region of space $-L/2 < z < L/2$. The velocity V of the liquid can be approximated as

$$V(z) = \begin{cases} V_0, & 0<z<L/2 \\ -V_0, & -L/2<z<0 \end{cases}, \qquad (5)$$

where V_0 could be positive or negative depending on the direction of the flow. We assume that the tube is a resonator for the electromagnetic field which for $V=0$ has the following even normal modes inside the cavity

$$E(\mathbf{r}) = E_0\cos(k_m z), \qquad (6)$$

where $k_m = n\omega_m/c$, n is the medium refractive index and ω_m are the resonator frequencies

$$\omega_m = \frac{(1+2m)\pi c}{nL}, \qquad (7)$$

and $m = 0, 1, 2, \ldots$ is an integer number. There are also odd normal modes in the cavity. However, they are not amplified in the present geometry and will not be considered here. Eq. (6) is a good approximation if orifice in the middle part of the cavity is sealed with a porous dielectric material with the medium flowing through. Under this condition the field inside the cavity approximately depends only on z.

Plugging Eq. (6) into Eq. (4) and integrating over the cavity volume we obtain the following expression for the gain per unit time

$$G = -i\delta\omega_m = 2\frac{(n^2-1)}{n^2}\frac{V_0}{L}, \tag{8}$$

which is given by the same formula for all even normal modes. In Eq. (8) the medium velocity V_0 could be positive or negative. $V_0 < 0$ means that the flow direction is opposite to those shown in Fig. 1.

For an arbitrary nonuniform velocity profile in the cavity $\mathbf{V}(\mathbf{r})$ the gain per unit time is determined by the Fourier component of $\mathbf{V}(\mathbf{r})$. Namely, Eq. (4) yields that gain for the cavity mode (6) is

$$G_m = \frac{(n^2-1)k_m}{n^2\mathcal{V}}\int d\mathbf{r}V_z(\mathbf{r})\sin(2k_m z), \tag{9}$$

where integration is taken over the cavity volume $\mathcal{V}$.

Eq. (8) shows that gain can be large for small enough refractive index n. For example, gain for the fundamental cavity mode with frequency $\omega_0 = \pi c/nL$ becomes comparable with ω_0 provided $n \simeq |V_0|/c$. If $L = 1$ cm and $n = 10^{-5}$ then $\omega_0 = 10^{16}$ s^{-1}, while for $V_0 = -10$ cm$/s$ Eq. (8) yields high value for the gain per unit time $G = 2\times 10^{11}$ s^{-1} comparable to the gain in active media of lasers [7]. In our case, however, the light amplification mechanism is different from those in lasers. Namely, in a laser light is amplified at the expense of the internal energy stored in atoms and laser generation usually requires population inversion. In the present mechanism the energy comes from the kinetic energy of the collective atomic motion and no initial population of atoms in the excited state is necessary. To achieve light amplification G must be greater than losses caused by atomic collisions and field leakage out of the cavity.

To check our analytical prediction of light amplification we solve Eq. (1) for $E(t,z)$ numerically assuming velocity profile $V(z) = V_0\sin(2\pi z/L)$, refractive index $n = 1.8$ and the following boundary $E(t, \pm L/2) = 0$ and initial $E(0,z) = E_0\cos(\pi z/L)$, $\partial E(0,z)/\partial t = 0$ conditions. In Fig. 2 we plot electric field at the middle of the cavity $E(t,0)$ as a function of time for $V_0 = 0.05c$ (a) and $V_0 = -0.05c$ (b). The simple one-dimensional velocity profile we use in numerical simulations is not realistic, however, it can be used to check correctness of the general analytical result (9). Numerical calculations show that motion of the medium yields light amplification if

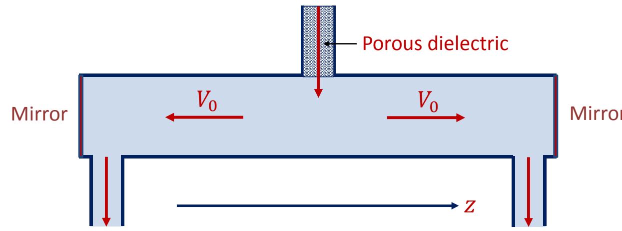

FIG. 1. Dielectric medium flows in a tube with mirrors at the edges. The system is a resonator for electromagnetic field. The orifice in the middle part of the cavity is sealed with a porous dielectric material with the medium flowing through.

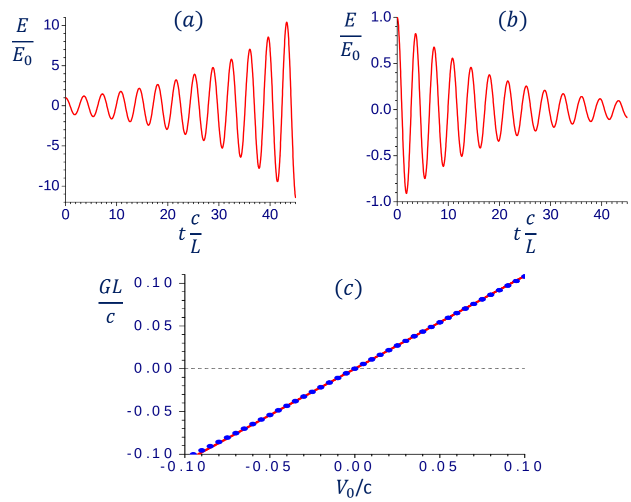

FIG. 2. Electric field at the cavity center as a function of time for the fundamental cavity mode obtained by numerical solution of Eq. (1) with refractive index $n = 1.8$ and velocity profile $V(z) = V_0\sin(2\pi z/L)$ for $V_0 = 0.05c$ (a) and $V_0 = -0.05c$ (b). (c) Gain per unit time G as a function of V_0 obtained by numerical solution of Eq. (1) (dots) and using analytical formula (9) for $m = 0$ (solid line).

$V_0 > 0$ (Fig. 2a) and attenuation for $V_0 < 0$ (Fig. 2b). In Fig. 2c we plot gain per unit time G as a function of V_0 obtained by numerical solution of Eq. (1) (dots) and using analytical formula (9) (solid line). Excellent agreement between the two results demonstrates validity of Eq. (9).

Inclusion of frequency dispersion. Next we investigate how dispersion modifies expression for the gain. From Maxwell's equations in a dielectric non magnetic medium, assuming no macroscopic currents and charges, we obtain the following wave equation for the transverse field

$$\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2} - \varepsilon_0\nabla^2\mathbf{E} = \mathbf{0},$$

where $\mathbf{D}$ is the electric displacement vector. Fourier transform yields

$$\frac{\omega^2}{c^2}\mathbf{D}(\omega,\mathbf{k})-\varepsilon_0 k^2\mathbf{E}(\omega,\mathbf{k})=\mathbf{0}.$$

If there is frequency dispersion then relation between $\mathbf{D}(\omega,\mathbf{k})$ and $\mathbf{E}(\omega,\mathbf{k})$ is

$$\mathbf{D}(\omega,\mathbf{k})=\varepsilon_0\varepsilon(\omega)\mathbf{E}(\omega,\mathbf{k})$$

which gives equation for the electric field

$$\frac{\omega^2}{c^2}\varepsilon(\omega)\mathbf{E}(\omega,\mathbf{k})-k^2\mathbf{E}(\omega,\mathbf{k})=\mathbf{0}.$$

If the medium is moving with nonrelativistic velocity $\mathbf{V}$ then, due to Doppler effect, ω and $\mathbf{k}$ must be replaced by [8]

$$\omega\to\omega-\mathbf{V}\cdot\mathbf{k},\qquad \mathbf{k}\to\mathbf{k}-\frac{\omega}{c^2}\mathbf{V},$$

and we obtain

$$\frac{1}{c^2}\left[\omega^2-2\mathbf{V}\cdot\mathbf{k}\omega\right]\varepsilon(\omega-\mathbf{V}\cdot\mathbf{k})\mathbf{E}(\omega,\mathbf{k})$$

$$-\left(k^2-2\frac{\omega}{c^2}\mathbf{V}\cdot\mathbf{k}\right)\mathbf{E}(\omega,\mathbf{k})=\mathbf{0}.$$

Approximating $\varepsilon(\omega-\mathbf{V}\cdot\mathbf{k})$ by the Taylor expansion in the first order

$$\varepsilon(\omega-\mathbf{V}\cdot\mathbf{k})\approx\varepsilon(\omega)-\frac{\partial\varepsilon(\omega)}{\partial\omega}\mathbf{V}\cdot\mathbf{k}$$

we find

$$\left[\omega^2\varepsilon(\omega)-2\omega\left(\varepsilon(\omega)-1+\frac{\omega}{2}\frac{\partial\varepsilon(\omega)}{\partial\omega}\right)\mathbf{V}\cdot\mathbf{k}\right]\mathbf{E}(\omega,\mathbf{k})$$

$$-c^2k^2\mathbf{E}(\omega,\mathbf{k})=\mathbf{0}.$$

Taking the inverse Fourier transform over $\mathbf{k}$ we obtain the following equation for $\mathbf{E}(\omega,\mathbf{r})$

$$\varepsilon(\omega)\omega^2\mathbf{E}(\omega,\mathbf{r})+c^2\nabla^2\mathbf{E}(\omega,\mathbf{r})+$$

$$2i\left(\varepsilon(\omega)-1+\frac{\omega}{2}\frac{\partial\varepsilon(\omega)}{\partial\omega}\right)\omega(\mathbf{V}\cdot\nabla)\mathbf{E}(\omega,\mathbf{r})=\mathbf{0}.\qquad(10)$$

Comparing this equation with Eq. (2) we see that frequency dispersion modifies expression for the gain (8) by replacing n^2-1 with $n^2-1+\frac{\omega}{2}\frac{\partial\varepsilon(\omega)}{\partial\omega}$. As a result, formula for the gain (8) becomes

$$G=\frac{2V_0}{n^2L}\left(n^2-1+\frac{\omega}{2}\frac{\partial\varepsilon(\omega)}{\partial\omega}\right).\qquad(11)$$

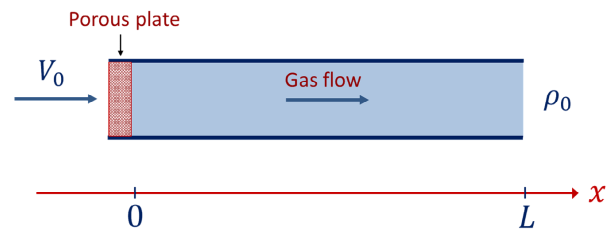

FIG. 3. Geometry of the closed-open tube with gas flowing along the tube axis. Porous plate at the end provides acoustically closed boundary condition.

Eq. (11) indicates that gain can be also large for the medium with strong dispersion and not only for the medium with small refractive index. Taking into account formula for the group velocity of light in a dispersive medium $c/V_g=\partial(n\omega)/\partial\omega$ and the relation $\varepsilon(\omega)=n^2(\omega)$ one can rewrite Eq. (11) as

$$G=\frac{2}{n^2}\left(\frac{nc}{V_g}-1\right)\frac{V_0}{L}.\qquad(12)$$

Thus, high gain can be achieved in a medium for which light group velocity $|V_g|\ll c$. If nc/V_g is a large positive (negative) number then there is gain if $V_0>0$ ($V_0<0$). Slow-light propagation effects typically originate from electromagnetically induced transparency (EIT). Such a phenomenon arises from quantum interference and is characterized by a strong enhancement of dispersion within a narrow frequency window around the medium resonance where absorption turns out to be largely quenched. EIT media are ideal candidates for the observation of extremely low or negative group velocities. In such media, the group velocity can in fact be readily tuned over a wide range of values directly by varying the coupling and probe detunings in a standard three-level Λ configuration.

Group velocity of about 8 m/s was achieved in a Teflon coated cell with rubidium vapor [9]. In this experiment the factor nc/V_g is of the order of 4×10^7. For such group velocity, $L=1$ cm and $V_0=10$ cm/s Eq. (12) yields gain per unit time $G\approx10^9$ s^{-1}. The EIT effect strongly suppresses absorption at the line center yielding nearly perfect transparency for strong coherent coupling beam. As a result, the loss rate can be made smaller than G leading to light amplification if the cavity frequency matches the atomic transition frequency.

Analogy with generation of sound waves. Similar mechanism can yield generation of sound waves due to motion of the medium. Since acoustic experiment is much easier to realize than its optical counterpart we next discuss the acoustic analogy in detail. We consider the flow of gas along a tube of the acoustically closed-open boundary type (see Fig. 3). The tube has length L and occupies a region of space $0 < x < L$. We disregard viscosity and assume that gas temperature is constant and uniform. The acoustically closed boundary condition can be realized by a flat porous plate (located at $x = 0$) of sintered material with a gas stream flowing through it at a constant velocity V_0 [2]. The opposite end of the tube (at $x = L$) is open.

Gas dynamics is described by the equations of mass conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial x} = 0 \tag{13}$$

and momentum conservation

$$\frac{\partial (\rho V)}{\partial t} + \frac{\partial \left(\rho V^2\right)}{\partial x} + \frac{\partial P}{\partial x} = 0, \tag{14}$$

where V is the gas velocity, ρ is the gas density and P is the pressure. We assume that gas flow is isothermal, then equation of state reads

$$P = u_s^2 \rho, \tag{15}$$

where u_s is the speed of sound.

We solve Eqs. (13)-(15) numerically subject to the following initial and boundary conditions

$$V(x,0) = V_0, \quad \rho(x,0) = \rho_0 \left[1 + 0.001 \cos\left(\frac{\pi x}{2L}\right)\right], \tag{16}$$

$$\frac{\partial \rho(0,t)}{\partial x} = 0, \quad \rho(L,t) = \rho_0. \tag{17}$$

We choose L as a unit of length and L/u_s as a unit of time. Gas velocity is then measured in u_s. Results of numerical simulations are shown in Fig. 4. We plot $\rho(0,t)$ as a function of time for $V_0 = 0.03u_s$ (a) and $V_0 = -0.03u_s$ (b). The figure shows that if flow velocity is directed from the closed to the open end of the tube the sound wave with wavelength $4L$ is excited. If flow velocity is the opposite the wave undergoes attenuation.

In summary, we show that steady nonuniform motion of a medium through an optical resonator can yield light amplification at the resonator frequencies. High gain can be achieved if at the generated frequency the medium refractive index is close to zero or the medium has a very strong frequency dispersion.

Liquid metals near plasma frequency are example of a medium with small refractive index. Recently, metamaterials with near-zero refractive index have drawn much attention. Over the past several years, zero-index structures have been experimentally demonstrated in the microwave [10, 11], mid-IR [12], and visible regimes [13].

Strong dispersion and slow group velocities in the m/s range have been achieved taking advantage of the electromagnetically induced transparency effect [9, 14]. The EIT effect suppresses absorption in the region of strong dispersion yielding nearly perfect transparency. One should mention that strongly dispersive medium supporting slow light also yields a remarkable enhancement of the transverse Fresnel-Fizeau light drag effect [8].

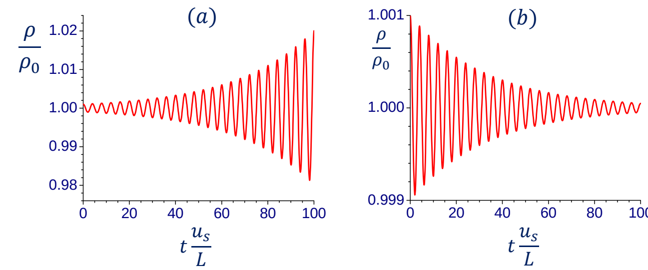

FIG. 4. Gas density at the closed end of the tube as a function of time obtained by numerical solution of Eqs. (13)-(15) with initial and boundary conditions (16), (17) for flow velocity $V_0 = 0.03u_s$ (a) and $V_0 = -0.03u_s$ (b).

The effect of light amplification by a moving medium has an analogy with sound generation when gas flows along a tube with acoustically closed-open boundaries. We show that for such geometry there is sound generation if flow velocity is directed from the closed to the open end of the tube. If flow velocity is the opposite the sound waves undergo attenuation. As one can see from Eq. (8) and Fig. 2, whether there is light amplification or attenuation in the optical system also depends on the flow direction. Energy for light generation is supplied from the kinetic energy of the moving medium (see Supplementary Material). This is different from the light amplification mechanism of a conventional laser where internal energy of the inverted medium is converted into coherent light. However, there is some resemblance with a free-electron laser in which a relativistic electron beam moving freely through a magnetic structure produces coherent radiation.

There is also a distant similarity with the Unruh radiation when a single neutral atom accelerated through vacuum is promoted to an excited state as if it was in contact with a blackbody thermal field [15]. If the ground-state atoms are accelerated through a high Q cavity the effect is enhanced by many orders of magnitude [16]. The reason is a fast nonadiabatic change of the atom-field coupling at the cavity boundaries. In the present problem nonadiabaticity also plays the key role. Namely, according to Eq. (9), high gain can be obtained only if $\mathbf{V}(\mathbf{r})$ substantially changes on the scale of the wavelength of electromagnetic oscillations. However, the present effect is collective and requires a medium with special properties to achieve high gain.

To the best of our knowledge the effect of light amplification by a medium moving through an optical cavity is new. It provides interesting insights on collective interaction between light and matter, and, in principle, could be useful for development of new coherent sources of radiation. We further investigate the effect using energy consideration and its acoustic analogy in Supplementary Material.

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