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Unity-Efficiency Parametric Down-Conversion via Amplitude Amplification

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We propose an optical scheme, employing optical parametric down-converters interlaced with nonlinear sign gates (NSGs), that completely converts an n-photon Fock-state pump to n signal-idler photon pairs when the down-converters' crystal lengths are chosen appropriately. The proof of this assertion relies on amplitude amplification, analogous to that employed in Grover search, applied to the full quantum dynamics of single-mode parametric down-conversion. When we require that all Grover iterations use the same crystal, and account for potential experimental limitations on crystal-length precision, our optimized conversion efficiencies reach unity for $1 \leq n \leq 5$, after which they decrease monotonically for n values up to 50, which is the upper limit of our numerical dynamics evaluations. Nevertheless, our conversion efficiencies remain higher than those for a conventional (no NSGs) down-converter.

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Nonclassical states of light, such as single-photon states [1–3], polarization-entangled states [4, 5] and multi-photon path-entangled states [6–9] are essential for linear-optical quantum computation [10], quantum communication [11–13], quantum metrology [14, 15], and experimental tests of quantum foundations [16–18]. Spontaneous parametric down-conversion (SPDC) employing the $\chi^{(2)}$ nonlinearity [4] is a standard tool for generating nonclassical light. As currently implemented, SPDC sources of nonclassical light rely on strong coherent-state pump beams. These pumps do not suffer appreciable depletion in the down-conversion process, meaning that their conversion efficiencies are exceedingly low. Moreover, the number of signal-idler pairs that are emitted in response to a pump pulse is random. To circumvent these drawbacks, we focus our attention on SPDC using n-photon Fock-state pumps [19]. We propose and analyze a scheme using such pumps that interlaces SPDC processes with nonlinear sign gates (NSGs) [10] to generate n signal-idler pairs with unity efficiency when the down-converters' crystal lengths are chosen appropriately. Our proof of unity-efficiency conversion presumes $n \gg 1$ and allows each Grover iteration to employ a different crystal length. Because the precision with which those crystal lengths must be realized becomes increasingly demanding as n increases, we evaluate the conversion efficiencies at a fixed crystal-length precision. Furthermore, to reduce our scheme's resource burden, we perform our efficiency evaluations assuming that all Grover iterations use the same crystal. We find that complete conversion is maintained for $1 \leq n \leq 5$, and that our approach's conversion efficiencies—although less than 100%—still exceed those of a conventional (no NSGs) down-converter for n values up to 50. Thus, even using the same crystal for all Grover iterations with finite crystal-length precision, our approach can efficiently prepare heralded single-photon states as well as dual-Fock ($|n\rangle|n\rangle$) states and multi-photon path-entangled states for $n \leq 5$ [20].

We begin by solving the full quantum dynamics for SPDC with single-mode signal, idler, and pump beams. Conventionally, SPDC dynamics are derived under the nondepleting-pump assumption, which treats a strong coherent-state pump as a constant-strength classical field throughout the nonlinear interaction. To date, SPDC with a quantized pump field [21, 22] has only been solved for pump-photon numbers up to 4 [23]. We construct the SPDC solution for an arbitrary single-mode pure-state pump as an iteration that we can evaluate numerically for pump photon numbers up to 50. From this result, we prove a fundamental bound on SPDC's conversion efficiency: no pure-state pump whose average photon number exceeds one can be completely converted to signal-idler photon pairs.

Inspired by the Grover search algorithm's use of amplitude amplification [24, 25], we show how the preceding limit on SPDC's conversion efficiency can be transcended by employing NSGs in between SPDC processes. In particular, we show that our method increases the efficiency with which all pump photons are converted to signal-idler pairs, enabling complete pump conversion to be achieved for Fock-state pumps when the down-converters' crystal lengths are chosen appropriately. This perfect conversion is deterministic if the NSGs are implemented using nonlinear optical elements. It is postselected—based on ancilla-photon detections—if the NSGs are realized with only linear optics.

Our technique for unity-efficiency parametric down-conversion (UPDC) has transformative applications in quantum metrology, quantum cryptography and quantum computation. In quantum metrology, an interferometer whose two input ports are illuminated by the signal and idler of the n-pair (dual-Fock) state $|n,n\rangle$ achieves a quadratic improvement in phase-sensing accuracy over what results from sending all $2n$ photons into one input port [14]. Single-mode SPDC yields a thermal distribution of $|n,n\rangle$ states, however, which erases the preceding entanglement-based advantage [15], whereas UPDC delivers the desired dual-Fock state for this purpose (Sec. II of [26]). The dual-Fock state turns out to be extremely valuable for preparing heralded Greenberger-Horne-Zeilinger (GHZ) and other path-entangled states with high probability, which are crucial resources for device-independent quantum cryptography [27, 28], quantum secret sharing [29], and testing quantum nonlocality [30].

Our development begins by addressing the $t \geq 0$ quantum dynamics for parametric down-conversion with single-mode signal, idler, and pump beams. The relevant three-wave-mixing interaction Hamiltonian is [21]

$$\hat{H} = \mathrm{i}\hbar\kappa\Big(\hat{a}_s^\dagger \hat{a}_i^\dagger \hat{a}_p - \hat{a}_p^\dagger \hat{a}_s \hat{a}_i\Big)\,, \tag{1}$$

where $\hat{a}_j^\dagger$ ($\hat{a}_j$) is the photon creation (annihilation) operator and $j = s, i, p$ denotes the signal, idler, and pump, respectively. The coefficient κ, which is assumed to be real valued, characterizes the nonlinear susceptibility $\chi^{(2)}$ of the down-conversion crystal [21]. We assume SPDC with type-II phase matching, so that the signal and idler beams are orthogonally polarized and the pump is co-polarized with the idler. This orthogonality is crucial to realizing the Grover iteration, as detailed below.

We restrict ourselves to initial states of the form $|\Psi(0)\rangle = \sum_{n=0}^{\infty} c_n\,|\Psi_n(0)\rangle$, where $\sum_{n=0}^{\infty} |c_n|^2 = 1$, and

$$|\Psi_n(0)\rangle = \sum_{k=0}^{n} f_k^{(n)}(0)\,|k,k,n-k\rangle\,, \tag{2}$$

with $\sum_{k=0}^{n} |f_k^{(n)}(0)|^2 = 1$, and $|n_s, n_i, n_p\rangle$ being the Fock state containing n_s signal photons, n_i idler photons, and n_p pump photons. For these initial states, the SPDC dynamics occur independently in the subspaces spanned by $\{|0,0,n\rangle\,, |1,1,n-1\rangle\,, \ldots, |n,n,0\rangle : 0 \leq n < \infty\}$, whose basis states comprise all possibilities from no conversion to complete conversion of pump photons into signal-idler photon pairs. The decoupling between these n-pump-photon subspaces allows us to solve the Schrödinger equation, $\mathrm{i}\hbar\,|\dot{\Psi}(t)\rangle = \hat{H}\,|\Psi(t)\rangle$ for $t \geq 0$, by solving the coupled ordinary differential equations

$$\dot{f}_k^{(n)}(t) = \begin{cases} -\kappa\sqrt{n} f_1^{(n)}(t), & k = 0 \\ \kappa\left[k\sqrt{n-k+1} f_{k-1}^{(n)}(t) - (k+1)\sqrt{n-k} f_{k+1}^{(n)}(t)\right], & k = 1,2,\ldots,n-1 \\ \kappa n f_{n-1}^{(n)}(t), & k = n, \end{cases} \tag{3}$$

given the initial conditions $\{f_k^{(n)}(0) : 0 \leq k \leq n\}$. We then get the n-pump-photon subspace's state evolution,

$$|\Psi_n(t)\rangle = \sum_{k=0}^{n} f_k^{(n)}(t)\,|k,k,n-k\rangle\,, \tag{4}$$

from which the full state evolution,

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} c_n\,|\Psi_n(t)\rangle\,, \tag{5}$$

follows. We have obtained analytical solutions to Eqs. (3) for $0 \leq n \leq 4$, and numerical solutions for $5 \leq n \leq 50$. The nth subspace's quantum conversion efficiency,

$$\mu_n(t) \equiv \sum_{k=1}^{n} \frac{k|f_k^{(n)}(t)|^2}{n},\ \text{when } |\Psi_n(0)\rangle = |0,0,n\rangle, \tag{6}$$

is the fraction of the initial n pump photons that are converted to signal-idler photon pairs. The down-converter's total quantum conversion efficiency is then

$$\mu(t) \equiv \frac{\sum_{n=0}^{\infty} |c_n|^2 n \mu_n(t)}{\sum_{n=0}^{\infty} |c_n|^2 n}. \tag{7}$$

Because $\sum_{k=0}^{n} |f_k^{(n)}(t)|^2 = 1$ for all n, neither $\mu_n(t)$ nor $\mu(t)$ can exceed unity. The central question for this paper is how to obtain unity-efficiency conversion, which occurs for $\mu_n(t)$ when $|f_n^{(n)}(t)| = 1$, and for $\mu(t)$ when $|f_n^{(n)}(t)| = 1$ for all n with nonzero c_n.

Our analytic solutions to Eqs. (3) for $1 \leq n \leq 4$ with $|\Psi_n(0)\rangle = |0,0,n\rangle$ show that $\max_t[\mu_n(t)]$ decreases with increasing n from $\max_t[\mu_1(t)] = 1$. This downward trend in conversion efficiency continues for $5 \leq n \leq 50$, where we employed numerical solutions because the Abel-Ruffini theorem shows that polynomial equations of fifth or higher order do not have universal analytic solutions. In other words, when the down-converter crystal is driven by vacuum signal and idler and an n-photon Fock-state pump, *only* the $n = 1$ case can yield unity efficiency. Moreover, because mixed states are convex combinations of pure states, exciting the down-converter with a mixture of $|0,0,n\rangle$ states also fails to realize complete conversion of pump photons to signal-idler photon pairs.

To overcome this fundamental limitation we interlace SPDC processes with NSGs. In Grover search [24], NSGs serve as quantum oracles that flip the sign of the marked state $|n\rangle$ by means of the unitary transformation

$$U_{\rm NSG}^{(n)} \sum_{j=0}^{n} \alpha_j\,|j\rangle = \sum_{j=0}^{n} (-1)^{\delta_{jn}} \alpha_j\,|j\rangle\,, \tag{8}$$

where δ_{jn} is the Kronecker delta function. The $U_{\rm NSG}^{(2)}$ gate, which is essential to linear-optical quantum computing's construction of a CNOT gate [10], has a nondeterministic implementation that only requires linear optics and single-photon detection. A deterministic realization of $U_{\rm NSG}^{(2)}$ is possible through use of a Kerr nonlinearity [31]. Nondeterministic $U_{\rm NSG}^{(n)}$ gates have postselection success probabilities with $O(1/n^2)$ scaling [32].

Grover search [24] finds the marked item in an unsorted data set of size N in the optimal [33] $O(\sqrt{N})$ steps, as opposed to the best classical algorithm's requirement of $O(N)$ steps. To reap Grover search's benefit in our context we perform it in the Fock basis. In particular, given a Fock-state input $|0,0,n\rangle$, with $n \geq 2$, our UPDC procedure uses $O(\sqrt{n})$ iterations of Grover search—in which an iteration consists of an NSG followed by SPDC—to convert that input to the dual-Fock-state output $|n,n,0\rangle$ with unity efficiency for n sufficiently large. (In Sec. I of [26] we show that unity-efficiency conversion of $|0,0,1\rangle$ to $|1,1,0\rangle$ can be realized with a single SPDC stage.) Our UPDC procedure is as follows.

I. *Initialization:* Initialize the UPDC procedure by sending signal, idler, and pump inputs in the joint state $|0,0,n\rangle$ into a length-L_0, type-II phase-matched $\chi^{(2)}$ crystal for an interaction time $t_0 = L_0/v$, where v is the *in situ* propagation velocity, to obtain the initial state [34]

$$|\Psi_0\rangle = \sum_{k=0}^{n} f_k^{(n,0)}(t_0)\,|k,k,n-k\rangle\,, \tag{9}$$

where the $\{f_k^{(n,0)}(t_0)\}$ are solutions to (3) for the initial conditions $f_k^{(n,0)}(0) = \delta_{k0}$.

II. *Sign flip on the marked state:* Begin the mth Grover iteration by sending the signal, idler, and pump outputs from the $(m-1)$th iteration—whose joint state is

$$|\Psi'_{m-1}\rangle = \sum_{k=0}^{n} f_k^{(n,m-1)}(t_{m-1})\,|k,k,n-k\rangle\,,\ \text{for } m \geq 1, \tag{10}$$

where $|\Psi'_0\rangle \equiv |\Psi_0\rangle$—through a polarization beam splitter (PBS) to separate the signal and idler into distinct spatial modes with the pump accompanying the idler. Then apply the $U_{\rm NSG}^{(n)}$ gate to the signal mode in Eq. (10) to produce the state

$$|\Psi_m\rangle = \sum_{k=0}^{n} f_k^{(n,m)}(0)\,|k,k,n-k\rangle\,, \tag{11}$$

where $f_k^{(n,m)}(0) = (-1)^{\delta_{kn}} f_k^{(n,m-1)}(t_{m-1})$, and use another PBS to recombine the signal, idler, and pump into a common spatial mode without changing their joint state.

III. *Rotation toward the marked state:* Complete the mth Grover iteration by sending the signal, idler, and pump in the joint state $|\Psi_m\rangle$ into a length-L_m, type-II phase-matched $\chi^{(2)}$ crystal for an interaction time $t_m = L_m/v$ to obtain the state

$$|\Psi'_m\rangle = \sum_{k=0}^{n} f_k^{(n,m)}(t_m)\,|k,k,n-k\rangle\,, \tag{12}$$

where the $\{f_k^{(n,m)}(t_m)\}$ are solutions to (3) for the initial conditions $\{f_k^{(n,m)}(0)\}$.

IV. *Termination:* Repeat Steps II and III until the probability that Step III's output beams are in the desired fully converted state is sufficiently close to unity.

Below we explain how Steps I–III can drive the conversion efficiency arbitrarily close to unity, and how, for n sufficiently large, this can be done in $O(\sqrt{n})$ Grover iterations.

For an initial state $|0,0,n\rangle$, the Fock-state amplitudes occurring in our UPDC procedure are real valued. Thus, for our present purposes, we can reduce the UPDC procedure's state evolution to SU(2) rotations by writing

$$|\Psi'_m\rangle = \sqrt{1-[f_n^{(n,m)}(t_m)]^2}\,|\mathbb{0}\rangle + f_n^{(n,m)}(t_m)|\mathbb{1}\rangle, \tag{13}$$

for $m \geq 0$, where $|\mathbb{1}\rangle \equiv |n,n,0\rangle$ is the fully converted state, and $|\mathbb{0}\rangle$ is the m-dependent, normalized state satisfying $\langle\mathbb{1}|\mathbb{0}\rangle = 0$. In Sec. I of [26] we show that with L_0 appropriately chosen we can realize

$$|\Psi'_0\rangle = \cos(\theta_g/2)\,|\mathbb{0}\rangle + \sin(\theta_g/2)\,|\mathbb{1}\rangle\,, \tag{14}$$

for small values of θ_g; e.g., $\theta_g \simeq 1/\sqrt{n}$ for large n. There we also prove that our UPDC procedure, with the $\{L_m\}$ appropriately chosen, can produce

$$|\Psi'_m\rangle = \cos[(2m+1)\theta_g/2]\,|0\rangle + \sin[(2m+1)\theta_g/2]\,|1\rangle\,, \tag{15}$$

for $m > 1$. Terminating the UPDC procedure after M Grover iterations, where M is the largest integer satisfying $(2M+1)\theta_g \le \pi$, then gives a $\sin^2[(2M+1)\theta_g/2]$ conversion efficiency. Rewriting this conversion efficiency as $1-\cos^2[(\pi-(2M+1)\theta_g)/2]$ and choosing L_0 such that $0 < (\pi-(2M+1)\theta_g)/2 \ll 1$, we find that $1-\cos^2[(\pi-(2M+1)\theta_g)/2] \approx (\pi-(2M+1)\theta_g)^2/8 \ll 1$. Moreover, for $\theta_g \simeq 1/\sqrt{n}$ with $n \gg 1$, we have that this near-unity conversion efficiency is realized with M being $O(\sqrt{n})$, meaning that $\sqrt{n}$ iterations suffice to achieve that performance.

Our proof that UPDC can achieve unity-efficiency conversion of an initial $|0,0,n\rangle$ state to a final $|n,n,0\rangle$ state for $n \gg 1$ allows each Grover iteration to use a crystal of a different length, making its required resources of order $O(\sqrt{n})$. Thus in our analytic (for $2 \le n \le 4$) and numerical (for $5 \le n \le 50$) conversion-efficiency evaluations we restricted our procedure's Grover iterations to recirculate the signal, idler, and pump beams through a single length-L_1 crystal, and we chose L_0 and L_1 to maximize the conversion efficiency. However, as Eqs. (3) evolutions have eigenmodes with associated eigenvalues whose magnitudes grow with increasing n, the precision to which the crystal lengths L_0 and L_1 must be cut grows with increasing n. Thus, for experimental feasibility, our conversion-efficiency optimizations took L_0 and L_1 to be integer multiples of $10^{-3}v/\kappa$ [35].

Available analytic solutions to Eqs. (3) for $n \le 4$ allowed us to verify that unity-efficiency conversion can be achieved for those pump-photon numbers; see Sec. III of [26] for a demonstration that a single Grover iteration suffices for $n = 2$. For $n \in \{2,3,4,5,6,7,8,10,20,40,50\}$ the optimized conversion efficiencies we obtained are shown in Fig. 1. Here we see that unity-efficiency conversion is possible for n values up to 5, using a single Grover-iteration crystal that is cut with the assumed length precision. Beyond $n = 5$, however, greater precision is presumably required. Figure 1 also includes similarly evaluated conversion efficiencies for a conventional SPDC setup, i.e., one in which a single nonlinear crystal is employed without any NSGs. As mentioned earlier, the conventional approach can only reach unity-efficiency conversion for $n = 1$, and Fig. 1 shows that the UPDC approach with finite crystal precision outperforms the conventional setup with the same crystal precision for $2 \le n \le 50$. Our UPDC conversion efficiencies presume the use of deterministic (unity efficiency) NSGs, such as can be realized under ideal conditions with a weak Kerr nonlinearity [31] or with trapped atoms governed by the Jaynes-Cummings Hamiltonian [36]. Now

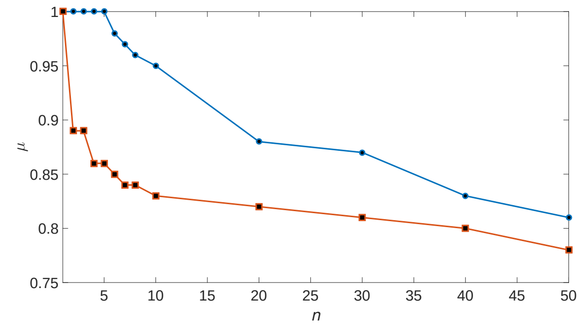

FIG. 1. Down-conversion efficiencies for n-photon Fock-state pumps optimized over nonlinear-crystal lengths cut to a precision of $10^{-3}v/\kappa$. Lower (red) curve: maximum conversion efficiencies for a $\chi^{(2)}$ crystal without Grover-search amplitude amplification. Upper (blue) curve: maximum UPDC conversion efficiencies, where the $n = 1$ point did not employ an NSG.

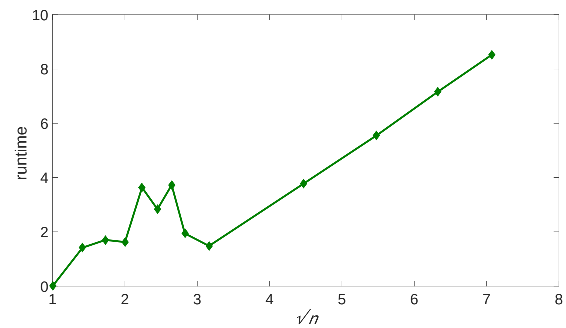

FIG. 2. UPDC runtime (defined to be M_nL_1/v with M_n being the number of Grover iterations used in Fig. 1 to achieve maximum efficiency for an n-photon pump) versus $\sqrt{n}$.

consider a UPDC procedure that employs $\sqrt{n}$ Grover iterations to transform an n-pump-photon Fock state to n signal-idler photon pairs using nondeterministic NSGs. Its conversion efficiency is reduced from our deterministic NSG result by a $(1/n^2)^{\sqrt{n}}$ factor, owing to each of its $\sqrt{n}$ NSG uses having an efficiency that is bounded above by $1/n^2$ [32]. Furthermore, each of these nondeterministic NSGs will require at least n single-photon ancillae [32].

The preceding efficiency optimization also permits us to determine the runtimes for our UPDC procedure at finite crystal-length precision, where runtime is defined to be M_nL_1/v with M_n being the number of Grover iterations needed to achieve the n-photon-pump's maximum efficiency from Fig. 1. These runtimes, which we have plotted in Fig. 2, show the expected $O(\sqrt{n})$ behavior for $3 \le \sqrt{n} \le 7$.

At this juncture, some discussion of implementation considerations is warranted. UPDC requires a very strong $\chi^{(2)}$ nonlinearity if it is to be practical. Probably the most promising candidate for implementation is the induced $\chi^{(2)}$ behavior of the $\chi^{(3)}$ nonlinearity in a photonic-crystal fiber [37]. Such an arrangement uses nondegenerate four-wave mixing with a strong, nondepleting pump beam at one wavelength whose presence induces a strong $\chi^{(2)}$ for a weak SPDC pump beam at another wavelength [38, 39]. Presuming that the induced $\chi^{(2)}$ value enables unity-efficiency conversion of the $|0,0,2\rangle$ input state to a $|2,2,0\rangle$ output state, a K-level cascade of these UPDC systems then enables unity-efficiency preparation of the $|2^K,2^K\rangle$ dual-Fock polarization state from the $|0,0,2\rangle$ input state, as shown in Sec. IV of [26]. This method requires efficient preparation of the two-photon Fock-state pump, which is experimentally challenging at present. Theoretical suggestions for such Fock-state preparation include Refs. [19, 40]. Microwave generation experiments include Refs. [41, 42], which could yield two-photon optical pumps by means of microwave-to-optical quantum-state frequency conversion (QSFC). See Refs. [43–47] for optical-to-optical QSFC.

In conclusion, we have studied the quantum theory of SPDC with single-mode signal, idler, and pump beams and Fock-state pumps. We found that the efficiency of converting pump photons into signal-idler photon pairs is unity only for the single-photon pump. In order to transcend this fundamental limit, we proposed using amplitude amplification, analogous to Grover search, of the completely-converted state by interlacing SPDC processes with NSGs. Our method can realize unity-efficiency conversion, with nonlinear crystals of the appropriate lengths, for all pump-photon numbers, but the required crystal-length precision becomes increasingly demanding with increasing pump-photon number. Nevertheless, unity-efficiency conversion should be possible for pump-photon numbers up to 5, even if the same crystal length is used for all Grover iterations.

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[26] See Supplemental Material at http://link.aps.org/... for: the quantum dynamics of parametric down-conversion with single-mode signal, idler, and pump beams; the proof that unity conversion efficiency is realized for n-pump-photon Fock-state pumps when $n \gg 1$; the two-photon-pump Grover search example; and the generation of dual-Fock states by means of cascaded two-pump-photon UPDC.
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