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# Size sorting on the rubble-pile asteroid Itokawa 

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#### Abstract

Photographs of the asteroid Itokawa reveal unexpectedly strong size segregation between lowlands populated almost entirely by small pebbles and highlands consisting of larger boulders. We propose that this segregation may be caused by a simple and unexplored effect: pebbles accreting onto the asteroid rebound from boulders, but sink into pebbly regions. By number, overwhelmingly more particles on Itokawa are pebbles, and collisions involving these pebbles must unavoidably cause pebbly regions to grow. We carry out experiments and simulations that demonstrate that this mechanism of size sorting based on simple counting of grains produces strong lateral segregation that reliably obeys an analytic formula.


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In 2005, the asteroid $25143 /$ Itokawa was visited by the JAXA spacecraft Hayabusa ${ }^{1}$, which surprisingly found strong lateral segregation between small and large particles, shown in Fig. 1(a). Itokawa, shown in Fig. 1(a), is about 540 m along its longest axis, and is believed to be a socalled "rubble pile," formed through gravitational accretion followed by collisional and weathering processes ${ }^{7}$. Raised areas ${ }^{2}$ on Itokawa are populated by boulders ranging in diameter ${ }^{3,4}$ from 5 to 40 m , while depressions are filled with smooth seas of smaller particles ${ }^{5}$ ranging from fine dust to centimeter-sized pebbles ${ }^{6}$. It is not known how this segregation came about, and understanding this may shed light on the processes that asteroids ${ }^{7,8,9}$ - and perhaps other bodies ${ }^{10,11}$ - undergo during formation and development.

This segregation has been attributed ${ }^{5,12,13}$ to the Brazil nut effect (BNE) ${ }^{14}$, in which particles differing in size separate during sustained vertical shaking in the presence of gravity ${ }^{15,16}$. In the BNE, smaller grains either subduct through convection ${ }^{17,18,19}$ or sift beneath larger neighbors ${ }^{20}$. Studies attributing Itokawa's segregation to convection ${ }^{5,12}$ are constrained by the observation ${ }^{21,22}$ that under its weak gravity $\left(\mathrm{g} / 10^{5}\right)$, convection would be very slow, and by calculations ${ }^{23,24}$ that show that required agitation velocities would be very close to escape velocities. Simulations investigating sifting ${ }^{21,25}$, on the other hand, produce surfaces uniformly dotted with boulders, in stark contrast the lateral segregation seen on Itokawa. Thus even if the BNE could explain boulders rising to the surface, it does not account for their pronounced lateral segregation. Finally, irrespective of the variety of BNE, it is perplexing that boulders do roll ${ }^{5}$, but do not make their way into gravitational valleys ${ }^{26}$, and instead perch on highlands ${ }^{2}$ leaving valleys populated almost entirely by pebbles ${ }^{4}$.

We propose here an alternative, and much simpler, mechanism of size segregation on rubble pile asteroids such as Itokawa. We observe that on Itokawa the volumes of gravitated pebbles and boulders are comparable (about 20\% of Itokawa's surface area and several percent of its volume consists of fine particles ${ }^{1,27}$ ). This implies that there must be overwhelmingly more small particles, by number, than large, and so most collisions that made up the asteroid must have been from small particles.


FIG. 1 - Segregation of large and small grains. (a) Asteroid 25143/Itokawa is shown in inset; enlargement highlights size segregation, especially in the "Muses sea". Credit: http://global/JAXA.jp. (b) Illustration of ballistic sorting: a small pebble rebounds from a larger boulder, but (c) is absorbed into a cluster of pebbles. Demonstration of effect in simple experiments: (d) small particles dropped independently onto a ceramic plate rebound leaving few residual particles, but (e) the same number of particles made to interact with one another by pouring them from the same height leave a substantial residue on the plate. (f) Similarly, if a small pile is initially placed on the plate (left), then particles sprinkled from above exactly as in panel (d) accumulate into a growing mound (right).

As an estimate, if we take pebbles on Itokawa to be of order ${ }^{5,6,28} 1 \mathrm{~cm}$ in diameter and boulders to be of order ${ }^{3} 10$ m , the diameters would differ by factor of a thousand. If there were equal volumes of pebbles and boulders, then there would be a billion times more pebbles than boulders. This estimate can be made more conservative by 1 or 2 orders of magnitude by accounting for actual volumes;
nevertheless overwhelmingly more collisions with the asteroid must have been by smaller particles than by larger.

This is significant because when a pebble hits a boulder, it rebounds (sketched in Fig. 1(b)), whereas when it hits a sea of other pebbles, its momentum dies (Fig. 1(c)). This has been understood since Bagnold's foundational work on granular physics ${ }^{29}$, and occurs because every collision reduces the normal speed by a constant restitution coefficient ${ }^{30}$. Since a pebble sea contains numerous pebbles, an incoming particle causes numerous collisions, which makes granular beds excellent impact absorbers ${ }^{31,32}$.

Thus simply by counting particles we can conclude that the collisions that made up rubble pile asteroids such as Itokawa were overwhelmingly by pebbles, and since pebbles bounce off of boulders and sink into pebble seas, it is inevitable that these seas will grow. Moreover as we will show, this mechanism leads to smooth seas of pebbles in valleys and pebble-free boulders on highlands. We term this mechanism "ballistic sorting," and acknowledge related work on ballistic deposition ${ }^{33}$ and collisional segregation ${ }^{34}$.

We test this model in several ways in this paper. As a first, qualitative, test, in Fig's 1(d)-(f) we demonstrate that pebbles colliding with a large "boulder" rebound, leaving essentially no residual pebbles on the boulder, while collisions between pebbles cause pebbles to aggregate.

In Fig. 1(d), we sprinkle 500 ml of 1 mm glass beads ("pebbles") onto a ceramic plate ("boulder" - see video in Supplement 1). The sprinkling technique distributes pebbles according to a random and spatially uncorrelated distribution as described in Supplement 2. Fig. 1(d) shows the end of the experiment, when almost every pebble has left the plate.

If instead we deposit the pebbles from the same height as before but by pouring them so that the pebbles collide with one another, we produce a growing mound as shown in Fig. 1(e) (see video in Supplement 3): this snapshot is taken just before the beaker is empty so as to show the pouring, but the mound remains after pouring has stopped.

Finally if we initially place 100 ml of the pebbles on the plate, shown on the left of Fig. 1(f), and then sprinkle the remaining 500 ml exactly as in panel (d), we produce a growing mound, shown to the right of panel (f) (also in video of Supplement 4).

So individual collisions between pebbles and a fixed boulder produce essentially no residual pebbles (Fig. 1(d)), while collective collisions either due to depositional conditions (Fig. 1(e)) or to the initial state (Fig. 1(f)) result in growing accumulations of pebbles.

To quantify this behavior, we note that ballistic sorting promotes the accumulation of pebbles in regions already occupied by pebbles. This behavior is termed cooperativity ${ }^{35}$, and is described by the Hill equation:

$$
\begin{equation*}
F(T)=\frac{100}{1+(k / T)^{n}}+f_{o} \tag{1}
\end{equation*}
$$

where $F$ defines the fraction of coverage by pebbles, $T$ is a timescale, and $k$ is a holding capacity. In our problem, $T$ is the number of sieve-loads (see Supplement 2) of deposited
pebbles, and $k$ is a timescale at which rapid filling of small interstices between boulders gives way to slow filling of large surface areas. Crucially, the exponent $n$ defines the cooperativity: for $n>1$, accumulations of pebbles reinforce further accumulation, and for $n<1$, they inhibit it. We include a constant coverage, $f_{o}$, to account for a minimum initial accumulation of pebbles that is needed to initiate cooperation (evident from comparing Fig's 1(d) and (f)).


FIG. 2 - Evaluation of cooperativity in deposition of small grains. (a) Typical snapshots after sprinkling of 1 mm glass beads from a height of 50 cm onto river stones of mean volume $140 \pm 110 \mathrm{~cm}^{3}$ on the bottom of a 45 cm X 45 cm box. Shown are 5, 8 and 15 sieve-loads (photos from Trial 3); small grains are false-colored to aid visualization. Measurements of sprinkling uniformity provided in Supplement 2. (b) Several trials using different initial substrates of river stones along with fit to their average from $E q$. (1) ( $f_{o}=5 \pm 1, k=8.0 \pm 0.2, n=2.15 \pm 0.06, r^{2}=$ 0.9994). (c) In the presence of pre-existing topography with a central valley, a nearly unbroken sand sea forms between higher peaks. Note that occasionally a stone may settle during an experiment, producing minor discrepancies between initial and final stone locations.

We evaluate whether pebble seas grow as predicted by Eq. (1) by performing trials in which we initially place river stones in different random arrangements for each trial, and uniformly sprinkle glass beads onto the stones from above. Typical snapshots are shown in Fig. 2(a), and evaluations of fractional areas occupied by beads are shown in Fig. 2(b)
for 3 trials. Details of imaging, measurement and sprinkling techniques are provided in Supplement 2. Data at each timepoint (i.e. every sieve-load of beads deposited) are averaged from Fig. 2(b) and fit to Eq. (1) with correlation coefficient $r^{2}=0.9994$, and exponent $n=2.15 \pm 0.05$. Since $n$ is significantly above 1 , we conclude that substantial cooperativity is present ${ }^{35}$, meaning that accumulations of pebbles promote further accumulation.

Small asteroids typically have topographies with substantial peaks and valleys, and as we have mentioned, pebble seas tend to occupy valleys ${ }^{5}$ on Itokawa. We therefore repeated our experiments with substrates of stones arranged into peaks and valleys. In multiple trials, we invariably find as shown in Fig. 2(c) that the valleys fill up to form nearly unbroken and flat pebble seas, leaving pebble-free stones in raised areas.

This raises the question of whether the observed size separation is due to ballistic sorting, or whether pebbles simply flow to the lowest points on irregular terrains. To address this question, we first briefly calculate the expected rate of growth of surface area of a valley due to downhill flow, and then perform simulations that track individual pebbles to determine whether or not they sort ballistically.

So first we calculate the surface area of flowing pebbles by setting the rate of change of volume of settled particles equal to the volume flux added: $d V / d t=N \cdot v$, where $V$ is the volume of a filled valley at time $t, N$ is the number of pebbles added per unit time, and $v$ is the volume of a single pebble. The simplest model for a 3D valley is a hemisphere of radius $R$. In this case the radius, $r$, of a valley at height $h$ is $r=\sqrt{2 R h-h^{2}}$. During the initial filling of a valley, $h \ll \mathrm{R}$, so $r \approx \sqrt{2 R h}$, and $V=\int_{0}^{h} \pi r^{2} d h \approx \pi r^{4} /(4 R)$. Integrating the volume over time gives $r^{2}=\sqrt{4 N R v / \pi} t^{1 / 2}$.

Thus the surface area of a hemispheric valley initially grows as $t^{1 / 2}$. By comparison, the Hill Eq. (1) for short times (i.e. small $T / k$ ) gives a fraction fill of $\left[F(T)-f_{o}\right] \approx 100 \cdot t^{n}$, where $t=T / k$ and $\mathrm{n}=2.15$. We have repeated this calculation for a hemicylindric valley, a V-shaped valley, and a valley surrounding a hemispheric mound by simply changing the formula for $V$ and integrating over time, and we obtain surface areas that grow as $t^{1 / 3}, t^{1 / 2}$ and $t^{2 / 3}$ respectively. In all cases, for early growth of a valley we obtain exponents, $n$, less than one, which are inconsistent with the positive cooperativity that we see experimentally. We conclude that our pebble seas do not grow by downhill flow into valleys.

Second, to explicitly confirm that the cooperative growth shown in Fig. 2(b) occurs due to ballistic sorting, we perform simulations that permit us to track trajectories, pebble by pebble, so as to quantify the extent to which pebbles bounce away or inelastically collapse after each collision. The simulations also allow us to evaluate the effect of gravity on the segregation observed.

Details are provided in Supplement 6, but in summary we form an initial substrate by dropping particles,
consisting either of 1 mm pebbles or larger stones, from a fixed height onto an irregular surface. We drop equal masses of pebbles and stones over time to produce a substrate of particles on this surface. Once the substrate has been established in this way, we vary gravity and quantify the ballistic sorting effect by dropping pebbles onto the substrate and tracking each pebble trajectory. The substrate used and typical trajectories obtained are provided in Supplements $6 \& 7$. We allow the pebbles to come to rest and evaluate lengths of trajectories from initial to final contact with the substrate.

Pebbles invariably come to rest in valleys, and we plot the trajectory lengths obtained using several hundred test particles in Fig. 3. In that figure, we plot distributions of trajectory lengths of pebbles that initially strike larger stones ("pebble-to-stone") alongside distances of pebbles that initially strike other pebbles ("pebble-to-pebble"). The ballistic sorting hypothesis holds that pebble-to-stone particles should travel much greater distances than pebble-to-pebble particles, and that both types of particles should ultimately be deposited in pebble seas. Qualitatively, this is seen both experimentally and computationally, shown respectively in videos of Supplements 4 and 7, which display no residual pebbles on stones. Quantitatively, our simulations confirm that pebble-to-stone particles travel a factor of greater than 5 further than pebble-to-pebble particles: this is shown in the main plot of Fig. 3 for Earth's gravity, $g$, using $N=702$ pebbles, and in the inset at $g / 10^{5}$ (approximately Itokawa's gravity) using $N=693$ pebbles. From these results, the sorting effect of ballistic sorting appears to be stronger at lower gravity.


FIG. 3 - Lengths of trajectories, obtained from tracking a large number of pebbles dropped in discrete element simulations onto irregular substrate (details provided in Supplement 6). Main plot shows factor of 5 greater distance for pebble-to-stone collisions than pebble-to-pebble ones using Earth's gravity, g. Inset shows factor of 13 using Itokawa's lower gravity, $g / 10^{5}$. In both cases, pebble-to-pebble collisions are transported much shorter distances than pebble-to-stone collisions.

Thus our simulations confirm that pebble seas grow because incoming pebbles rebound from stones but collide inelastically with other pebbles, and this finding does not
diminish at low gravity. Additionally, in both experiments and simulations we find that ballistic sorting leads to the formation of flat pebble seas in gravitational valleys.

Based on both experiments and simulations, it appears that low speed deposition of pebbles results in a predictable growth of pebble seas. On asteroids, however, high-speed collisions are known to occur ${ }^{36}$. As we have mentioned, granular beds are excellent impact absorbers, and so even high-speed micrometeorites can be expected to be captured by pebble seas, causing them to grow - sandbags are used to stop rifle bullets for this reason. Moreover, in boulderrich regions away from pebble seas, smaller particles produced by fracture, comminution or thermal fatigue ${ }^{37}$ can be expected to be ejected skyward by high-speed impacts. Those particles ejected faster than the escape velocity will be cleared from boulder-rich regions, while those traveling slower will undergo ballistic sorting when they return to the asteroid.

To be sure, sufficiently energetic impacts can destroy landscapes produced by any process; likewise landscapes will be remodeled by many effects ${ }^{7,37,38}$, including tidal disruption, Yarkovsky and YORP effects, rotational losses, gardening, ice sublimation, electrostatics, radiation pressure sweeping, etc. In common with other analyses of segregation on asteroids (e.g. the $\mathrm{BNE}^{12,21,23,24,25}$ ), we focus on the results of a single mechanism while acknowledging that it is only one of several processes involved.

Ballistic sorting can be expected to have greater influence on smaller asteroids that present a small cross section to high-speed projectiles and that form through gravitational accretion. We note that Itokawa's escape velocity is about $0.2 \mathrm{~m} / \mathrm{s}$, and at this re-entrant speed, rubble wouldn't fracture or substantially rearrange previously deposited material. Consequently we expect that small rubble piles should be especially subject to ballistic sorting and should generically exhibit strong lateral segregation between deposits of small and large particles. Indeed, analysis of spectroscopic ${ }^{39}$ and thermal imaging ${ }^{40}$ data indicate that the comparably-sized asteroid 101955 Bennu (selected for visitation beginning in 2018 by the NASA explorer OSIRIS-REx) also exhibits lateral size segregation, with fines smaller than 1 cm and a $10-20 \mathrm{~m}$ boulder on its surface. Larger rubble piles may also undergo ballistic sorting, however their increased cross sections and re-entrant rubble speeds can be expected to give rise to fracture and other complicating effects ${ }^{41}$.

We can estimate the asteroid size at which ballistic sorting will lose its influence by observing that the specific impact energy at which chondrite ${ }^{41}$ and similar rocks ${ }^{8}$ fracture is above $2 \cdot 10^{6} \mathrm{erg} / \mathrm{g}$. This implies that the impact speed that will fracture a projectile must exceed 2000

[^0]$\mathrm{cm} / \mathrm{sec}$, which is close to the escape velocities of the asteroids 253 Mathilde or 243 Ida. We therefore predict that sand seas will grow on rubble piles smaller than these asteroids, but that ballistic sorting will give way to other effects on larger asteroids. Larger asteroids may experience ballistic sorting during their evolutions, but they will also suffer significant fracture and rearrangement, reducing the effects of segregation over their histories.

Beyond small rubble-pile asteroids, it is enticing to note that even the large and consolidated asteroids Vesta ${ }^{42}$ and Eros ${ }^{43}$ possess flat "ponds" believed to consist of fine particles. Mechanisms such as electrostatic levitation and micrometeorite abrasion have been proposed for the origin of these ponds. Whatever their origin, all existing models appeal to a secondary mechanism such as seismic shaking to impose flatness ${ }^{44}$. As we have mentioned, both experiments and simulations of ballistic sorting lead to flat deposits, and so we speculate that this effect may play a minor role on large asteroids as well.

In conclusion, we have hypothesized that rubble-pile asteroids such as Itokawa may be size segregated simply because they have accumulated rubble by collisions that were predominantly, by number, by smaller particles. We have argued that the predominance of small particle collisions leads to a growth of pebble seas caused by inelasticity of collisions. We have also shown that this mechanism accounts for the formation of pebble seas in valleys and an absence of pebbles in highlands. We have performed experiments and simulations to assess each part of this hypothesis. Experimentally, we have found that isolated pebbles that strike a large object rebound to end up far away from the object (Fig. 1(d)), while collisions between multiple pebbles lead to growing aggregates (Fig's $1(\mathrm{e})-(\mathrm{f})$ ). We have further determined that this growth is well described by a cooperative "Hill" equation (Fig. 2). Computationally, we have evaluated distances traveled following an initial collision with either a large stone or a pebble sea, under gravities ranging from Earth's down to Itokawa's, and we have confirmed that pebbles consistently rebound much further from larger stones than from pebble seas. While it remains to be seen how broadly this phenomenon may be applied, we propose that the underlying mechanism is simple and general enough that it may play a significant role in asteroid geomorphology.

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