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Current noise from a magnetic moment in a helical edge

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We calculate the two-terminal current noise generated by a magnetic moment coupled to a helical edge of a two-dimensional topological insulator. When the system is symmetric with respect to in-plane spin rotation, the noise is dominated by the Nyquist component even in the presence of a voltage bias V . The corresponding noise spectrum $S(V, \omega)$ is determined by a modified fluctuation-dissipation theorem with the differential conductance $G(V, \omega)$ in place of the linear one. The differential noise $\partial S/\partial V$, commonly measured in experiments, is strongly dependent on frequency on a small scale $\tau_K^{-1} \ll T$ set by the Korringa relaxation rate of the local moment. This is in stark contrast with the case of conventional mesoscopic conductors where $\partial S/\partial V$ is frequency-independent and defined by the shot noise. In a helical edge, a violation of the spin-rotation symmetry leads to the shot noise, which becomes important only at a high bias. Uncharacteristically for a fermion system, this noise in the backscattered current is super-Poissonian.

Introduction. The edge conduction [1–7] in two-dimensional topological insulators [8–11] (2D TI) such as HgTe and InAs/GaSb still poses unresolved questions. The lack of topological protection [1–3, 12–15] of the modes in both materials and the edge conductance in InAs/GaSb structures having similar values in the trivial and topological phases [16, 17], are not understood. The former question has created a plethora of theoretical activity in trying to identify the relevant time-reversal symmetric backscattering mechanism [18–34]. Charge puddles, ubiquitous in narrow-gap semiconductors and capable of having a magnetic moment, are a plausible candidate to explain the phenomena [35, 36] and may have been observed in local measurements [37].

A natural continuation to the measurements of average current is to measure correlations in it, *i.e.*, noise. Theoretical studies of noise in 2D TIs have focused on point-contacts and more complicated geometries [38–43]. The simpler case of two-terminal noise of a long single edge has been measured only recently [44]. The observations were explained in Ref. [45] where the authors found the conventional noise of a diffusive conductor at temperatures exceeding the charging energy of a near-edge puddle.

At temperature T below the charging energy, the puddle can develop a spin which gives rise to a new source of noise. In this paper we identify the signatures in the noise caused by such a local magnetic moment. We calculate both equilibrium noise and shot noise. Due to a peculiar property of the local moment in a spin-orbit coupled system, the equilibrium noise is unconventional, and depends on the applied bias, see Fig. (1). The most notable feature of the shot noise (which becomes important only at bias substantially exceeding T) is that it gives rise to the Fano factor $F_{bs} > 1$ in the backscattered current. The super-Poissonian nature of the noise is unusual for fermions and stems from the kinetics of the local spin flips which control the electron backscattering. Throughout this paper we use units $\hbar = k_B = 1$.

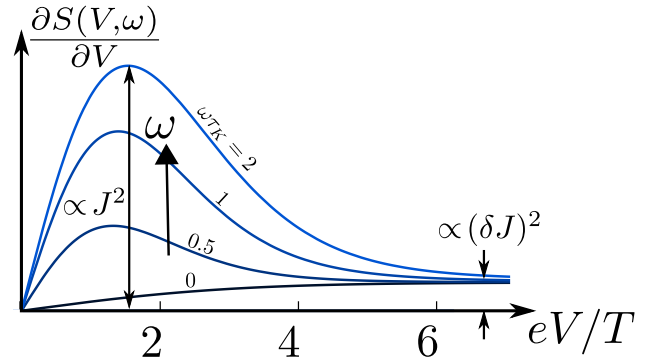


Figure 1. The low-frequency differential noise $\partial S/\partial V$, commonly measured by using bias modulation technique. At moderately low bias $eV \lesssim 4T$ and $\omega \gtrsim \tau_K^{-1}$, the main contribution to $\partial S/\partial V$ comes from the equilibrium noise, Eq. (1); the shot noise is smaller by a factor $(\delta J/J)^2$. At higher bias, the equilibrium noise contribution falls off exponentially with eV/T , and $\partial S/\partial V$ is dominated by the shot noise, Eq. (7). Here $\delta J = 0.5J$, and the spin-flip rate $\tau_K^{-1} = 0.025T$ at $V = 0$.

The coupling of the moment and the 2D TI helical edge can be described by a weak exchange interaction. The full exchange tensor consists of two parts, $J_{\text{full}} = J + \delta J$. The larger piece J is symmetric with respect to in-plane spin rotations [36] (hereafter we denote this symmetry as $U(1)_s$ [46]); it does not lead to any dissipation of dc current [21]. The exchange terms that break $U(1)_s$ are much smaller, $\delta J \ll J$, but still important as they give rise to dc dissipation.

The two parts of a generic exchange interaction contribute to the total current noise $S(V, \omega)$. Similarly, the noise can be divided into two pieces, $S = S_{\text{eq}} + S_{\text{non-eq}}$.

The larger $U(1)_s$ symmetric part J of the exchange contributes only to S_{eq} . (We call this component “equilibrium noise” since it satisfies a modified fluctuation-dissipation theorem.) Even though the exchange term J does not modify the dc conductance of the edge, it gives rise to spin flips of the local moment. These local

moment spin fluctuations are accompanied by backscattering events on the edge and therefore contribute to the equilibrium current noise. The spin flip rate $\tau_K^{-1} \propto J^2 T$ defines the characteristic frequency of $S_{\text{eq}}(V, \omega)$. The application of a bias affects the distribution function of electrons in helical edge and results in its net spin density, which in turn polarizes the local moment. That polarization affects the noise power, making it bias-dependent. At high bias $eV \gg T$, the “frozen” local moment does not fluctuate and as a result the J -dependent part of S_{eq} vanishes. The dependence on V and the sensitivity to low frequencies, $\omega \ll T$, make S_{eq} strikingly different from the conventional Johnson-Nyquist noise produced by electron elastic scattering [47]. Since S_{eq} depends on the bias voltage, it contributes to the differential noise $\partial S / \partial V$.

A common experimental technique to measure shot noise is the so-called bias modulation technique [48, 49] where the bias V is modulated slowly in time leading in effect to a measurement of $\partial S / \partial V$. In a conventional setup this eliminates the equilibrium (or thermal) noise, which is bias-independent. In contrast to this, in the case of a magnetic moment in a helical edge it is in fact the equilibrium noise S_{eq} that dominates finite-frequency differential noise. We find the differential noise at $\omega \ll T$,

$$\frac{\partial S(V, \omega)}{\partial V} \approx \frac{\partial S_{\text{eq}}(V, \omega)}{\partial V} = \frac{e^3}{2T\tau_K} \frac{(\omega\tau_K)^2}{1 + (\omega\tau_K)^2} \frac{\tanh \frac{eV}{2T}}{\cosh^2 \frac{eV}{2T}}. \quad (1)$$

The second factor demonstrates the above-mentioned sensitivity of S_{eq} to low frequencies $\omega \sim \tau_K^{-1}$, see also Fig. 1.

In the limit of high bias voltage, $eV \gg T$, the right-hand-side of Eq. (1) vanishes. The differential noise is then dominated by the non-equilibrium component $S_{\text{non-eq}}$, which arises from the combination of the $U(1)_s$ breaking interaction δJ and finite bias voltage. In the above limit, it becomes conventional shot noise $S_{\text{non-eq}} = eF_{bs}\langle\delta I\rangle$, where $\langle\delta I\rangle \propto \delta J^2 G_0 V$ is the backscattering current (we denote $G_0 = e^2/h$ the edge ideal conductance). The backscattering current arises from rare random backscatterings of one electron which happen with or without a local moment spin flip. In the former case, the subsequent backscattering event will happen fast, in time τ_K , and is correlated with the first one, see Fig. 2. This correlation results in super-Poissonian noise as demonstrated by the backscattering Fano factor $F_{bs} > 1$.

Next, we move to a more detailed discussion of noise caused by a local magnetic moment on a helical edge. We start from the equilibrium noise and derive Eq. (1).

Equilibrium noise. We start by considering the $U(1)_s$ symmetric system. The exchange Hamiltonian is then $H_{\text{ex}} = \frac{1}{2}J_{\perp}(S_-s_+ + S_+s_-) + J_z S_z s_z$. Here \mathbf{S} is the spin-operator of the localized moment, and $\mathbf{s} = \frac{1}{2} \sum_{\alpha\beta} \psi_{\alpha}^{\dagger} \boldsymbol{\sigma}_{\alpha\beta} \psi_{\beta}$ is the spin density of the helical

edge electrons. We denote $S_{\pm} = S_x \pm iS_y$. The Hamiltonian conserves the total spin $S_z^{\text{tot}} = S_z + \int dx s_z(x)$. In presence of an applied dc bias voltage V , the system is described by a generalized equilibrium Gibbs distribution where the bias couples to the conserved quantity S_z^{tot} . Because of spin-momentum locking, $\int dx s_z(x)$ is equal to the difference between the total number of right and left movers on the edge. The equilibrium density matrix

$$\varrho \sim e^{-\beta(H - eV S_z^{\text{tot}})} \quad (2)$$

thus correctly assigns a chemical potential difference eV between left and right movers. Having an equilibrium state, one may use a generalized fluctuation-dissipation theorem (FDT):

$$S_{\text{eq}}(V, \omega) = \text{Re}G(V, \omega) \omega \coth \frac{\omega}{2T}. \quad (3)$$

Here $G(V, \omega)$ is the differential ac conductance of the helical edge defined as the linear current response to a weak ac modulation $V_1 e^{-i\omega t}$ on top of dc bias V . To calculate the noise $S_{\text{eq}}(\omega)$, it is thus sufficient to evaluate $G(V, \omega)$, outlined next.

The current operator on the edge can be split into two parts [50, 51], $I(t) = I_{\text{in}}(t) - \delta I(t)$, where I_{in} is the current incoming from the leads. The backscattering current δI is given by the rate of change of the total number of right movers on the edge. By using the conservation of S_z^{tot} and the total number of electrons [52], we can write the operator as $\delta I(t) = e\partial_t S_z$ [21]. By employing the Kubo formula, this last form allows us to write the ac conductance as

$$\text{Re}G(V, \omega) = G_0 - e^2 \omega \chi''(V, \omega), \quad (4)$$

where the second term χ'' is the dissipative (imaginary) part of the local moment longitudinal susceptibility [53].

The local moment susceptibility can be calculated by using the Bloch equation for $\langle \mathbf{S} \rangle$ [50, 54]. We find for $\omega \ll T$,

$$\chi''(V, \omega) = \frac{1}{4T \cosh^2 \frac{eV}{2T}} \frac{\omega\tau_K}{1 + (\omega\tau_K)^2}. \quad (5)$$

Here $\tau_K^{-1} = \pi T (\rho J_{\perp})^2 \frac{eV}{2T} \coth \frac{eV}{2T}$ is the Korringa relaxation rate of the local moment, with ρ being the density of states of the edge per spin per length (the low-bias limit of τ_K^{-1} was already used in the Introduction).

Using Eq. (5) for χ'' and its relation to $\text{Re}G$, we find the ac conductance (valid at $\omega \ll T$)

$$\text{Re}G(V, \omega) = G_0 - \frac{e^2}{4T\tau_K} \frac{1}{\cosh^2 \frac{eV}{2T}} \frac{(\omega\tau_K)^2}{1 + (\omega\tau_K)^2}, \quad (6)$$

which generalizes the $V = 0$ result of Ref. [21] to arbitrary V . Substitution of Eq. (6) into the FDT result, Eq. (3), and subsequent differentiation yields Eq. (1).

At high bias voltage, $eV \gg T$, the noise contribution from a $U(1)_s$ symmetric part of exchange disappears, see Eqs. (6) and (3). This effect can be understood as follows. In the $U(1)_s$ symmetric case helical electron backscattering is always associated with the local moment spin flip. At large dc bias the moment is strongly polarized and has therefore exponentially small susceptibility to spin flips caused by the time-varying bias, see Eq. (5). This limitation does not concern $U(1)_s$ symmetry-breaking perturbations which, for instance, enable backscattering of helical electrons without a flip of the local moment. Such perturbations reduce the ideal zero bias conductance G_0 in Eq. (6) by an amount $\sim (\rho\delta J)^2$ [36, 55], without affecting Eq. (1). More importantly, the very same perturbation leads to the shot noise, $S_{\text{non-eq}} \propto (\delta J)^2 V$, dominant in $S(V, \omega)$ at high bias.

Non-equilibrium noise; shot noise and Fano factor. At finite bias V , the perturbation δJ drives the density matrix out of the equilibrium state Eq. (2). We focus on the limit of high bias voltage, $eV \gg T$, and evaluate the backscattered current noise within the perturbation theory in δJ . The operator of backscattered current is $\delta I(t) = e\partial_t S_z - e\partial_t S_z^{\text{tot}}$, which generalizes the earlier form $\delta I(t) = e\partial_t S_z$ to the case of broken $U(1)_s$ symmetry. The full symmetrized current noise is $S(t) = \text{Re} \langle I(t+t') : I(t') : \rangle$, where $:I(t) := I(t) - \langle I \rangle$ and $I = I_{in} - \delta I$.

At high bias, $eV \gg T$ the full noise is dominated by the shot noise, obtained from the δI - δI correlation function. The term $e\partial_t S_z$ in the operator δI does not contribute to the zero-frequency noise [50], and we find [56],

$$S(V, 0) \approx S_{\text{non-eq}}(V, 0) = e \sum_{n=1,2} n \langle \delta I^{(n)} \rangle. \quad (7)$$

Here $\langle \delta I^{(n)} \rangle$ is the contribution to backscattered current from the terms of the exchange interaction which change S_z^{tot} by $n\hbar$. The origin of the prefactor n is explained below (see also Fig. 2 which illustrates the terms with $n = 2$).

The resulting Fano factor, $F = S(V, 0)/e\langle I \rangle$, to lowest order in δJ is $F \sim (\rho\delta J)^2 \ll 1$ as expected for weak backscattering [47]. (Small Fano factors, $F \lesssim 0.26$, were found in the experiment [44] in HgTe and in the theoretical paper Ref. [45].) The backscattering Fano factor defined as $F_{bs} = S(V, 0)/e\langle \delta I \rangle$ lies between 1 and 2:

$$F_{bs} = 1 + \frac{|\delta J_{--}|^2}{|\delta J_{--}|^2 + \frac{1}{2} \sum_{i=x,y} [\delta J_{zi} + \delta J_{iz}]^2}. \quad (8)$$

Here $\delta J_{--} = \delta J_{xx} - \delta J_{yy} - i(\delta J_{xy} + \delta J_{yx})$ corresponds to the exchange component that changes S_z^{tot} by $2\hbar$ (while δJ_{zi} and δJ_{iz} change it by \hbar) [55].

It is tempting to interpret F_{bs} as the effective charge (in units of e) of the backscattered particles [57]. In our considered system this interpretation must be used with caution: we find a super-Poissonian value $F_{bs} > 1$, Eq. (8),

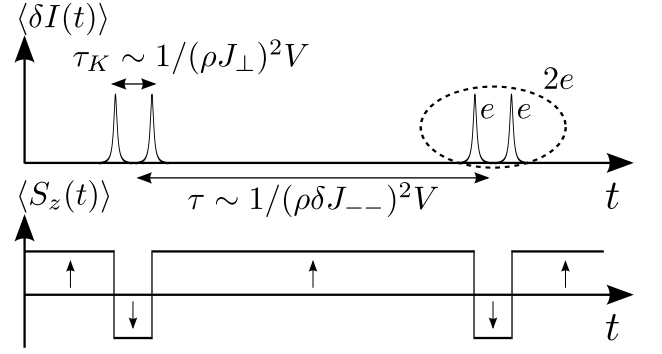


Figure 2. Super-Poissonian shot noise in the backscattered current δI . At large bias $eV \gg T$, the local spin spends most of its time in the quasiequilibrium state \uparrow , $\langle S_z \rangle = 1/2$. At random times, on average once per time $\tau \propto 1/J_{--}^2$, the exchange term $\delta J_{--} S_{-s_-} + h.c.$ backscatters one electron and flips the localized spin ($\uparrow \rightarrow \downarrow$ in figure). After a short time $\tau_K \ll \tau$, the local moment relaxes to its quasiequilibrium state ($\downarrow \rightarrow \uparrow$). In this process, another electron is scattered back. Since $\tau \gg \tau_K$, the time-averaged backscattered current is $2e/\tau$ and the effective backscattered charge is $2e$ when probed at frequencies $\omega \ll 1/\tau_K$.

even though the exchange Hamiltonian only backscatters a single electron at a time. The reason behind $F_{bs} > 1$ is correlated backscattering that employs the exchange terms $\delta J_{--} S_{-s_-} + h.c.$ and $\frac{1}{2} J_{\perp} S_{+s_+} + h.c.$ sequentially, as demonstrated in Fig. 2. The corresponding times $\tau \sim 1/(\rho\delta J_{--})^2 V$ and $\tau_K \sim 1/(\rho J_{\perp})^2 V$ are vastly different, $\tau_K \ll \tau$, because $\delta J/J \ll 1$. As the result, the spin-flip event caused by δJ_{--} which drives system out of the equilibrium, is quickly followed by a flip restoring the dominant spin direction, see the lower panel of Fig. 2. Each of the flips sends one electron back. Therefore, electrons are backscattered in pairs. The characteristic time τ_K separates the electrons in one pair, while τ is the time between pairs (see the upper panel of Fig 2). These paired events are uncorrelated with each other, forming a Poissonian process with the average backscattered current $2e/\tau$ and the corresponding Fano factor 2. Inclusion of the backscattering processes without a flip of the local moment, *e.g.*, due to term $S_z s_x$ in the exchange interaction, yields $1 < F_{bs} < 2$, Eq. (8).

The exchange interaction J of a local moment with itinerant electrons leads to the Kondo effect [58] which, in the absence of external perturbations, results in screening of the local spin at low temperatures $T \lesssim T_K$. The Kondo effect is a crossover rather than a phase transition occurring at some temperature. The screening builds up gradually upon reducing T towards T_K , and at high temperatures may be accounted for by means of the renormalization group (RG) technique. Within RG, the “bare” exchange constants are replaced by running ones, $\rho J(D) = 1/\ln(D/T_K)$ and $\rho\delta J(D) = \rho\delta J(D_0) \ln(D/T_K)/\ln(D_0/T_K)$ which are functions of

energy scale D . (The bare constants depend on the microscopic origin of the exchange interaction; the corresponding “bare” Hamiltonian is defined in the band of width $D_0 > D$.) As long as $D_0 \gtrsim \max(T, eV) \gtrsim T_K$, we may account for the Kondo effect in noise by replacing $J, \delta J \rightarrow J(D), \delta J(D)$ with $D = \max(T, eV)$. Going to lower T, eV requires a more careful consideration, discussed next.

Noise at low temperatures $T < T_K$. Let us begin our discussion from the $U(1)_s$ symmetric case, $\delta J = 0$, in which the noise, Eq. (3), is determined by the spin susceptibility $\chi''(V, \omega)$, Eq. (4). At low frequencies and temperature ($\omega, T \ll T_K$) the susceptibility is known from the Shiba relation [59] generalized to finite magnetic field [60], whose role here is played by the bias voltage. We find [50] $\chi''(V, \omega) = c_1 \frac{\omega}{T_K^2} f(eV/T_K)$. The differential noise at $\omega, T \ll T_K$ is then [61],

$$\frac{\partial S}{\partial V} = c_1 e^3 \frac{\omega^3}{T_K^3} \coth \frac{\omega}{2T} \left[-f' \left(\frac{eV}{T_K} \right) \right], \quad (9)$$

$$f'(x) = - \begin{cases} 2c_2 x, & x \ll 1, \\ 2[W^2 x^3 \ln^2 x]^{-1}, & x \gg 1. \end{cases} \quad (10)$$

The $\partial S/\partial V$ vs. V dependence is qualitatively similar to the one in Fig. (1) with the replacement $T \rightarrow T_K$ on the horizontal axis.

The equilibrium differential noise described above vanishes at $\omega = 0$, as well as in the limit of high bias $eV \gg T_K$; in these regimes the noise is dominated by the non-equilibrium component coming from exchange interaction terms breaking the $U(1)_s$ symmetry. In the high-bias limit the discussion of the previous section, including Eq. (7), remains valid. In the opposite regime, $eV \ll T_K$, we can use an effective local interaction Hamiltonian [22, 24, 62] to describe the breaking of $U(1)_s$ symmetry. The most important irrelevant perturbation backscatters one particle inelastically. The low-frequency shot noise, Eq. (7), is then given by the $n = 1$ term: at $T, eV \ll T_K$ we have $\langle \delta I^{(1)} \rangle \gg \langle \delta I^{(2)} \rangle$ [22]. As a result, the backscattering Fano factor crosses over from $F_{bs} > 1$, Eq. (8), to $F_{bs} = 1$ as V is decreased below T_K . This V -dependence of F_{bs} is not visible in Eq. (8) in the leading-order RG, since the corresponding running couplings δJ have the same V -dependence. On the contrary, the Fano factor F of the total current noise shows reduction from $F_\infty \sim [\rho \delta J(D_0)]^2$ to even smaller values $F(V) \sim F_\infty [\ln(eV/T_K)/\ln(D_0/T_K)]^2$ when the bias is reduced from $\sim D_0$ to $eV \gtrsim T_K$. Matching this RG result with the known [22] power-law-dependence in the low-bias regime, we find $F(V) \sim F_\infty [\ln(D_0/T_K)]^{-2} (eV/T_K)^4$ at $eV \lesssim T_K$.

Discussion. Measurement of noise provides ways to distinguish between different sources of backscattering on the helical edge. In particular, it may tell the difference between the effects of a local magnetic moment and

strong electron-electron interactions on the edge; both explanations are consistent with recent conductance measurements [7, 55]. The zero-frequency shot noise provides a clear difference between the two backscattering mechanisms. In both cases there is a crossover in the bias-voltage dependence of the Fano factor $F(V)$. In the case of strong electron-electron interactions, $F(V)$ is a monotonically-decreasing function with a fairly weak bias dependence: $1 - F \propto V^\alpha$ below some crossover bias value ($V \ll V^*$), and $F \propto V^{-\alpha}$ above the crossover, with $0 < \alpha \ll 1$. In the case of a local magnetic moment, the Fano factor F monotonically increases with V , exhibiting a highly asymmetric crossover around $eV \sim T_K$. Below the crossover scale, we have a strong power-law $F(V) \propto V^4$ (see end of the previous paragraph) while above it the V -dependence is weak, $F(V) \propto \ln^2(eV/T_K)$. (Weak electron-electron interaction may change the logarithmic function here to a weak power-law [55].) Moreover, the Fano factor remains small, $F \ll 1$, at any bias.

Another clear distinction between magnetic moments and other backscattering mechanisms is in the finite-frequency noise power $S(V, \omega)$. In the case of a magnetic moment, the differential noise $\partial S/\partial V$ at frequencies $\omega \sim \tau_K^{-1}$ is dominated by the bias-dependent equilibrium component, see Fig. (1). The characteristic time τ_K here is determined by the relaxation dynamics of the local moment. In the conventional case of a scatterer having no internal dynamics, $\partial S/\partial V$ is produced by the shot noise which is insensitive to frequencies all the way up to $\omega \sim eV$ [49]. In the case of magnetic moment, the shot noise contribution is much smaller than the equilibrium one, as it requires a perturbation breaking the $U(1)_s$ symmetry. For a spin-carrying charge puddle, it is generated by spin non-conserving interaction in it; one can estimate the ratio of the corresponding exchange constant to the one respecting the $U(1)_s$ symmetry as $\delta J/J \sim 1/g$ where $g \geq 1$ is the dimensionless conductance of the puddle [36]. (We took $g = 2$ in Fig. 1 to illustrate that even a fairly small value for g gives a sizable effect.)

Besides magnetic moments due to charge puddles, our theory is applicable to the case of magnetic doping, *e.g.*, Mn impurities in InAs. Such impurities have an out-of-plane magnetic easy axis which does not violate the $U(1)_s$ symmetry [63]. Therefore we expect the equilibrium noise to remain the most important noise component. Furthermore, the magnetic anisotropy energy is estimated to be small, $\mathcal{K} \approx 0.08$ K in InAs host [64]; noise-producing spin flips are thus expected to occur even at fairly low temperatures. To estimate the spin-flip rate τ_K^{-1} we can use the value $\rho J(D_0) = 0.03$ [55] which gives the frequency scale $\tau_K^{-1} \approx 80$ MHz at $T = 1$ K. This frequency is safely above $1/f$ noise; in measurements [44] of the shot noise in a HgTe heterostructure even lower frequencies, 10 – 20 MHz, were successfully used.

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