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## **Orbital-Wise Coordination Number for Predicting Adsorption Properties of Metal Nanocatalysts**

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We present the orbital-wise coordination number  $CN^{\alpha}$  ( $\alpha = s$  or d) as a reactivity descriptor for metal nanocatalysts. With the noble metal Au ( $5d^{10}6s^1$ ) as a specific case, the  $CN^s$  computed using the two-center *s*-electron hopping integrals to neighboring atoms provides an accurate and robust description of the trends in CO and O adsorption energies on extended surfaces terminated with different facets and nanoparticles of varying size and shape, outperforming existing bond-counting methods. Importantly, the  $CN^s$  has a solid physiochemical basis via a direct connection to the moment characteristics of projected density of states onto the *s*-orbital of a Au adsorption site. Furthermore, the  $CN^s$  shows promise as a viable descriptor for predicting adsorption properties of Au alloy nanoparticles with size-dependent lattice strains and coinage metal ligands.

Metal nanoparticles exhibit distinct physiochemical properties from their bulk counterparts. This phenomenon manifests itself in numerous applications such as chemical and biological sensing [1], targeted drug delivery [2], and heterogeneous catalysis [3]. Understanding effects of particle size, shape, and composition on the binding affinity of molecules to surface metal atoms is of key importance for rational design of functional materials with the utmost efficiency and durability. In recent years, many efforts have been made aiming to correlate the binding energies of an adsorbate on metal surfaces with the electronic or geometric factors of the adsorption site (termed *descriptor*) [4–11]. The moment characteristics of projected density of states onto valence orbitals of a transitionmetal atom, e.g., d-band center [4], have been successfully used as reactivity descriptors for planar transition-metal surfaces, while transferring the concept to nanometer-sized systems is problematic due to the discrete nature of electronic states and/or a formidable computational cost. Mpourmpakis et al. proposed a dual-descriptor model with the coordination number and curvature angle of surface metal atoms for capturing structural and reactivity properties of Au nanoclusters [9]. Recent work by Calle-Vallejo et al. introduced the generalized coordination number  $\overline{CN}$  as a single reactivity descriptor for Pt nanocatalysts that can be easily computed from the local atomic arrangement of an adsorption site [11–13]. Although both revised bond-counting methods beyond 1<sup>st</sup> nearest-neighbor atoms captures the general trend of adsorption properties of planar surfaces and shape-specific nanoparticles of pristine metals, their application to complex systems with varying lattice strains and metal ligands is out of reach due to the lack of an explicit consideration of interatomic interactions.

In this *Letter*, we propose the orbital-wise coordination number  $CN^{\alpha}$  ( $\alpha = s$  or d) as a reactivity descriptor for metal nanocatalysts. The  $CN^{\alpha}$  quantifies the degree of coordinative saturation of metal atoms and their inclination to form new bonds via the  $\alpha$ -orbital of an adsorption site. In surface chemistry, the coordinative saturation is an often-used concept for understanding adsorption site preferences [14], trends of surface reactivity [15], and scaling relations among adsorption energies of hydrogen-containing species [16]. For example, hydrogenating an electronegative adatom (A) that initially adsorbs at a hollow site of a metal surface by creating an A-H bond will decrease its binding strength to surface atoms and likely shift the adsorbate to a less coordinated bridge or atop site, obeying the bond-order conservation principle [14, 15, 17]. In reverse thinking, saturating a surface metal atom by an introduction of extra neighbors of the same type or by a replacement with more reactive metal atoms will decrease its chemical reactivity towards adsorbing species. Intuitively, the variation in adsorbate binding energies across non-uniform sites of a metal surface are strongly dependent on the local coordination environment. Nevertheless, a rigorous definition of the coordination-based reactivity descriptor that is rooted on a solid theoretical basis and ambiguously distinguishes between inequivalent sites of metal nanocatalysts remains elusive.

To unravel intrinsic reactivity trends of metal nanocatalysts, we use a series of freestanding Au nanoparticles with varying size (diameters from  $\sim 0.5$  nm to  $\sim 2$  nm) and shape (cuboctahedron, truncated octahedron, tetrahedron, and cube) as model systems. The surface of metal nanoparticles studied here is composed of the  $\{111\}$  and  $\{100\}$  terrace atoms, and the step and kink atoms at the intersection of neighboring facets [18]. We first probe the chemical reactivity of individual surface atoms of Au metal nanoparticles with molecularly adsorbed carbon monoxide (CO) in a straight up configuration at the atop site. All adsorption energy calculations were performed on the basis of density functional theory (DFT) using the generalized gradient approximation (GGA-PBE) [19] and the projector augmented-wave (PAW) method implemented in real-space GPAW [18, 20, 21]. Figure S1(a) shows that the  $\overline{CN}$  of inequivalent surface Au atoms of truncated octahedral nanoparticles linearly correlates with CO adsorption energies ( $R^2$ =0.90), similar to previous studies on Pt nanocatalysts [11]. Here, the  $\overline{CN}_i$  is computed as a sum of the weighted metal-metal coordination  $CN_i/CN_{\infty}$ , where the  $CN_i$  represents the regular coordination number of the atom j, the index j goes through the  $1^{st}$  nearest neighbors of the surface atom *i*, and the normalization factor  $CN_{\infty}$  is 12 and 8 for FCC/HCP and BCC metals, respectively [11, 12]. Nevertheless, the correlation coefficient  $(R^2)$  significantly degrades if different shapes of Au nanoparticles are also included in the regression ( $R^2=0.75$ ), see Fig. 1(a). Extending the  $\overline{CN}$  to fur-

TABLE I. The  $\alpha$ -electron hopping integral matrix  $t_{ij}^{\alpha\beta}$  between the two atoms aligned along the *z*-axis [24, 27].

| $t_{ij}^{\alpha\beta}$     | s          | $d_{xy}$   | $d_{yz}$ | $d_{xz}$ | $d_{x^2-y^2}$ | $d_{z^2}$  |
|----------------------------|------------|------------|----------|----------|---------------|------------|
| S                          | $ss\sigma$ | 0          | 0        | 0        | 0             | $sd\sigma$ |
| $d_{xy}$                   | 0          | $dd\delta$ | 0        | 0        | 0             | 0          |
| $d_{xy} \ d_{yz} \ d_{xz}$ | 0          | 0          | $dd\pi$  | 0        | 0             | 0          |
| $d_{xz}$                   | 0          | 0          | 0        | $dd\pi$  | 0             | 0          |
| $d_{x^2-y^2}$              | 0          | 0          | 0        | 0        | $dd\delta$    | 0          |
| $d_{z^2}$                  | $ds\sigma$ | 0          | 0        | 0        | 0             | $dd\sigma$ |

ther neighbors only slightly improves its correlation with CO binding energies, see Fig. S1(b).

To go beyond the simple bond-counting scheme and explicitly consider interatomic interactions in defining the local coordination, we are in search of pair-additive physical factors that quantify the influence of neighboring atoms on the chemical reactivity of an adsorption site. In light of the importance of the electronic structure of surface metal atoms in determining the nature and strength of chemical bonding, we resort to moment characteristics of projected density of states onto the transition or noble metal atom *i* with a given eigenspectrum  $\{\varepsilon_i^{\alpha}\}$  ( $\alpha$ =s or d) and aim to unearth their connection to the coordination environment. From the moment's theorem [22], the  $0^{th}$  and  $1^{st}$  moments  $(M_{0,i}^{\alpha} \text{ and } M_{1,i}^{\alpha})$  of the distribution  $\{\varepsilon_i^{\alpha}\}$  are the total number of atomic orbitals  $N_{\alpha}$  and the average band energy  $\varepsilon_{\alpha}$  (zero if it is set as the energy reference), respectively, thus giving no distinctive electronic structure information of surface atoms. Arguably, the  $2^{nd}$  moment  $M_{2i}^{\alpha}$  characterizing the width of the distribution  $\{\varepsilon_i^{\alpha}\}$ mainly governs the variation in the local chemical reactivity of an adsorption site perturbed by neighboring atoms. Within the Linear Combination of Atomic Orbital (LCAO) formalism [18, 23, 24], the second moment  $M_{2,i}^{\alpha}$  of projected density of states onto the  $\alpha$ -orbital at the site *i* is pair-additive with respect to the square of two-center  $\alpha$ -electron hopping integrals  $(t_{ii}^{\alpha\beta})^2$  from the  $\alpha$ -orbital at the atom *i* to all  $\beta$ -orbitals of the neighboring atom j within a cutoff radius  $r_c$ ,

$$M_{2,i}^{\alpha} = \sum_{j}^{r_{ij} < r_c} \sum_{\beta} (t_{ij}^{\alpha\beta})^2 = \sum_{j}^{r_{ij} < r_c} \sum_{\beta} \left| \left\langle \psi_i^{\alpha} \middle| \mathscr{H} \middle| \psi_j^{\beta} \right\rangle \right|^2, \quad (1)$$

where  $\psi_i^{\alpha}$  and  $\psi_j^{\beta}$  are the wave functions of the  $\alpha$  and  $\beta$ atomic orbitals at the site *i* and *j*, respectively,  $\mathscr{H}$  is the oneelectron Hückel Hamiltonian [25], and the summation *j* extends over all neighbors of the atom *i* within a cutoff radius  $r_c$ . The angular variation of hopping integrals between atomic orbitals as one atom is rotated around the other is quite complicated, and was tabulated by Slater and Koster [26]. Thanks to the invariance of object shape under translation and rotation, the  $2^{nd}$  moment of the distribution  $\{\varepsilon_i^{\alpha}\}$  can be directly computed using the  $\alpha$ -electron hopping integral matrix  $(t_{ij}^{\alpha\beta})$ where the two atoms are aligned along the *z*-axis [18, 24], see Table I. With the tight-binding approximation, the twocenter  $\alpha$ -electron hopping integrals depend on the orbital size/shape and internucleus distance, *i.e.*,  $t_{ij}^{ss} = \eta_{ss}\hbar^2/md_{ij}^2$ ,  $t_{ij}^{sd} = \eta_{sd}\hbar^2 r_{d,j}^{3/2}/md_{ij}^{7/2}$ , and  $t_{id}^{dd} = \eta_{dd}\hbar^2 r_{d,i}^{3/2} r_{d,j}^{3/2}/md_{ij}^5$ , where the symmetry-dependent coefficient  $\eta$  and the spatial extent of the *d*-orbital  $r_d$  of transition and noble metals are tabulated [27]. Rooted on the pair-additivity of the  $2^{nd}$  moment in Eq. 1, we introduce the orbital-wise coordination numbers  $CN^{\alpha}$  ( $\alpha$ =s or *d*) of the atom *i*, written as [18, 23]

$$CN_i^{\alpha} = \frac{M_{2,i}^{\alpha}}{\left(t_{nn}^{\alpha,\infty}\right)^2}, \text{ where } \alpha = s \text{ or } d$$
 (2)

where  $(t_{nn}^{\alpha,\infty})^2$  is a sum of the square of the  $\alpha$ -electron hopping integrals to relevant valence orbitals of a 1<sup>st</sup> nearest-neighbor atom in the optimized bulk [18].

With the minimal basis set in an LCAO description of the electronic structure for Au  $(5d^{10}6s^1)$ , we found that the  $CN^s$ of surface Au atoms of extended surfaces ({111}, {100}, {211}, and {532}) and nanoparticles (varying size and shape) computed using the two-center s-electron hopping integrals to neighboring s- and d-orbitals ( $r_c=5.5$  Å or equivalently up to  $2^{nd}$  nearest neighbors) shows a strong linear correlation with the CO adsorption energies ( $R^2=0.94$ ). As expected, if a surface Au atom becomes less (more) coordinated with its neighbors, *i.e.*, the CN<sup>s</sup> is smaller (larger), the CO adsorption would be stronger (weaker). The  $CN^d$ , obtained using the two-center d-electron hopping integrals to neighboring sand d-orbitals [18], and the simple summation of interatomic coupling matrix defined previously [6] still show a good linear correlation to CO adsorption energies because of the similar power-law dependence of s- and d-electron hopping integrals on the bond distance [27]. However, the *d*-states contribution to the surface chemical bonding is expected to be less important since the *d*-orbitals of a Au atom are fully occupied and the *d*-band center is further down the Fermi level, see Fig. S2a. Not surprisingly, the  $CN^d$  outperforms  $CN^s$  for transition metals with partially filled *d*-bands, such as Pt (to be discussed in a future publication). To simplify the argument, we will focus on the CN<sup>s</sup> in the following analysis for Au. Considering

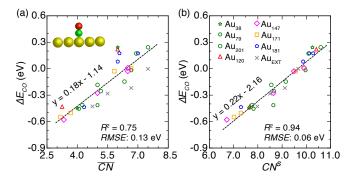


FIG. 1. (color) Adsorption energies of CO atop on Au nanoparticles and extended surfaces described by (a) the generalized coordination number  $\overline{CN}$  [11], and (b) the orbital-wise coordination number  $CN^s$ computed using the two-center *s*-electron hopping integrals. Linear regression lines (dashed) and related statistics are also given.

the simplicity of the descriptor, the observed linear relationship shown in Fig. 1(b) is striking. Compared with the purely geometry-based  $\overline{CN}$  [11, 12] as shown in Fig. 1(a), the model with the  $CN^s$  as a reactivity descriptor shows improved performance in terms of the correlation coefficient and statistical errors. If the bulk geometry of nanoparticles without relaxation is used for computing the  $CN^s$ , its correlation with CO adsorption energies decreases and becomes similar to that using the  $\overline{CN}$ . Therefore, we attributed the improvement of the new reactivity descriptor  $CN^{\alpha}$  over the  $\overline{CN}$  to the explicit consideration of lattice strains that are highly dependent on the size and shape of metal nanoparticles [28].

To unravel underlying electronic factors governing variations in the  $CN^s$  and thus further understand reactivity trends across surface Au atoms, we show in Fig. 2(a) the density of states projected onto the s-orbital of the terrace, step, and kink atoms of the truncated octahedral Au<sub>201</sub> nanoparticle. Clearly, as a surface Au atom becomes less coordinated with its neighbors, the width of the s-band measured by the  $2^{nd}$  moment  $M_{2i}^{s}$  decreases, which results in an upshift of the center of gravity of occupied s-states  $\varepsilon_s^*$ . Since the cohesion of noble metal atoms is mainly determined by the interaction of valence s-electrons with neighboring s- and d-electrons, the cohesive energies of surface Au atoms can be directly linked to the one-electron energy distribution of the s-band with the aid of the moment's theorem [22, 24] and to the orbital-wise coordination number  $CN^s$  using the bond-cutting model [29, 30]. The theoretical relation between the  $\varepsilon_s^*$  and the  $CN^s$  can be written as [18],

$$\boldsymbol{\varepsilon}_{s}^{*} = \boldsymbol{\varepsilon}_{s}^{*,\infty} + \frac{E_{coh}^{\infty}}{2\boldsymbol{\theta}_{s}} \left(\frac{CN^{s}}{CN_{\infty}^{s}} - 1\right), \tag{3}$$

where  $\mathcal{E}_s^{*,\infty}$  is the center of gravity of occupied *s*-states projected onto a bulk atom,  $\theta_s$  is the theoretical filling of the *s*-band (0.5 for Au), and  $E_{coh}^{\infty}$  (negative sign convention) is the cohesive energy of a bulk atom. Using DFT-calculated  $\mathcal{E}_s^{*,\infty}$  (-5.3 eV),  $E_{coh}^{\infty}$  (-3.19 eV) of a Au atom in the FCC bulk, and the  $CN_{\infty}^{s}$  (13.3), the theoretical line between the  $\mathcal{E}_s^{*}$  and the  $CN^{s}$  is presented in Fig. 2(b). A linear regression of the  $\mathcal{E}_s^{*}$  and  $CN^{s}$  is performed for inequivalent surface Au atoms of extended surfaces and nanoparticles, which shows an excellent agreement with the theoretical model. The observation shown in Fig. 2 sheds light on the electronic origin of the  $CN^{s}$  and provides a solid theoretical basis for using the orbital-wise coordination number  $CN^{\alpha}$  as a reactivity descriptor for metal nanocatalysts.

The question that still remains to be answered is why there is a linear correlation between the coordination numbers (e.g., CN,  $\overline{CN}$ , and  $CN^{\alpha}$ ) of a surface metal atom and its local chemical reactivity. This is often assumed based on intuition rather than rigorously proved. According to the bond-order conservation principle in chemisorption [14, 15, 17], the total bond order  $\chi$  of a surface metal atom *i*, which is coordinated with its nearest neighbors and a covalently bonded adsorbate *A* at an atop site, for example, is conserved and normalized to unity

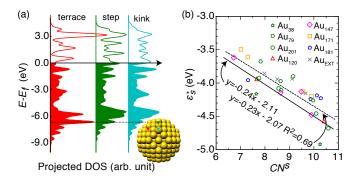


FIG. 2. (color) (a) Density of states projected onto the *s*-orbital of surface Au atoms on the terrace, step, and kink atoms of the Au<sub>201</sub> nanoparticle, and (b) the relationship between the  $\varepsilon_s^*$  and the  $CN^s$  for various surface atoms of extended Au surfaces and nanoparticles. The dashed line is the least-square fit; the sold line is the theoretical correlation, corresponding to the Eq. 3.

along all surface metal atoms,

$$\chi_{im} + \chi_{iA} = 1, \tag{4}$$

where  $\chi_{im}$  and  $\chi_{iA}$  are the bond order of the atom *i* with the surrounding metal atoms and with an adsorbate *A*, respectively. We argue that the  $\chi_{im}$  is not pair-additive with respect to the coordination number *CN*, but rather has a weaker dependence, following  $\chi_{im} \propto \sqrt{CN}$ . This is in analogy to the square-root bond-cutting model for describing variations in the cohesive energies of surface metal atoms  $(E_{coh}^i \propto \sqrt{CN_i})$  [31]. If the two-center interatomic potential of the adsorbate *A* with the metal atom *i* can be represented by a morse-type potential,  $\Delta E_A = -Q_0(2\chi_{iA} - \chi_{iA}^2)$ , where  $Q_0$  is the equilibrium bond energy of the adsorbate *A* with an isolated atom *i*, the adsorption energy  $\Delta E_A$ , to a first-order approximation without a consideration of the metal-metal and intra-adsorbate relaxations upon adsorption, is then linearly proportional to the  $\sqrt{CN}$ , or more generally to the  $\sqrt{CN}$  and  $\sqrt{CN^{\alpha}}$ .

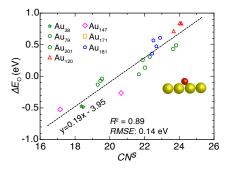


FIG. 3. (color) Description of adsorption energies of atomic O on hollow sites of Au nanoparticles by the orbital-wise coordination number  $CN^{s}$ . Related statistics of the linear regression is shown.

Thanks to the generality of the bond-order conservation principle discussed above, the  $CN^s$  of the hollow sites comprised by multiple atoms (*e.g.*, 3 atoms for 111-type hollow sites) should also reflect the trend of their intrinsic chemical

reactivity to probing species. As a straightforward extension, the  $CN^s$  of a hollow Au site can be computed by a sum of the square of normalized *s*-electron hopping integrals to all neighbors within the cutoff radius  $r_c$  while explicitly excluding the atom-atom hopping within the adsorption site. With this definition, the  $CN^s$  of a fcc, and hcp hollow site on Au{111} is 23.4 and 23.8, respectively, which are larger than the CN(15 and 16) due to the broken-bond strains and long-range interactions explicitly considered in Eq. 2. We use the atomic O as the probing species for the reactivity of hollow sites on Au nanocatalysts. In Fig. 3, we showed the correlation between the  $CN^s$  of hollow sites of Au metal nanoparticles with adsorption energies of the atomic O. Compared with the  $\overline{CN}$ (see Fig. S4), the  $CN^s$  exhibits improved linear relationships with the local chemical reactivity of hollow sites ( $R^2$ : 0.89).

The key advantage of the orbital-wise coordination number  $CN^{\alpha}$  compared to the CN and  $\overline{CN}$  as a reactivity descriptor is that it can be naturally extended to alloy systems. This extension is guaranteed in Eq. 2 since the electron hopping integrals are dependent not only on the geometric arrangement but also on the metal identity of an adsorption site. Given the linear relationships between the  $CN^s$  of a surface site *i* on pure Au nanocatalysts and the CO and O binding energies as shown in Fig. 1(b) and Fig. 3, respectively, we aim to predict the surface reactivity of Au alloy nanoparticles where the local coordination environment of Au surface atoms is fine-tuned by metal ligands. We chose Au core-shell alloy nanoparticles (Ag@Au and Cu@Au) as the test systems because of their superior activity for low-temperature CO oxidation [32, 33]. Geometric structures of a series of truncated octahedral nanoparticles with 38, 79, and 201 metal atoms were directly optimized with DFT and then were used for computing the  $CN^s$  of inequivalent surface Au sites. Figure 4 shows the parity plot between the model-predicted CO and O adsorption energies on coreshell alloy NPs (Ag@Au and Cu@Au) and self-consistent DFT calculations. The agreement between the model and the full DFT calculations (RMSE<0.2 eV) indicates that the  $CN^{\alpha}$ has the power to describe surface reactivity of both the pristine and alloy nanocatalysts within a consistent framework, and can be easily generalized to other materials.

To conclude, we propose the orbital-wise coordination number  $CN^{\alpha}$  of an adsorption site as a reactivity descriptor for metal nanocatalysts. Compared with the semi-empirical bond-counting methods, the new descriptor showed improved performance for describing the surface reactivity of metal nanoparticles with varying size, shape, and composition, attributed to its explicit consideration of lattice strains and metal ligands. Moreover, the  $CN^{\alpha}$  has a solid physical foundation via a direct connection to the electronic structure of an adsorption site, and thus provides an intuitive link to the complex nature of chemical bonding at surfaces. This study opens up the possibility of developing adsorbate-specific descriptors and a new way for unraveling trends of the surface reactivity of metal nanocatalysts with defects, impurities, alloy additions, supports, etc.

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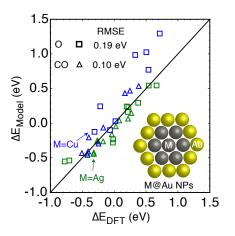


FIG. 4. (color) The prediction of CO and O adsorption energies on a series of M@Au (M=Cu or Ag) core-shell NPs (38, 79, and 201 atoms) using the  $CN^s$  as a reactivity descriptor and theoretical relationships developed from pure Au nanocatalysts.

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