Stochastic Gravitational-Wave Background due to Primordial Binary Black Hole Mergers

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Recent Advanced LIGO detections of binary black hole mergers have prompted multiple studies investigating the possibility that the heavy GW150914 binary system was of primordial origin, and hence could be evidence for dark matter in the form of black holes. We compute the stochastic background arising from the incoherent superposition of such primordial binary black hole systems in the universe and compare it to the similar background spectrum due to binary black hole systems of stellar origin. We investigate the possibility of detecting this background with future gravitational wave detectors, and conclude that constraining dark matter component in the form of black holes using stochastic gravitational-wave background measurements will be very challenging.

PACS numbers:

Introduction: Advanced LIGO detectors [1, 2] recently recorded the first two gravitational wave events: GW150914 [3] and GW151226 [4]. Both events had waveforms consistent with the mergers of binary black hole (BBH) systems, providing the first evidence that such binary systems exist and that they can merge within the lifetime of the universe. While the GW151226 system was characterized with black hole masses of $14^{+8}_{-4}M_\odot$ and $7^{+2}_{-2}M_\odot$ [4], which were consistent with the dynamical black hole mass estimates in x-ray binaries, GW150914 was characterized with rather heavy individual black hole masses: $36^{+5}_{-4}M_\odot$ and $29^{+4}_{-4}M_\odot$ [3, 5]. Such large black hole masses are still consistent with the (relatively uncertain) high mass stellar formation process, possibly in low-metallicity environments, with binaries formed in either the field or dynamical formation scenarios [6–15].

It has also been argued that black holes with large masses could be of primordial origin and contribute to the dark matter content of the universe [16–21]. Black hole masses below $20M_\odot$ have been excluded as a significant contributor to dark matter via microlensing surveys [22–24]. Similarly, masses above $100M_\odot$ would disrupt wide binaries [25–27]. While the mass range $20 - 100M_\odot$ has also been constrained by CMB observations [28, 29], these constraints are subject to significant uncertainties, as discussed in [20]. Furthermore, it has been argued [20] that under the assumption that dark matter is made up of $\sim 30M_\odot$ primordial black holes, the rate of mergers of primordial black hole binary systems would be consistent with the rate observed by Advanced LIGO detectors.

In this letter we investigate whether the origin of heavy BBH systems could be determined using the stochastic gravitational wave background measurements. We extend the study presented in [20] to compute the stochastic background as an incoherent superposition of gravitational waves emitted by all primordial black hole binaries in the universe. We investigate the possibility of detecting this background in the future runs of Advanced LIGO and of future gravitational wave detectors. Measuring this background could therefore be used to constrain the fraction of dark matter that is in the form of primordial black holes. We note that the stochastic background due to sub-solar primordial black-hole MACHO binaries has been investigated in [30] at frequencies below the sensitive band of terrestrial detectors.

Merger Rate per Halo: Following [20], we assume that dark matter consists of black holes of mass M, resulting in the number density of such black holes in a given dark matter halo. The primordial black holes interact with each other via the emission of gravitational waves and occasionally become bound and form binaries. The cross section for this process has been computed (see Eq. 1 of [20] and [31, 32]). Combining the black hole number density with the capture cross section and integrating over a dark matter halo yields the rate of primordial BBH mergers per halo [20]:

$$R_{\rm halo}(z) = \left(\frac{85\pi}{6\sqrt{2}}\right)^{2/7} \frac{2\pi}{3} \frac{G^2 M_{vir}^2 D(v)\lambda^2}{R_S^3 c g^2(C)} \left[1 - \frac{1}{(1+C)^3}\right].$$
(1)

Here, z is the redshift, G is the Newton constant, c is the speed of light, λ is the fraction of dark matter energy density in the form of black holes, and we make the following definitions and assumptions:

- We assume the Navarro-Frank-White (NFW) halo density model, which takes the form:

$$\rho(r) = \frac{4\rho_s}{\frac{r}{R_S}(1 + \frac{r}{R_S})^2}$$
(2)

where ρ_s and R_S are the characteristic density and radius of the halo profile, and r is the distance from the center of the halo.

- The virial radius R_{vir} is defined to be the radius at which the NFW profile reaches 200 times the critical density of the universe (which is redshift dependent). The virial mass M_{vir} is defined to be the mass inside the virial radius.

- The concentration parameter is defined as $C = R_{vir}/R_S$ and is extracted from the fits to numerical simulations [33, 34]. Our calculations below will require extrapolating the concentration models outside of the mass and redshift range considered in simulations. To avoid divergent behavior of C in such parts of the parameter space, we clip the value of C at 0.5 (minimum) and 1000 (maximum). We have verified that the impact of such clipping on the estimate of the stochastic gravitational-wave spectrum is negligibly small.

- We define the following function of concentration:

$$g(C) = \ln(1+C) - \frac{C}{1+C}. \tag{3}$$

- Finally, we average over black hole velocities, using the Maxwell-Boltzmann distribution for the dark matter halo cut-off at the virial velocity v_{vir}:

$$P(v, v_{dm}) = F_0 \left[\exp\left(-\frac{v^2}{v_{dm}^2}\right) - \exp\left(-\frac{v_{vir}^2}{v_{dm}^2}\right) \right] \tag{4}$$

$$v_{dm} = \sqrt{\frac{GM_{vir}g(C_m)}{R_{\max}g(C)}} \tag{5}$$

$$v_{dm} = \frac{v_{vir}}{\sqrt{2}} \sqrt{\frac{C}{C_m}\frac{g(C_m)}{g(C)}} \tag{6}$$

$$D(v) = \int_0^{v_{vir}} P(v, v_{dm}) \left(\frac{2v}{c}\right)^{3/7} dv. \tag{7}$$

Here, F_0 is the normalization of the Maxwell-Boltzmann distribution, $C_m = 2.1626$, and $R_{\max} = R_S C_m$.

With the above definitions, we can compute the primordial BBH merger rate per halo, as a function of the halo (virial) mass. This is shown in Figure 1 which compares two concentration models, Prada et al. [34] and Ludlow et al. [33], for several values of redshift. The two concentration models yield similar merger rates for the low halo masses, but the difference could be substantial at higher masses. As discussed below, the high end of the halo mass distribution contributes little to the gravitational wave spectrum, so this disagreement between concentration models (arising because of the finite box size of the underlying simulations) for high halo mass is not critical.

Merger Rate per Comoving Volume: To calculate the merger rate per comoving volume we use the halo mass function dn/dM_{vir}, i.e. the number density
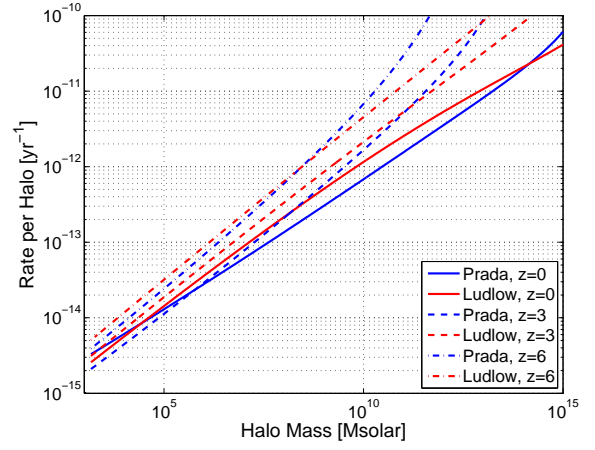

FIG. 1: Primordial BBH merger rate per halo as a function of the halo virial mass for the Prada et al. [34] and Ludlow et al. [33] concentration models, assuming $\lambda = 1$, and for several values of redshift. The local $z = 0$ curves are to be compared to the Figure 1 of [20].

of dark matter halos between M_{vir} and $M_{vir} + dM_{vir}$, which is a function of both halo mass and redshift and has units $\mathrm{Gpc}^{-3}M_\odot^{-1}$. The halo mass function is also extracted from simulations, and we will compare several halo mass function models from the hmf package [35], specifically Watson et al. [36], Press-Schechter [37], and Tinker et al. [38]. Several other models available within the hmf package were also tested to confirm that they yield results consistent with those presented here. We therefore define:

$$R(z) = \int R_{\mathrm{halo}}(z)\frac{dn}{dM_{vir}}dM_{vir}. \tag{8}$$

Hence, $R(z)$ is the merger rate per comoving volume as a function of redshift in units of $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$. Figure 2 shows the merger rate obtained using several halo mass functions and two concentration models, compared to the stellar BBH model defined as the fiducial model in [39]. Note that the primordial BBH merger rate has a very different redshift profile from the stellar one: while the stellar model follows the star formation rate (and hence peaks at redshifts 1-2), the primordial models weakly increase with redshift. Furthermore, different halo mass function models and uncertainty in the mass of the smallest halos, yield estimates of the merger rate differing by a factor of ~ 3.

Gravitational-Wave Energy Density: The energy density arising from an incoherent superposition of BBH systems throughout the universe has been calculated by multiple authors under the assumption that the BBH systems are of stellar origin [39–49]. We now extend this calculation to primordial BBH systems, following the formalism of [39]:
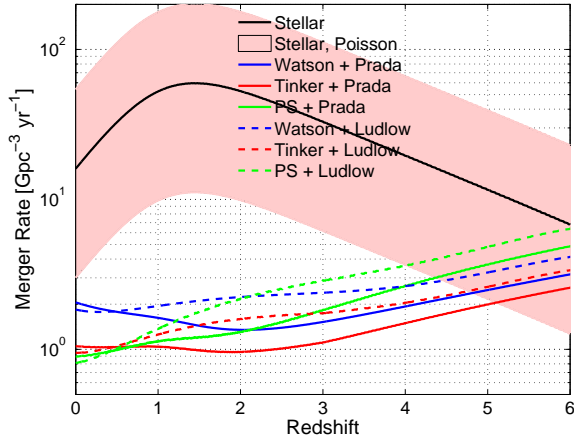
FIG. 2: Primordial BBH merger rate per comoving volume as a function of redshift, using the Prada et al. [34] and Ludlow et al. [33] concentration models, assuming $\lambda = 1$, and for several halo mass function models [36–38]. Note that the fiducial stellar BBH model is computed using black hole binaries which trace the cosmic star formation rate, and thus peaks around $z \sim 1 - 2$ [39]. The Poisson band around the fiducial stellar model represents the statistical uncertainty in the local rate of BBH mergers [39]. The primordial BBH merger rate in all considered models is weakly dependent on redshift and slightly increases with redshift.

$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c H_0} \int_0^{z_{\text{max}}} dz \frac{R(z)\frac{dE_{\text{GW}}(f_s,z)}{df}}{(1+z)E(\Omega_{\text{M}},\Omega_{\Lambda},z)}. \quad (9)$$

Here, H_0 is the present value of the Hubble parameter, ρ_c is the critical energy density of the universe, and $E(\Omega_{\text{M}},\Omega_{\Lambda},z) = \sqrt{\Omega_{\text{M}}(1+z)^3 + \Omega_{\Lambda}}$ captures the dependence of the comoving volume on redshift for the standard flat cosmology model, with $\Omega_{\text{M}} = 0.307$ and $\Omega_{\Lambda} = 1 - \Omega_{\text{M}}$. The energy spectrum emitted by a single binary, dE_{GW}/df, is evaluated at the source frequency $f_s = f(1 + z)$. We follow the formalism of [47, 50] with the modifications from [51] to calculate the contributions of the inspiral, merger, and ringdown parts of the BBH waveform. Specifically, the low-frequency part of the emitted spectrum is generated during the inspiral phase of the merger and leads to $\Omega_{\text{GW}}(f) \sim f^{2/3}$, while merger and ringdown phases lead to more complex spectral behavior.

Figure 3 shows the gravitational wave spectra for the stellar fiducial model of [39] and for the primordial model for several combinations of halo mass functions and concentration models. All spectra assume the chirp mass of $M_{\text{chirp}} = 30 M_\odot$. For the stellar model we make the same assumptions as in [39]: the local merger rate is taken to be $R_0 = 16 \, \text{Gpc}^{-3} \, \text{yr}^{-1}$, and we assume that the merger rate as a function of redshift follows the star formation rate at metallicity $Z < Z_\odot/2$, convolved with the distribution of time-delays (between BBH formation
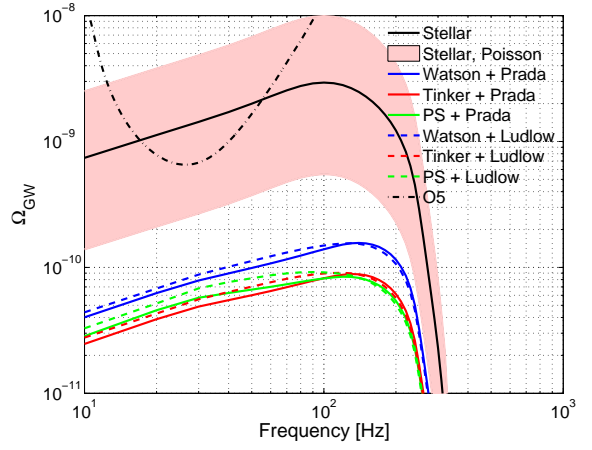

FIG. 3: Gravitational-wave energy density as a function of frequency for the same models of the halo mass function and concentration as considered in Figure 2 and assuming $\lambda = 1$. While different primordial models agree with each other within a factor of ~ 2, the fiducial stellar model is significantly louder. We note that the amplitude of the stellar fiducial model is currently uncertain due to the large errors on the local rate of BBH mergers, as denoted by the Poisson band [39]. Also shown is the projected final sensitivity of advanced detectors, denoted O5 [39].

and merger) that obeys $P(t) \sim t^{-1}$, with the minimum time delay of 50 Myr. The stellar model spectrum is characterized by a larger overall amplitude, due to both the larger local merger rate assumed in the calculation and because of the different redshift distribution of the BBH systems in the two models. It should be noted, however, that there is currently a large uncertainty in this amplitude due to the large uncertainty in the local BBH merger rate [52]. We also note that the different choices of the halo mass function or the concentration model yield variations in the primordial GW spectrum at the level of a factor of 2.

The gravitational-wave spectrum due to BBH mergers also depends on the assumed BBH chirp mass, as discussed in [39] for the stellar BBH systems and demonstrated for the primordial BBH systems in Figure 4. Specifically, for smaller black hole masses, the binaries would merge at a higher frequency, hence shifting the entire spectrum to higher frequencies. Furthermore, since the amplitude of the spectrum scales as $M_{\text{chirp}}^{5/3}$, lower chirp mass results in a lower spectral amplitude at a given frequency. In reality, the BBH chirp mass distribution is likely broader than the single value assumed in our calculations above, leading to a broader peak of the spectrum.

The simulations used to calibrate the mass function and concentration models are most reliable at low redshift. Thus the uncertainty in the halo mass function increases for $z > 4$. To examine how much this affects our results, we investigate how large a contribution high-redshift mergers make to the gravitational wave energy
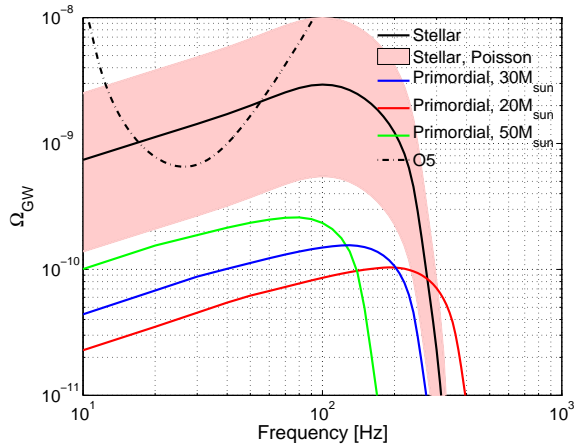
FIG. 4: Gravitational-wave energy density for the primordial BBH model is shown as a function of frequency for several values of the black hole mass, assuming the Ludlow et al. concentration model [33] and the Watson et al. model of the halo mass function [36]. Also shown is the projected final sensitivity of advanced detectors, denoted O5, as well as the fiducial stellar model and its Poisson error band [39].
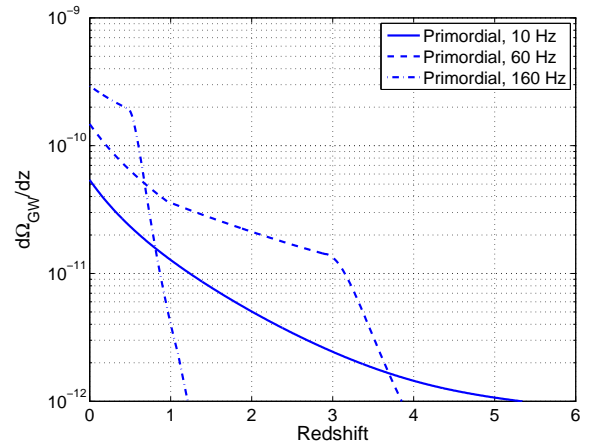

FIG. 5: Gravitational-wave energy density for the primordial BBH model is shown as a function of redshift for three selected frequencies: 10 Hz, 60 Hz, and 160 Hz. For all curves we assume the Ludlow et al. concentration model [33] and the Watson et al. model of the halo mass function [36]. The majority of the signal is generated at redshifts below ~ 3, implying that the uncertainties in the concentration and halo mass function models at high redshifts have a small impact on the gravitational wave energy density spectrum.

density spectrum. Figure 5 shows the integrand of Eq. 9, i.e. the energy density as a function of redshift for three frequencies: 10 Hz, 60 Hz, and 160 Hz. It is evident that most of the signal at the three frequencies (and hence in the sensitive frequency band of terrestrial gravitational-wave detectors) comes from redshifts smaller than ~ 3. Hence, lack of information about the concentration and halo mass function at high redshifts does not have a significant impact on the BBH stochastic background spectrum in the frequency band of terrestrial detectors and on our conclusions. Furthermore, as the signal is dominated by the smallest halos, which cannot form stars, baryonic effects are unlikely to change our results.

Detectability and Distinguishing Between the Stellar and Primordial Models: Having computed the amplitude and the spectral shape of the primordial BBH stochastic background, we now investigate whether this background could be detected by future gravitational wave detectors and whether it could be distinguished from other expected stochastic backgrounds. We consider two detector pairs with one year of exposure: two Advanced LIGO detectors at their final sensitivity [1, 2] and two co-located detectors at the Einstein Telescope ET-D sensitivity [53, 54]. We use the detection statistic defined by [55], following past searches for the stochastic gravitational-wave background [56, 57], for which the signal to noise ratio is defined as:

$$\text{SNR} = \frac{3H_0^2}{10\pi^2}\sqrt{2T}\left[\int_0^\infty df\, \frac{\gamma^2(f)\Omega_{\text{GW}}^2(f)}{f^6 P_1(f)P_1(f)}\right]^{1/2}, \quad (10)$$

where H_0 is the present value of the Hubble parameter, T is the observation time (set to 1 year in our case), $\gamma(f)$

is the overlap reduction function for the chosen detector pair, arising from the different locations and orientations of the detectors [55], and $P_1(f)$ and $P_2(f)$ are the strain power spectral densities of the two detectors.

Inserting the primordial BBH spectrum for $\Omega_{\text{GW}}(f)$, assuming the Watson et al. halo mass function [36], the Ludlow concentration model [33], and $\lambda = 1$ (i.e. that all of dark matter is in the form of black holes) yields SNR < 2 for the Advanced LIGO final detector sensitivity. Hence, the primordial BBH stochastic background cannot be measured by Advanced LIGO detectors. However, a pair of co-located Einstein Telescope detectors would have sufficient sensitivity to measure the primordial BBH background with SNR > 2 even if the fraction of dark matter in the form of black holes is as low as $\lambda \approx 0.2$.

Although the primordial BBH background should be strong enough to be measurable by the Einstein Telescope, distinguishing it from other types of stochastic background will be very challenging. As shown in Figures 3 and 4, both primordial and stellar BBH spectra have the same frequency dependence $f^{2/3}$ in the sensitive frequency band of the Einstein Telescope. While there is currently a large uncertainty in the stellar BBH merger rate, and hence in the amplitude of the corresponding stochastic background, even the lower end of the allowed range for the merger rate yields the stochastic background spectrum that is louder than the primordial one (see [39]). Hence, in order to detect the primordial component, the stellar component will have to be subtracted from the overall measurement, which would re-

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quire a very good estimate of the local merger rate of stellar BBH systems as well as a very good understanding of the formation and evolution of these systems with time, with statistical and systematic errors smaller than the amplitude of the primordial spectrum. This is challenging: as discussed in [39], the BBH formation mechanism (field vs dynamical), the dependence of black hole mass on metallicity at the time of black hole formation, and the distribution of the time delay between the formation and the merger of a BBH system, each lead to a factor of ~ 2 uncertainty in the stellar background spectral amplitude in the frequency band of terrestrial gravitational-wave detectors. In addition, both the stellar and the primordial BBH populations may be characterized by broad black hole mass distributions, implying that the corresponding spectra may be further modified relative to the spectra shown in Figures 3 and 4.

Conclusions: In this paper we have computed the stochastic gravitational wave spectrum arising from the incoherent superposition of many primordial BBH mergers. We have shown that the amplitude of this spectrum is significantly lower than that arising from the stellar BBH mergers, although there is currently a large uncertainty in the local merger rate for stellar BBH systems. Our calculation is not very sensitive to the uncertainties in the primordial BBH model, such as the halo mass function and the concentration model. Consequently, the stochastic GW background measurement with Advanced LIGO detectors is unlikely to detect this background. A similar measurement with future detectors, for example Einstein Telescope, may be sufficiently sensitive to detect the primordial BBH gravitational-wave background. However, distinguishing this background from other sources of stochastic background will be very challenging. Regardless, the stochastic gravitational-wave background measurements with future gravitational wave detectors may potentially yield new information on the dark matter problem.

[1] B. Abbott et al., *GW150914: The Advanced LIGO Detectors in the Era of First Discoveries*, Phys. Rev. Lett. 116, 131103 (2016).

[2] J. Aasi et al. *Advanced ligo*, Class. Quant. Grav. 32, 074001 (2015).

[3] B. Abbott et al., *Observation of Gravitational Waves from a Binary Black Hole Merger*, Phys. Rev. Lett. 116, 061102 (2016).

[4] B. Abbott et al., *GW151226: Observation of gravitational waves from a 22-solar-mass binary black hole coalescence*, Phys. Rev. Lett. 116, 241103 (2016).

[5] B. Abbott et al., *Properties of the binary black hole merger GW150914*, Phys. Rev. Lett. 116, 241102 (2016).

[6] K. Belczynski et al., *On The Maximum Mass of Stellar Black Holes*, Astrop. J. 714, 1217 (2010).

[7] M. Mapelli et al., *Dynamics of stellar black holes in young star clusters with different metallicities I. Implications for X-ray binaries*, MNRAS 429, 2298 (2013).

[8] M. Spera et al., *The mass spectrum of compact remnants from the PARSEC stellar evolution tracks*, MNRAS 451, 4086 (2015).

[9] I. Mandel and S.E. de Mink, *Merging binary black holes formed through chemically homogeneous evolution in short-period stellar binaries*, MNRAS 458, 2634 (2016).

[10] P. Marchant et al., *A new route towards merging massive black holes*, Astron. and Astrop. 588, A50 (2016).

[11] K. Belczynski et al., *Compact Binary Merger Rates: Comparison with LIGO/Virgo Upper Limits*, Astrophys. J. 819, 108 (2016).

[12] M. Morscher et al., *The Dynamical Evolution of Stellar Black Holes in Globular Clusters*, ApJ 800, 9 (2015).

[13] C.L. Rodriguez et al., *Binary Black Hole Mergers from Globular Clusters: Implications for Advanced LIGO*, Phys. Rev. Lett. 115, 051101 (2015).

[14] C.L. Rodriguez et al., *Erratum: Binary Black Hole Mergers from Globular Clusters: Implications for Advanced LIGO*, Phys. Rev. Lett. 116, 029901 (2016).

[15] B. Abbott et al., *Astrophysical Implications of the Binary Black-Hole Merger GW150914*, Astrop. J. Lett. 818, L22 (2016).

[16] J.R. Bond and B.J. Carr, *Gravitational waves from a population of binary black holes*, MNRAS 207, 585 (1984).

[17] S.G. Rubin, A.S. Sakharov, and M.Y. Khlopov, *The Formation of Primary Galactic Nuclei During Phase Transitions in the Early Universe*, J. Exp. Theor. Phys. 91, 921 (2001).

[18] M.Y. Khlopov, S.G. Rubin, and A.S. Sakharov, *Strong Primordial Inhomogeneities and Galaxy Formation*, Grav. Cosmol. 8, 57 (2002).

[19] M.Y. Khlopov, S.G. Rubin, and A.S. Sakharov, *Primordial Structure of Massive Black Hole Clusters*, Astropart. Phys. 23, 265 (2005).

[20] S. Bird et al., *Did LIGO Detect Dark Matter?*, Phys. Rev. Lett. 116, 201301 (2016).

[21] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, *Primordial black hole scenario for the gravitational wave event GW150914*, arXiv:1603.08338 (2016).

[22] R. A. Allsman et al. [Macho Collaboration], Astrophys. J. 550, L169 (2001).

[23] L. Wyrzykowski et al., Mon. Not. Roy. Astron. Soc. 416, 2949 (2011).

[24] P. Tisserand et al. [EROS-2 Collaboration], Astron. Astrophys. 469, 387 (2007).

[25] M. A. Monroy-Rodriguez and C. Allen, Astrophys. J. 790, 159 (2014).

[26] J. Yoo, J. Chaname and A. Gould, Astrophys. J. 601, 311 (2004).

[27] D. P. Quinn et al., Mon. Not. Roy. Astron. Soc. 396, L11 (2009).

[28] M. Ricotti, J. P. Ostriker and K. J. Mack, *Effect of Primordial Black Holes on the Cosmic Microwave Back-*

ground and Cosmological Parameter Estimates, Astrophys. J. 680, 829 (2008).

[29] M. Ricotti, *Bondi accretion in the early universe* Astrophys. J. 662, 53 (2007).

[30] K. Ioka, T. Tanaka, and T. Nakamura, *Low frequency gravitational waves from black hole MACHO binaries*, Phys. Rev. D 60, 083512 (1999).

[31] G. D. Quinlan and S. L. Shapiro, Astrophys. J. 343, 725 (1989).

[32] H. Mouri and Y. Taniguchi, Astrophys. J. 566, L17 (2002).

[33] A.D. Ludlow et al., *The Mass-Concentration-Redshit Relation of Cold and Warm Dark Matter Halos*, arXiv:1601.02624.

[34] F. Prada et al., *Halo Concentrations in the Standard ΛCDM Cosmology*, MNRAS **423**, 3018 (2011).

[35] S.G. Murray et al., *HMFcalc: An Online Tool for Calculating Dark Matter Halo Mass Functions*, arXiv:1306.6721. http://hmf.icrar.org/hmf_finder/form/create/

[36] W.A. Watson et al., *The halo mass function through the cosmic ages*, MNRAS **433**, 1230W (2013).

[37] W.H. Press and P. Schechter, *Formation of galaxies and clusters of galaxies by self-similar gravitational condensation*, Astrop. J. 187, 425 (1974).

[38] J.L. Tinker et al., *The Large Scale Bias of Dark Matter Halos: Numerical Calibration and Model Tests*, ApJ 724, 878 (2010).

[39] B. Abbott et al., *GW150914: Implications for the stochastic gravitational wave background from binary black holes*, Phys. Rev. Lett. 116, 131102 (2016).

[40] E.S. Phinney, *The Rate of Neutron Star Binary Mergers in the Universe: Minimal Predictions for Gravity Wave Detectors*, Astrop. J. **380**, L17 (1991).

[41] D.I. Kosenko and K.A. Postnov, *On the gravitational wave noise from unresolved extragalactic binaries*, Astron. & Astrop. **336**, 786 (1998).

[42] R. Schneider et al., *Low-frequency gravitational waves from cosmological compact binaries*, Mon. Not. Roy. Astr. Soc. **324**, 797 (2001).

[43] T. Regimbau and J.A. de Freitas Pacheco, *Stochastic Background from Coalescences of Neutron Star-Neutron Star Binaries*, Astrop. J. **642**, 455 (2006).

[44] T. Regimbau and V. Mandic, *Astrophysical sources of stochastic gravitational-wave background*, Class. Quantum Grav. **25**, 184018 (2008) ; arXiv:0806.2794 (revised version)

[45] T. Regimbau, *The astrophysical gravitational wave stochastic background* , Res. Astr. Astrop. **11**, 369 (2011); arXiv:1101.2762 (revised version)

[46] P.A. Rosado, *Gravitational wave background from binary systems*, Phys. Rev. D **84**, 084004 (2011); arXiv:1106.5795.

[47] X.-J. Zhu et al, *Stochastic Gravitational Wave Background from Coalescing Binary Black Holes*, Astrophys. J. **739**, 86 (2011); arXiv:1104.3565 (2011).

[48] S. Marassi et al., *Imprint of the merger and ring-down on the gravitational wave background from black hole binaries coalescence*, Phys. Rev. D **84**, 124037 (2011).

[49] C. Wu, V. Mandic and T. Regimbau, *Accessibility of the Gravitational-Wave Background due to Binary Coalescences to Second and Third Generation Gravitational-Wave Detectors*, Phys. Rev. D. **85**, 104024 (2012).

[50] P. Ajith et al., *Template bank for gravitational waveforms from coalescing binary black holes: Nonspinning binaries*, Phys. Rev. D 77, 104017 (2008).

[51] P. Ajith et al.,*Inspiral-Merger-Ringdown Waveforms for Black-Hole Binaries with Nonprecessing Spins*, Phys. Rev. Lett. 106, 241101 (2011).

[52] B. Abbott et al., *The Rate of Binary Black Hole Mergers Inferred from Advanced LIGO Observations Surrounding GW150914*, arXiv:1602.03842 (2016).

[53] Einstein Telescope: http://www.et-gw.eu/.

[54] M. Punturo et al, *The third generation of gravitational wave observatories and their science reach*, Class. Quant. Grav. **27**, 084007 (2010).

[55] B. Allen and J. Romano, *Detecting a stochastic background of gravitational radiation: Signal processing strategies and sensitivities*, Phys. Rev. D **59**, 102001 (1999); gr-qc/9710117.

[56] B. Abbott et al., *An Upper Limit on the Amplitude of Stochastic Gravitational-Wave Background of Cosmological Origin*, Nature **460**, 990 (2009).

[57] J. Aasi et al., *Improved Upper Limits on the Stochastic Gravitational-Wave Background from 2009-2010 LIGO and Virgo Data*, Phys. Rev. Lett. 113, 231101 (2014).