# Pairing Phase Transitions of Matter under Rotation

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# Pairing Phase Transitions of Matter under Rotation

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The phases and properties of matter under global rotation have attracted much interest recently. In this paper we investigate the pairing phenomena in a system of fermions under the presence of rotation. We find that there is a generic suppression effect on pairing states with zero angular momentum. We demonstrate this effect with the chiral condensation and the color superconductivity in hot dense QCD matter as explicit examples. In the case of chiral condensation, a new phase diagram in the temperature-rotation parameter space is found, with a nontrivial critical point.

*Introduction.*—The phases and properties of matter can become highly nontrivial under rotation, and have attracted a lot of interest recently. Such studies bear particular relevance for the strongly interacting matter of Quantum Chromodynamics (QCD). For example, astrophysical objects like neutron stars, made of dense QCD matter, can be rapidly spinning [1, 2]. In relativistic heavy ion collision experiments, the typical collision events are off-central and the created QCD matter will carry a nonzero angular momentum on the order of $10^4 \sim 10^5\hbar$ with local angular velocity in the range $0.01 \sim 0.1$GeV [3–8]. There has also been impressive progress to study the rotating QCD matter using lattice gauge theory simulations [9].

In rotating matter, many interesting transport phenomena could occur. For example, fluid rotation can induce anomalous transport processes in a system of chiral fermions, such as the chiral vortical effect [10–12] and chiral vortical wave [13]. These can lead to measurable experimental signals (see e.g. reviews in [14, 15]). In such anomalous transport, it is found that the fluid rotation plays a very analogous role to a magnetic field. Indeed there appears to be interesting similarity between the chiral vortical effect and the chiral magnetic effect [10, 16], as well as between the chiral vortical wave and the chiral magnetic wave [17, 18].

Apart from transport properties, it is of significant interest to explore the effects of rotation on the phase transitions of matter in both relativistic and non-relativistic cases. It is well known that a magnetic field can bring interesting effects on the thermodynamics and phase diagram of e.g. QCD matter [19–24], such as the magnetic catalysis and inverse catalysis (see reviews in [25, 26]) on the chiral condensation. Given the close analogy between rotation and magnetic field, it is tempting to ask how the rotation could influence phase transitions. In this paper, we investigate the pairing phenomena in a system of fermions under rotation. We find a generic suppression effect on pairing states with zero angular momentum, which will be demonstrated with the concrete examples of chiral condensation and color superconductivity in hot dense QCD matter.

*Rotational Suppression Effect on Scalar Pairing States.*— Let us first explain, in an intuitive way, the generic rotational suppression effect on scalar pairing states. We consider a general system of spin-$\frac{1}{2}$ fermions. They could be e.g. the dense quark or nucleon matter in compact stars [27–29] or cold atomic gases [30–32], or the electrons/holes in solid state systems, or liquid helium-3, etc. The pairing phenomenon between fermions under suitable conditions encompasses a wide range of systems. Examples include e.g. electron-electron pairing in superconductors, atom-atom pairing in helium-3 or cold fermi gases, nucleon-nucleon pairing in large nuclei or dense nuclear matter, quark-anti-quark pairing in the chiral condensate of QCD, or quark-quark pairing in color superconductivity, etc. We focus on *the scalar pairing states*, i.e. states with zero total angular momentum. For a pair of spin-$\frac{1}{2}$ fermions, there are different ways of forming a spin-0 pairing state: either, the pair could have both nonzero orbital angular momentum $L$ and nonzero total spin $S$, with $L$ and $S$ being opposite thus resulting in total $J = 0$; or the pair could have zero orbital angular momentum, and have opposite individual spin configurations for the two fermions.

As we will show below, for such a system under rotation, there is a generic rotational suppression effect on the scalar pairing states. Intuitively this can be understood as follows. The global rotation, implying a nonzero macroscopic angular momentum of the whole system, will induce a *rotational polarization effect* which tends to "force" all microscopic angular momentum to be aligned with the global angular momentum. So for a pair of fermions, their relative orbital angular momentum $L$ as well as their individual spins would prefer to be parallel to the global angular momentum rather than to arrange themselves into a scalar state with zero angular momentum. This therefore leads to a generic suppression effect on the scalar pairing states. It also implies that pairing states with nonzero angular momentum could become favorable. In the following, we quantitatively demonstrate this effect with two nontrivial examples in the QCD mat-

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ter: the chiral condensate (of quark-anti-quark pairing states with $L = S = 1$ but $J = 0$) and the diquark condensate (of quark-quark pairing state with $L = S = 0$).

*Description in Rotating Frame.*— Let us consider a system of spinor particles under slow rotation with a constant angular velocity $\vec{\omega}$ along a certain fixed axis. This system can be equivalently described as a system at rest in a *rotating reference frame*, see e.g. discussions in [9, 30]. We denote space-time as $(t, \vec{x})$ with flat Minkowski metric $\eta_{\mu\nu} = Diag(1, -1, -1, -1)$. The local velocity of this rotating frame (with respect to the non-rotating frame) is given by $\vec{v} = \vec{\omega} \times \vec{x}$. The space-time metric of the rotating frame thus becomes a curved one:

$$g_{\mu\nu} = \begin{pmatrix} 1 - \vec{v}^2 & -v_1 & -v_2 & -v_3 \\ -v_1 & -1 & 0 & 0 \\ -v_2 & 0 & -1 & 0 \\ -v_3 & 0 & 0 & -1 \end{pmatrix} \quad (1)$$

In such description, the free Dirac Lagrangian for spinor gets modified to be:

$$\mathcal{L} = \bar{\psi} \left[ i\bar{\gamma}^{\mu}(\partial_\mu + \Gamma_\mu) - m \right] \psi \quad (2)$$

where $m$ is the fermion mass. The $\bar{\gamma}^{\mu} = e_a^{\ \mu}\gamma^a$ with $e_a^{\ \mu}$ the tetrads for spinors and $\gamma^a$ the usual Dirac $\gamma$ matrices. The spinor connection is given by $\Gamma_\mu = \frac{1}{4} \times \frac{1}{2}[\gamma^a, \gamma^b]\,\Gamma_{ab\mu}$ where $\Gamma_{ab\mu} = \eta_{ac}(e^c_{\ \sigma} G^{\sigma}_{\ \mu\nu} e_b^{\ \nu} - e_b^{\ \nu}\partial_\mu e^c_{\ \nu})$ and $G^{\sigma}_{\ \mu\nu}$ is the affine connection determined by $g^{\mu\nu}$. For the tetrads we choose $e^a_{\ \mu} = \delta^a_{\ \mu} + \delta^a_{\ i}\delta^0_{\ \mu}\, v_i$ and $e_a^{\ \mu} = \delta_a^{\ \mu} - \delta_a^{\ 0}\delta_i^{\ \mu}\, v_i$.

We next consider the limit of slow rotation, i.e. with $\omega$ small compared with $1/R$ where $R$ is the system's transverse size [9, 30], and expand the Lagrangian up to the order of $\hat{O}(\omega)$. After some lengthy but straightforward calculations, one arrives at the following result:

$$\mathcal{L} = \psi^{\dagger} \left[ i\partial_0 + i\gamma^0 \vec{\gamma} \cdot \vec{\partial} + (\vec{\omega} \times \vec{x}) \cdot (-i\vec{\partial}) + \vec{\omega} \cdot \vec{S}_{4\times4} \right] \psi \quad (3)$$

where $\vec{S}_{4\times4} = \frac{1}{2} \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$ is the spin operator with $\vec{\sigma}$ the Pauli matrices. The last two terms in the above bracket may be interpreted as effective polarization term $\vec{\omega} \cdot \vec{J}$, with total angular momentum $\vec{J}$ consisting of an orbital term and a spin term. The rotational velocity $\vec{\omega}$ serves as an effective "chemical potential" for angular momentum.

The next step is to find the "natural" eigenstates in this rotating frame, in parallel to the plane-wave spinor eigenstates in normal frame [33–35]. The corresponding Hamiltonian in momentum space reads:

$$\hat{H} = \gamma^0(\vec{\gamma} \cdot \vec{p} + m) - \vec{\omega} \cdot (\vec{x} \times \vec{p} + \vec{S}_{4\times4}) = \hat{H}_0 - \vec{\omega} \cdot \hat{\vec{J}} \quad (4)$$

We use the cylindrical spatial coordinates $(r, \theta, z)$ with $\vec{\omega} = \omega\hat{z}$ and with $r, \theta$ being transverse radial position and azimuthal angle. The complete set of commutating operators consists of $\hat{H}$, $\hat{p}_z$, $\hat{\vec{p}}_t^{\,2}$, $\hat{J}_z$, and $\hat{h}_t \equiv \gamma^5\gamma^3 \vec{p}_t \cdot \vec{S}_{4\times4}$[36]. The last one is a sort of reduced helicity operator on transverse plane. One can therefore label the eigenstates of Hamiltonian by a set of corresponding eigenvalues: energy $E$, z-momentum $k_z$, transverse momentum magnitude $k_t$, z-angular-momentum quantum number $n = 0, \pm1, ...$, and "transverse helicity" $s = \pm$. The four solutions of spinor eigenstates are given by:

$$u_{k_z,k_t,n,s} = \sqrt{\frac{E_k + m}{4E_k}}\, e^{ik_z z} e^{in\theta} \begin{pmatrix} J_n(k_t r) \\ s\, e^{i\theta} J_{n+1}(k_t r) \\ \frac{k_z - is\,k_t}{E_k + m} J_n(k_t r) \\ \frac{-s\,k_z + ik_t}{E_k + m} e^{i\theta} J_{n+1}(k_t r) \end{pmatrix} \quad (5)$$

$$v_{k_z,k_t,n,s} = \sqrt{\frac{E_k + m}{4E_k}}\, e^{-ik_z z} e^{in\theta} \begin{pmatrix} \frac{k_z - is\,k_t}{E_k + m} J_n(k_t r) \\ \frac{s\,k_z - ik_t}{E_k + m} e^{i\theta} J_{n+1}(k_t r) \\ J_n(k_t r) \\ -s\, e^{i\theta} J_{n+1}(k_t r) \end{pmatrix} \quad (6)$$

where $E_k \equiv \sqrt{k_z^2 + k_t^2 + m^2}$ and $J_n(x)$ are $n$-th Bessel functions of the first kind. The energy eigenvalues are $E = \pm E_k - (n + 1/2)\omega$ with the plus (minus) for $u$ ($v$) spinor states respectively. The last term, i.e. $-(n + 1/2)\omega$, is the "rotational polarization energy". These results are the counterpart in rotating frame of the usual plane wave spinor states in non-rotating frame. With these states as basis one can then compute various quantities of interest using the standard method.

Finally we introduce an effective interaction that takes the generic form of four-fermion contact vertex:

$$\mathcal{L}_{I_{eff}} = G(\bar{\psi}\psi)^2 + G_d(i\psi^T C\gamma^5\psi)(i\psi^{\dagger} C\gamma^5\psi^*) \quad (7)$$

The first term is a fermion-anti-fermion scalar-channel coupling while the second term is a di-fermion scalar-channel coupling, with $G$ and $G_d$ the corresponding coupling constants. The above relativistic form of effective interaction is the Nambu-Jona-Lasinio (NJL) model. It shall be emphasized that essentially the same physics is applicable to many other fermion systems (such as pairing in cold fermionic gases and conventional superconductor, etc). For specific application to chiral condensation and color superconductivity in QCD matter, the pertinent color/flavor indices and structures can be easily added to the above interaction (see e.g. [37]).

*Chiral Condensation in Rotating Matter.*— We first consider the chiral condensation which is a fermion-anti-fermion pairing phenomenon. For this pairing state, the spatial angular momentum (for the relative orbital motion) $L = 1$ while the spin $S = 1$, with total angular momentum $J = 0$ for the pair. Following standard mean-field method, one introduces the expectation value $\langle\bar{\psi}\psi\rangle$ that leads to a mean-field mass gap $M = m - 2G\langle\bar{\psi}\psi\rangle$. Note that due to rotation, the system is no longer homogeneous and the $M$ as well as $\langle\bar{\psi}\psi\rangle$ become dependent on $r$ by virtue of symmetry. Using mean-field propagator one can obtain the grand potential as:

$$\Omega = \int d^3\vec{r}\Big\{\frac{(M-m)^2}{4G} - \frac{N_f N_c}{16\pi^2}\sum_n \int dk_t^2 \int dk_z \times [J_n(k_t r)^2 + J_{n+1}(k_t r)^2] \times T\Big[\ln\left(1+e^{(\epsilon_n-\mu)/T}\right) + \ln\left(1+e^{-(\epsilon_n-\mu)/T}\right) + \ln\left(1+e^{(\epsilon_n+\mu)/T}\right) + \ln\left(1+e^{-(\epsilon_n+\mu)/T}\right)\Big]\Big\} \quad (8)$$

In the above the mean-field quasiparticle dispersion $\epsilon_n$ is given by $\epsilon_n = \sqrt{k_z^2 + k_t^2 + M^2} - (n+\frac{1}{2})\omega$. The mean-field chiral condensate (or equivalently the mass gap $M$) at given values of temperature $T$, chemical potential $\mu$ and rotation $\omega$, can then be determined from the usual gap equation: $\frac{\delta\Omega}{\delta M(r)} = 0$ and $\frac{\delta^2\Omega}{\delta M(r)^2} > 0$. We will numerically solve the gap equation for the case of $N_f = 2$ and $N_c = 3$ and present the results below. For the parameters $G$, $G_d$ and a cutoff scale $\Lambda$ of this model, we choose the standard values: $G = 4.93\text{GeV}^{-2}$, $G_d = (3/4)G$, $m = 0.005$GeVand $\Lambda = 0.65$GeV (see e.g. [37]).

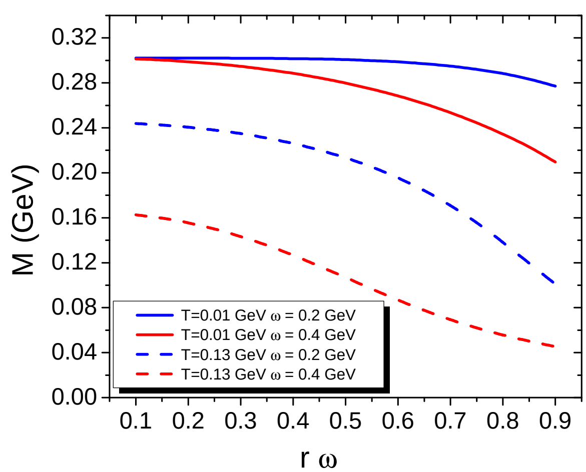


FIG. 1: The dependence of mean-field mass gap $M$ on radial coordinate $r\omega$ for several fixed values of $\omega$ and $T$.

Let us focus on the zero density case (i.e. $\mu = 0$) and study how the mass gap changes with various parameters. For the transverse radial coordinate $r$, we have found that the mass gap $M$ smoothly decreases with $r$, see Fig. 1. This dependence could be understood as follows: the orbital angular momentum for the same $k_t$ scale becomes larger at larger $r$ thus experiencing more suppression effect. we next show results for a particular value of $r$ for simplicity. In Fig. 2 we show $M$ (at $r = 0.1\text{GeV}^{-1}$) as a function of $\omega$ for several value of $T$. At all temperature, $M$ decreases with increasing $\omega$: this clearly confirms the rotational suppression effect on the chiral condensate. At low temperature the chiral condensate experiences a first-order transition when $\omega$ exceeds a critical value $\omega_c$, while at high temperature the chiral condensate vanishes with increasing $\omega$ via a smooth crossover. The $\omega_c$ decreases with increasing temperature. From Fig. 2 one could also infer the dependence of $M$ on $T$. At small $\omega$, the mass gap decreases smoothly toward zero with increasing $T$, indicating a crossover transition. At large $\omega$ the transition becomes stronger and eventually a first-order transition. The transition temperature $T_c$ becomes smaller at larger $\omega$. These results could be understood by considering $\omega$ as a sort of "chemical potential" for angular momentum. Indeed this is evident from Eq.(4): the term $\vec{\omega}\cdot\hat{\vec{J}}$ is in direct analogy to a term $\mu\cdot\hat{Q}$ for a conserved charge $\hat{Q}$. Therefore the phase transition behavior at finite $\omega$ is similar to that at finite $\mu$ in the same model.

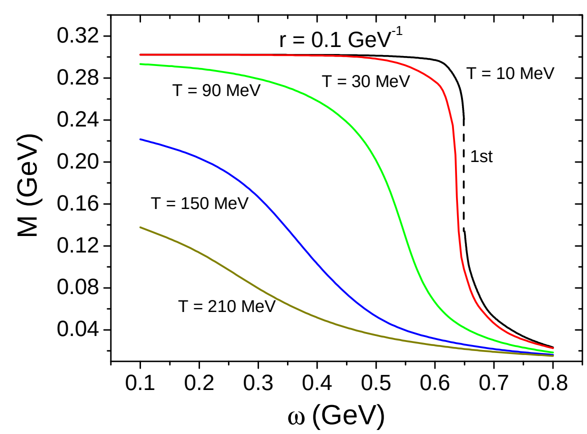


FIG. 2: The mean-field mass gap $M$ (at radial coordinate $r = 0.1\text{GeV}^{-1}$) versus $\omega$ for various fixed values of $T$.

With the above observation, it is tempting to envision a new phase diagram of the chiral phase transition on the $T-\omega$ parameter space: see Fig. 3. It features a chiral-symmetry-broken phase at low temperature and slow rotation while a chiral-symmetry-restored phase at high temperature and/or rapid rotation. A smooth crossover transition region at high $T$ and low $\omega$ and a first-order transition line at low $T$ and high $\omega$ are connected by a new *critical end point*. Given the present model parameters, this critical point is located at $T_{CEP} = 0.020$GeV and $\omega_{CEP} = 0.644$GeV. As the "rotational suppression" of scalar condensate is a quite generic effect, similar phase transition behaviors under rotation should also occur in other models for chiral condensate.

*Superconducting Pairing in Rotating Matter.*— To demonstrate that the "rotational suppression" is a generic effect, we also study another quite different type of scalar pairing: the fermion-fermion superconducting pairing phenomenon in the presence of rotation. In the QCD context, this is the color superconductivity at high density and low temperature (see e.g. [38] for a recent review). Different from the chiral condensate, the diquark pairing state has orbital angular momentum $L = 0$ while total spin $S = 0$ (i.e. antisymmetric combination

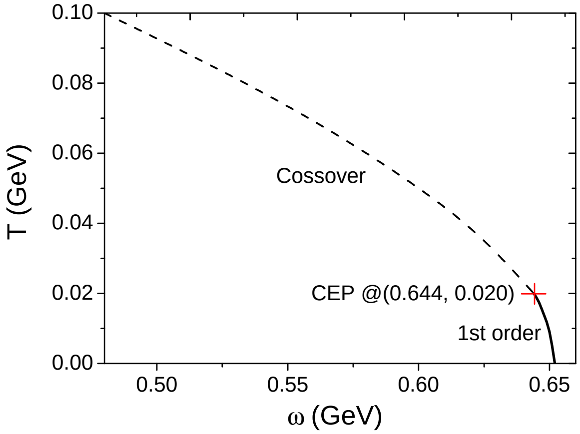


FIG. 3: The phase diagram on $T$-$\omega$ plane (see text).

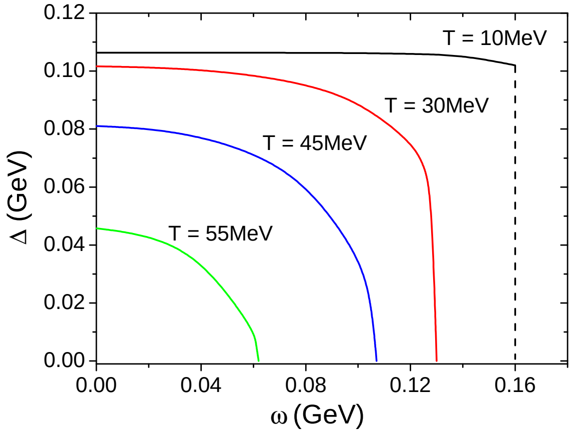


FIG. 4: The mean-field diquark condensate $\Delta$ (at radial coordinate $r = 0.1\text{GeV}^{-1}$) as a function of $\omega$ for several values of $T$ and fixed value of $\mu = 400\text{MeV}$.

of individual quark spins), again with the total angular momentum $J = 0$. We use the same NJL model and for simplicity we focus on the low-temperature high-density region where the chiral symmetry is already restored. Assuming a mean-field 2SC diquark condensate $\Delta\epsilon^{\alpha\beta 3}\epsilon_{ij} = -2G_d \left\langle i\psi_i^\alpha C\gamma^5\psi_j^\beta \right\rangle$ the grand potential is given by (with $N_f = 2$ and $N_c = 3$):

$$\begin{aligned}
\Omega \;=\; & \int d^3\vec{r} \Big\{ \frac{\Delta^2}{4G_d} - \frac{1}{4\pi^2}\sum_n \int dk_t^2 \int dk_z \\
& \times \quad [J_n(k_t r)^2 + J_{n+1}(k_t r)^2] \\
& \times N_f T \Big[ (N_c - 2) \left( \ln\left(1 + e^{\epsilon_n^+/T}\right) + \ln\left(1 + e^{-\epsilon_n^+/T}\right) \right. \\
& \left. + \ln\left(1 + e^{\epsilon_n^-/T}\right) + \ln\left(1 + e^{-\epsilon_n^-/T}\right) \right) \\
& + 2 \left( \ln\left(1 + e^{\epsilon_n^{\Delta+}/T}\right) + \ln\left(1 + e^{-\epsilon_n^{\Delta+}/T}\right) \right. \\
& \left. + \ln\left(1 + e^{\epsilon_n^{\Delta-}/T}\right) + \ln\left(1 + e^{-\epsilon_n^{\Delta-}/T}\right) \right) \Big] \Big\} \qquad (9)
\end{aligned}$$

In the above the mean-field quasiparticle dispersion $\epsilon_n^\pm$ and $\epsilon_n^{\Delta\pm}$ is given by $\epsilon_n^\pm = (\sqrt{k_z^2 + k_t^2 + m^2} \pm \mu) - (n + \frac{1}{2})\omega$ and $\epsilon_n^{\Delta\pm} = [(\sqrt{k_z^2 + k_t^2 + m^2} \pm \mu)^2 + \Delta^2]^{\frac{1}{2}} - (n + \frac{1}{2})\omega$. The diquark condensate $\Delta$ at given values of $T$, $\mu$ and $\omega$, can then be determined from the gap equation for the order parameter: $\frac{\delta\Omega}{\delta\Delta(r)} = 0$ and $\frac{\delta^2\Omega}{\delta\Delta(r)^2} > 0$. By numerically solving the equation, we show in Fig. 4 the $\Delta$ (at $r = 0.1\text{GeV}^{-1}$) as a function of $\omega$ for several values of $T$ and fixed $\mu = 400\text{MeV}$. With increasing $\omega$, the diquark condensate always decreases toward zero, through a 1st-order transition at low $T$ while a smooth crossover at higher $T$. This result again confirms the generic rotational suppression effect on the scalar diquark pairing.

*Summary and Discussions.*— In summary, we have found a generic rotational suppression effect on the fermion pairing state with zero angular momentum. This effect is demonstrated for two well-known pairing phenomena in QCD matter, namely the chiral condensate and the color superconductivity. The scalar pairing states in these two examples, while different in many aspects, are both found to be reduced with increasing rotation of the system. In the case of chiral phase transition, we have identified the phase boundary with a critical point on the $T - \omega$ parameter space.

Apart from significant theoretical interests, it is tempting to discuss potential implications of the rotational suppression effect for several physics systems. The phase diagram of QCD matter on $T - \omega$ plane could be quantitatively explored by ab initio lattice simulations which has recently become feasible [9]. In heavy ion collisions there is sizable global angular momentum ($\sim 10^5\hbar$) carried by the fireball (see e.g. [6]) with $\omega$ reaching the order of 0.1GeV, which may possibly help restore chiral symmetry at lower temperature. For the dense matter in neutron stars, the global rotation has a frequency up to $\omega \sim 10^3\text{s}^{-1}$ with $\omega r \sim 0.1c$ (where $c$ is speed of light) at outer crust which might influence the nucleon mass or nucleon-nucleon pairings as well as the moment of inertia for such stars [28, 29]. To see whether any measurable consequence in these QCD systems may result therein, requires quantitative studies. In the non-relativistic domain, the cold fermionic gas is an ideal place to study the rotational suppression effect on the fermion pairing and the BCS-BEC crossover phenomenon [39–43]. A realistic investigation of potential phenomenological applications, as well as a very detailed discussion of the present theoretical study, will be reported elsewhere in the future.

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