Extended Propagation of Powerful Laser Pulses in Focusing Kerr Media

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Extended propagation of powerful laser pulses in focusing Kerr media

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Powerful incoherent laser pulses can propagate in focusing Kerr media much longer distances than can coherent pulses, due to the fast phase mixing that prevents transverse filamentation. This distance is limited by 4-wave scattering, which accumulates waves at small transverse wavenumbers, where phase mixing is too slow to retain the incoherence and thus prevent the filamentation. However, we identify how this theoretical limit can be overcome by countering this accumulation through transverse heating of the pulse by random fluctuations of the refractive index. Thus, the laser pulse propagation distances are significantly extended, making feasible, in particular, generation of unprecedentedly intense and powerful short laser pulses in plasma by means of backward Raman amplification in new random laser regimes.

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Propagation of powerful laser pulses in focusing nonlinear media depends on the competition between the transverse dispersive spreading and focusing nonlinearity [1–6]. For negligible nonlinearity, the dispersion would double the cross section of a coherent pulse of transverse size $L_\perp$ within the Rayleigh length $L_R \sim L_\perp^2/\lambda$, where λ is the laser wavelength. A strong enough nonlinearity would noticeably reduce the pulse cross section, or even cause transverse filamentation, within the length $L_{SF} \sim v_g/\omega_{nl}$, where ω_{nl} is the nonlinear frequency shift and v_g is the group velocity of the pulse. For $L_R < L_{SF}$, the dispersive spreading outruns the self-focusing and suppresses it. We will consider focusing Kerr-like media for which the nonlinear frequency shift is proportional to the pulse intensity I, namely, $\omega_{nl} = -\alpha I$ with the positive coefficient $\alpha > 0$. Then, $L_R/L_{SF} \sim P_{ch}/P_{cr}$, where $P_{ch} \sim IL_\perp^2$ is the coherent pulse power and $P_{cr} \sim \lambda v_g/\alpha$ is the critical power of self-focusing.

A coherent laser pulse of power P much greater than the critical power, $P \gg P_{cr}$, would experience transverse filamentation within a propagation length not much exceeding the self-focusing length L_{SF}. In contrast to this, incoherent laser pulses could traverse focusing Kerr-like media, remaining statistically uniform in the transverse directions, at arbitrarily large powers. What actually matters for non-filamentation of incoherent pulses is not the total power P, but the coherent sub-power $P_{ch} \sim IL_\perp^2$ located within the transverse correlation length $L_\perp$. If this sub-power is much smaller than the critical power, $P_{ch} \ll P_{cr}$, then the Rayleigh length is much shorter than the self-focusing length, so that nonlinear phase shift accumulated within the Rayleigh length is small $\phi_{nl} \sim \omega_{nl} L_R/v_g \sim P_{ch}/P_{cr} \ll 1$.

The small parameter $P_{ch}/P_{cr} \ll 1$ enables a kind of perturbation theory. Physically, this small parameter implies that the phase mixing of different waves occurs much faster than the nonlinear interaction. In zero-order approximation, the phases of different waves are random. Statistical averaging over nearly random phases leads to a closed evolution equation for the pair correlation function of waves. This equation is usually referred to as a kinetic equation for waves. The pair correlation function of waves in Wigner representation is usually referred to as the wave spectral density. In general, the kinetic equation for waves contains terms linear, quadratic, cubic, *etc.* in the wave spectral density.

The linear term describes wave propagation with group velocities and scattering on inhomogeneities of the medium. For a pulse with negligible spread of group velocities in a conservative uniform medium, the wave spectral density does not change in the reference frame moving with the group velocity of the pulse.

The quadratic term in kinetic equation for waves gives linear contribution to the wave spectral density variation rate. This contribution comes from the statistically averaged nonlinear frequency shift, proportional to the pulse intensity. The corresponding nonlinear phase shift can bend phase fronts and cause self-focusing of incoherent pulses in a way similar to that of coherent pulses. The evolution of transverse size (radius) of an incoherent laser pulse within such a model was discussed in many papers (see, for instance, Refs. [7, 8]). This approximation is sufficient for Kerr media in which the nonlinear frequency shift is proportional to the laser field intensity integrated over time or space, because such an integration effectively accomplishes statistical averaging (see, for instance, Ref. [9]). However, for Kerr media of interest here in which the nonlinear frequency shift is proportional to the laser field intensity itself, higher order nonlinear processes can be important.

In fact, the higher order nonlinear processes appear to be dominant for sufficiently powerful incoherent laser pulses statistically nearly uniform in the transverse directions over many correlation lengths $L_\perp$. For statistically uniform pulses, the averaged nonlinear frequency shift does not vary in the transverse directions, so that there is no bending of the phase fronts, and the quadratic term in the kinetic equation for waves is zero. The dominant nonlinear term in the kinetic equation for the waves is then the cubic term.

More accurately: The cubic term in the kinetic equation for waves, that gives the quadratic contribution to the wave spectral density variation rate, contains the small factor P_{ch}^2/P_{cr}^2. The quadratic term in kinetic equation for waves, that gives the linear contribution to the wave spectral density variation rate, contains the small factor $f_I P_{ch}/P_{cr}$, where f_I is the small relative variation of the wave spectral density within the transverse correlation length $L_\perp$. For smooth wave spectral density non-modulated in the transverse directions, the typical f_I is about the ratio of $L_\perp$ to the pulse aperture, $f_I \sim \sqrt{P_{ch}/P}$. The cubic process is much faster than the quadratic process for $P \gg P_{cr}^2/P_{ch}$. Of interest here are pulses of very high powers P, perhaps exceeding the critical power P_{cr} by a factor of a million, while, to secure the pulse incoherence, the ratio P_{cr}/P_{ch} needs to be just somewhat larger than 1. The effect of the quadratic term in the kinetic equation for the waves on the propagation of such powerful pulses is then negligible.

The cubic term in kinetic equation for waves is associated with the 4-wave scattering. The evolution of wave spectral density due to the 4-wave scattering was subject to a longstanding theoretical controversy, as recounted and resolved analytically in Ref. [10]. That analytical resolution was recently supported numerically [11]. Being the same for focusing and defocusing Kerr nonlinearities, the 4-wave kinetic equation cannot produce self-focusing effects directly. However, it tends to accumulate waves at small transverse wavenumbers, where phase mixing is too slow to retain the incoherence. Therefore, the input pulse randomization (that might be arranged, say, by techniques of the type [12–14]) is not sufficient to achieve pulse propagation over distances much exceeding the 4-wave scattering length.

Here we propose to use an ongoing randomization to prevent wave accumulation at small transverse wavenumbers, thus extending the propagation of powerful laser pulses in focusing Kerr media much beyond the known theoretical limit [10]. The ongoing randomization can be accomplished through the natural ray diffusion in media with random fluctuations of the refractive index. The substantially extended propagation lengths could make feasible a new class of random lasers [15–17], based on backward Raman amplification [18–22] and capable of reaching relativistic non-focused intensities in plasmas. This would exploit random density fluctuation inherently present in plasmas and hitherto considered harmful to the backward Raman amplification.

Since the wave spectral density variation rate, associated with the 4-wave scattering, contains the small factor P_{ch}^2/P_{cr}^2 compared to the rate of phase mixing, the inverse length of 4-wave scattering is

$$L_{4w}^{-1} \sim L_R^{-1}\, P_{ch}^2\,/P_{cr}^2 \sim L_R\,/L_{SF}^2\ . \tag{1}$$

The 4-wave scattering of two waves with transverse wavenumbers $k_\perp \sim 2\pi/L_\perp$ into two new waves typically produces a wave of larger transverse wavenumber along with a wave of smaller transverse wavenumber. Further 4-wave scattering produces waves of even larger transverse wavenumbers, so that the wave distribution spreads over a circle of growing radius $k_{\perp M}$ in the $\boldsymbol{k}_\perp$-plane. At the same time, the number of waves with small $k_\perp$ increases. For smaller $k_\perp$, the 4-wave scattering is faster $L_{4w}^{-1} \propto L_R \propto L_\perp^2 \propto k_\perp^{-2}$, and it tends to establish the wave spectral density close to the equilibrium Rayleigh-Jeans distribution inside the circle $k_\perp \lesssim k_{\perp M}$:

$$N_{\boldsymbol{k}_\perp} \approx \frac{T\,(\Lambda_{k_\perp})}{k_\perp^2 + k_{\perp m}^2}\,, \quad \Lambda_{k_\perp} = \ln\left(1 + \frac{k_\perp^2}{k_{\perp m}^2}\right)\,. \tag{2}$$

Here $k_{\perp m}^2$ is the "chemical potential" and $T(\Lambda_{k_\perp})$ is the "local temperature" which can logarithmic-slowly depend on $k_\perp$ to provide nonzero fluxes across the spectrum. This multi-scale distribution has roughly $\Lambda_{k_{\perp M}}$ different populated scales each of which carries just a small fraction of total pulse intensity I. Such an optical turbulence does not satisfy the classical Kolmogorov hypothesis of spectral locality of interactions. There is, however, a more general kind of locality [10] that enables expressing $\boldsymbol{k}_\perp$-integrals in the 4-wave kinetic equation by explicit formulas local in $k_\perp$, so that Eq. (1) is modified for the multi-scale spectrum as follows:

$$L_{4wk_\perp}^{-1} \sim f'(\Lambda_{k_\perp}) f(\Lambda_{k_\perp}) L_{Rk_\perp}\,/L_{SF}^2 \tag{3}$$
$$f(\Lambda_{k_\perp}) = \int_{\xi \lesssim \Lambda_{k_\perp}} T(\xi) d\xi \Bigg/ \int_{\xi \lesssim \Lambda_{k_{\perp M}}} T(\xi) d\xi\ .$$

Here $f_{k_\perp}$ is the fraction of pulse intensity carried by waves with transverse wave-numbers not exceeding $k_\perp$. A simplified explanation of the factor $f'f$ entering here is that, being quadratic in the wave intensity, the rate of 4-wave scattering contains the product of two wave intensities; one of the contributing waves has wavenumber about $k_\perp$, and thus carries the intensity $\sim f'(\Lambda_{k_\perp})I$, while the second wave may have any wavenumber smaller than $k_\perp$, and thus carries the intensity $\sim f(\Lambda_{k_\perp})I$. Therefore, the rate of 4-wave scattering is modified by the factor $\sim f'f$. The 4-wave scattering inverse length $L_{4wk_\perp}^{-1}$ relates to its initial value L_{40}^{-1} as

$$\frac{L_{40}}{L_{4wk_\perp}} \sim \frac{I^2}{I_0^2}\frac{k_{\perp 0}^2}{k_\perp^2} f'(\Lambda_{k_\perp}) f(\Lambda_{k_\perp})\,, \quad L_{40} \sim \frac{L_{SF0}^2}{L_{Rk_{\perp 0}}}\,. \tag{4}$$

The applicability condition of random phase approximation, $L_{4wk_\perp}^{-1} < L_{Rk_\perp}^{-1}$, is the most restrictive at $k_\perp \sim k_{\perp m}$. There $f(\Lambda_{k_{\perp m}}) \sim f'(\Lambda_{k_{\perp m}})$ and $L_{4wk_{\perp m}}^{-1} \sim f'^2(\Lambda_{k_{\perp m}}) L_{Rk_{\perp m}}/L_{SF}^2$, so that the applicability condition reduces to $f'(\Lambda_{k_{\perp m}}) L_{Rk_{\perp m}}/L_{SF} < 1$. It simply means that the Rayleigh length of waves with $k_\perp \sim k_{\perp m}$ must be shorter than the self-focusing length of these waves themselves, regardless to other parts of the spectrum. This can be rewritten in the form

$$f'(\Lambda_{k_{\perp m}})(k_{\perp 0}^2/k_{\perp m}^2)(I/I_0) < P_{cr}/P_{ch0}\,. \quad (5)$$

The growth of $k_{\perp M}^2$ is described by the equation

$$\frac{dk_{\perp M}^2}{dz} \sim \frac{k_{\perp M}^2}{L_{4wk_{\perp M}}} \sim \frac{I^2}{I_0^2}\frac{k_{\perp 0}^2}{L_{40}} f'(\Lambda_{k_\perp M})\,. \quad (6)$$

As long as other effects do not cause fluxes, the 4-wave scattering establishes nearly the same "local temperature" across the spectrum, $T(\Lambda_{k_\perp}) \approx T(\Lambda_{k_{\perp M}}) = T$. It follows then $f'(\Lambda_{k_\perp}) \approx 1/\Lambda_{k_{\perp M}}$ and $f(\Lambda_{k_\perp}) \approx \Lambda_{k_\perp}/\Lambda_{k_{\perp M}}$. The intensity I may change due to the longitudinal stretching/contraction, or pumping/damping of the pulse. Even absent the group velocity dispersion and pumping/damping, the short pulse duration may be strongly affected by small non-paraxial corrections to the longitudinal component of group velocity $v_{g\|}$. The relative reduction of $v_{g\|}$, $\delta v_{g\|}/v_{g\|} \approx k_\perp^2/(2k_\|^2)$, is the largest for waves with $k_\perp \sim k_{\perp M}$. It may cause pulse stretching $\delta L \sim zk_{\perp M}^2/(2k_\|^2)$, which can be neglected only for pulses of sufficiently large length L,

$$L > zk_{\perp M}^2/(2k_\|^2)\,. \quad (7)$$

Consider, first, regimes with $I \sim I_0$. Since $I = \int d\boldsymbol{k}_\perp N_{\boldsymbol{k}_\perp} \sim T\Lambda_{k_{\perp M}}$, it follows $T \sim I_0/\Lambda_{k_{\perp M}}$. The 4-wave scattering itself conserves also the "transverse energy" $\mathcal{E}_\perp = \int d\boldsymbol{k}_\perp N_{\boldsymbol{k}_\perp} k_\perp^2$. The major contribution to this integral comes from waves with $k_\perp \sim k_{\perp M}$, so that $\mathcal{E}_\perp \sim k_{\perp M}^2 T \sim I_0 k_{\perp M}^2/\Lambda_{k_{\perp M}}$. As long as $\mathcal{E}_\perp$ is conserved, it follows $\Lambda_{k_{\perp M}} \sim k_{\perp M}^2/k_{\perp 0}^2$. Eq. (6) gives then $k_{\perp M}^4/k_{\perp 0}^4 \sim 2z/L_{40}$, so that

$$\ln\left(k_{\perp M}^2/k_{\perp m}^2\right) \approx \Lambda_{k_{\perp M}} \sim k_{\perp M}^2/k_{\perp 0}^2 \sim (2z/L_{40})^{1/2}\,. \quad (8)$$

The applicability condition (5) takes now the form

$$\Lambda_{k_{\perp M}} < \Lambda_*\,, \quad \exp(\Lambda_*)/\Lambda_*^2 \sim P_{cr}/P_{ch0}\,. \quad (9)$$

It can be satisfied only up to z not much exceeding L_{40}, namely $z < z_* \sim L_{40}\Lambda_*^2/2$. To overcome this previous theoretical limit [10], the ray diffusion on the medium inhomogeneities should stop the exponential decrease of $k_{\perp m}$ before the condition (9) is violated. Diffusion spreads the ray transverse wave-vectors according to $(\delta k_\perp)^2 = Dz$. It starts affecting the previous regime when $(\delta k_\perp)$ matches $k_{\perp m}$. This occurs at $\Lambda_{k_{\perp M}} = \Lambda_D$, such that $\Lambda_D \exp(\Lambda_D) \sim 2k_{\perp 0}^2 L_{40}^{-1}/D$. The applicability condition (9) is satisfied for

$$D > D_* \sim 2k_{\perp 0}^2 L_{40}^{-1}\Lambda_*^{-1}\exp(-\Lambda_*)\,. \quad (10)$$

For such D, condensate does not form, rather $k_{\perp m}^2$ passes the minimum Dz_D at $z_D \sim L_{40}\Lambda_D^2/2 < z_*$, and then grows at $z > z_D$ according to

$$k_{\perp m}^2 \sim Dz\,. \quad (11)$$

This increases the "local temperature" at $k_\perp \sim k_{\perp m}$, but the increase is quickly saturated by the faster 4-wave scattering which produces a flux of waves towards larger $k_\perp$ up to $k_{\perp M}$. As a result, a fixed profile of the "local temperature" $T(\Lambda_{k_\perp})$ sets across the spectrum. The entire spectrum evolves then in self-similar way, as a function of just $k_\perp/k_{\perp m} \propto k_\perp/\sqrt{z}$, rather than $k_\perp$ and z separately. It implies $L_{4wk_\perp} \sim z \propto k_\perp^2$ across the spectrum, which, according to (4), means $f'(\Lambda_{k_\perp})f(\Lambda_{k_\perp}) = const$. The solution of this equation is

$$f(\Lambda_{k_\perp}) \approx \Lambda_{k_\perp}^{1/2}\Lambda_{k_{\perp M}}^{-1/2}\,. \quad (12)$$

Integration of (6) gives

$$k_{\perp M}^2/k_{\perp 0}^2 \sim \Lambda_{k_{\perp M}}^{-1} z/L_{40}\,, \quad \Lambda_{k_{\perp M}} \approx \Lambda_D\,. \quad (13)$$

This regime extends until the applicability condition (7) is violated and the longitudinal stretching of the pulse becomes important. At larger z, the pulse intensity decreases due to the longitudinal stretching, but the fluence $w = IL$ does not change, absent pumping/damping. The stretching is described by the equation

$$dL/dz \sim k_{\perp M}^2/(2k_\|^2)\,. \quad (14)$$

Though the stretching is mainly due to the waves with $k_\perp \sim k_{\perp M}$ trailing, the 4-wave scattering quickly sets nearly the same transverse spectrum at all cross sections within the pulse length. Therefore, formula (12) remains valid. The variation of $k_{\perp M}^2$ is described by (6). For a fixed fluence w, the relative variation of $k_{\perp M}^2$ is small, so that the stretching proceeds linearly $L \propto z$. For a fixed pumping, the fluence grows linearly $w \propto z$, which gives $L \propto z^{4/3}$ and $k_{\perp M}^2 \propto z^{1/3}$. Waves with $k_\perp \sim k_{\perp M}$ are not directly affected by the ray diffusion, so that the spectrum remains dominated by the 4-wave scattering and multi-scale, as long as $k_{\perp M}^2 > Dz$. When this condition is violated, the 4-wave scattering becomes relatively small and the propagation regime becomes nearly linear.

Consider now calculation of the ray diffusion coefficient D for a medium with statistically uniform random inhomogeneities of the refractive index having typical relative amplitude of fluctuations f_r, transverse correlation length $L_{\perp r}$, and longitudinal correlation length $L_{\| r}$. Let the group and phase velocities of the laser pulse be comparable. Then, according to the Hamilton equations for rays, the transverse wavenumber typically changes, during the ray passing a single fluctuation, by $\delta k_\perp \sim k_\| f_r L_{\| r}/L_{\perp r}$, for not too small aspect ratio $L_{\perp r}/L_{\| r} \gtrsim k_\perp/k_\|$. The respective diffusion coefficient is

$$D \sim \delta k_\perp^2/L_{\| r} \sim k_\|^2 f_r^2 L_{\| r}/L_{\perp r}^2\,. \quad (15)$$

To prevent Bose-Einstein condensation, this diffusion coefficient should exceed the threshold D_*. The fluctuation amplitude needed is then

$$f_r > f_{r_*} \sim \frac{L_{\perp r} k_{\perp 0}\sqrt{2}\exp(-\Lambda_*/2)}{k_\| \sqrt{L_{\| r} L_{40}\Lambda_*}}\,. \quad (16)$$

In particular, for a plasma with fluctuations of electron concentration δn and plasma-to-laser frequency ratio $f_e = \omega_e/\omega = \lambda/\lambda_e \ll 1$ (where $\lambda_e = 2\pi c/\omega_e$), the refractive index fluctuation amplitude is $f_r = f_e^2 \delta n/(2n_0)$. The fluctuations δn needed to prevent Bose-Einstein condensate formation is then

$$\frac{\delta n}{n_0} > \frac{\delta n_*}{n_0} \sim \frac{L_{\perp r}\lambda_e^2 (2\Lambda_*)^{3/2}\exp(-3\Lambda_*/2)}{(L_{\parallel r}\lambda)^{1/2} L_{\perp 0}^2} . \tag{17}$$

Introducing such fluctuations is key to extending the laser propagation in plasma, making possible amplification of short laser pulses to unprecedented fluences. The pulses of durations too short to noticeably move ions, or to heat plasma, can be amplified to ultrahigh intensities, since the ponderomotive and thermal nonlinearities, leading to self-focusing and filamentation, do not develop. What limits the propagation and amplification of such short pulses in plasma is the relativistic electron nonlinearity which comes from the electron mass dependence of the electron quiver velocity, $m_e = m/\sqrt{1-v^2/c^2} \approx m\left(1+0.5v^2/c^2\right)$. The normalized quiver energy, averaged over the laser period, $\overline{v^2}/c^2$ can be expressed in the terms of laser intensity I as $\overline{v^2}/c^2 = 4\pi e^2 I/(m^2c^3\omega^2)$. The laser frequency shift due to the relativistic electron nonlinearity is $\omega_{nl} \approx -\overline{v^2}\omega_e^2/(4c^2\omega) = -\alpha I$, $\alpha = \pi e^2\omega_e^2/(m^2c^3\omega^3) = e^2\lambda^3/(2m^2c^4\lambda_e^2)$. The critical self-focusing power is then

$$P_{cr} \sim \lambda v_g/\alpha \approx \mathcal{P}\lambda_e^2/\lambda^2, \;\; \mathcal{P} = 2m^2c^5/e^2 \approx 17\,\mathrm{GW}, \tag{18}$$

which well agrees with the standard value of P_{cr} for axisymmetric laser pulses in plasma [23–25].

The relativistic electron nonlinearity limits the classical π-pulse regime of backward Raman amplification in plasma [20, 21, 26–29], during which the pulse amplitude grows proportionally to z and the pulse length contracts inversely. This ends when the self-focusing length, decreasing in the π-pulse regime like z^{-2}, matches z. Then the leading spike growth saturates and the transverse filamentation instability becomes dangerous. High-quality coherent pulses may propagate somewhat further without the filamentation, but no more than a few times this distance, until the filamentation instability will make enough exponentiations to grow from small initial imperfections of wave fronts. Such a modestly extended propagation allows to amplify coherent pulses to higher fluences if not intensities [28]. However, incoherent pulses can propagate much longer distances, remaining statistically uniform in the transverse directions. Since wave fronts of incoherent pulses are not smooth at all, the filamentation instability must be suppressed even before the leading amplified spike saturates at the intensity [27]

$$I_0 \approx \frac{\mathcal{P}}{\lambda_e\lambda}\left(\frac{I_{p0}}{I_{br}}\right)^{2/3}, \quad I_{br} = \frac{\pi\mathcal{P}\lambda}{32\lambda_e^3} . \tag{19}$$

Here I_{p0} is the incident pump pulse intensity, and I_{br} is its value at which wavebreaking occurs of the resonant Langmuir wave mediating energy transfer from the pump to the amplified pulse; it is assumed $I_{br} > I_{p0}$, because the energy transfer efficiency significantly drops at $I_{br} \ll I_{p0}$ [20, 26, 30, 31]. As seen from (19), the energy compression ratio is $I_0/I_{p0} \approx 10(I_{br}/I_{p0})^{1/3}\lambda_e^2/\lambda^2 > 1000$, for $\lambda_e/\lambda > 10$. This is also the ratio of the consumed pump length $2z_0$ to the compressed pulse length $L_0 \sim \lambda_e$. Within the propagation length z_0, the pulse longitudinal stretching due to the finite angular spread is $\delta L_0 \sim z_0\lambda^2/(2L_{\perp 0}^2)$. Since $z_0 \sim L_0 I_0/I_{p0}$ and $L_{\perp 0}^{-2} \sim I_0/P_{ch0}$, it follows $\delta L_0/L_0 \sim 5P_{cr}\lambda/(2P_{ch0}\lambda_e)\,(I_{p0}/I_{br})^{1/3}$. For $P_{ch0}/P_{cr} > 5\lambda/(2\lambda_e)\,(I_{p0}/I_{br})^{1/3}$, this stretching is small, so that the π-pulse stage is not much affected by the finite angular spread. Substitution of $L_{\perp 0}^{-2} \sim I_{p0}/P_{ch0} \sim \lambda\lambda_e^{-3}(I_{p0}/I_{br})^{2/3}P_{cr}/P_{ch0}$ into (17), and using (9) to express P_{cr}/P_{ch0} in the terms of Λ_* gives

$$\frac{\delta n}{n_0} > \frac{\delta n_*}{n_0} \sim \frac{2^{3/2}\exp(-\Lambda_*/2)L_{\perp r}\lambda^{1/2}}{\Lambda_*^{1/2}L_{\parallel r}^{1/2}\lambda_e}\left(\frac{I_{p0}}{I_{br}}\right)^{2/3} . \tag{20}$$

If, for example, $L_{\parallel r} = L_{\perp r} = \lambda_e = 25\lambda$, and $I_{p0} \sim I_{br}$, then $\delta n_*/n_0 \sim 15\%$ for $\Lambda_* = 2$, $\delta n_*/n_0 \sim 4\%$ for $\Lambda_* = 4$, and $\delta n_*/n_0 \sim 1\%$ for $\Lambda_* = 6$. For $\lambda = 0.3\,\mu$m, the intensity I_0 is $I_0 \approx 0.8\times 10^{18}\,\mathrm{W/cm^2}$, and the plasma concentration is $n_0 = \pi mc^2(e\lambda_e)^{-2} \approx 2\times 10^{19}\,\mathrm{cm^{-3}}$. Such plasma might be produced by ionization of dense aerosols with the droplets as small as a few microns [32, 33].

With the filamentation instability suppressed by the pulse incoherence, the backward Raman amplification can proceed over much larger distances and produce pulses of much higher fluences, if not intensities, than thought earlier. The high-energy output can be converted into an unprecedentedly intense and well-focused coherent laser pulse by means of the backward Raman amplification in a thin dense plasma layer, as described in the two-step scheme [22]. The 1st step output is used as the pump at the 2nd step. The transverse filamentation instability of coherent amplified pulse does not pose a serious problem at the 2nd step, because of the plasma short length and high density.

In summary, the key findings here are: one, powerful laser pulse scattering on random fluctuations of the refractive index suppresses the transverse filamentation instability and enables propagation distances significantly exceeding the previous theoretical limit in focusing Kerr media; and, two, this scattering may now be used to overcome the transverse filamentation instability, the hitherto key limitation to producing laser pulses of unprecedented high intensities and powers through the backward Raman amplification in plasma.

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