Dense Axion Stars

Eric Braaten, Abhishek Mohapatra, and Hong Zhang

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Eric Braaten,[*] Abhishek Mohapatra,[†] and Hong Zhang[‡]
Department of Physics, The Ohio State University,
Columbus, OH 43210, USA

If the dark matter particles are axions, gravity can cause them to coalesce into axion stars, which are stable gravitationally bound systems of axions. In the previously known solutions for axion stars, gravity and the attractive force between pairs of axions are balanced by the kinetic pressure. The mass of these dilute axion stars cannot exceed a critical mass, which is about $10^{-14} M_\odot$ if the axion mass is 10^{-4} eV. We study axion stars using a simple approximation to the effective potential of the nonrelativistic effective field theory for axions. We find a new branch of dense axion stars in which gravity is balanced by the mean-field pressure of the axion Bose-Einstein condensate. The mass on this branch ranges from about $10^{-20} M_\odot$ to about $M_\odot$. If a dilute axion star with the critical mass accretes additional axions and collapses, it could produce a bosenova, leaving a dense axion star as the remnant.

Introduction. The particle nature of the dark matter of the universe remains one of the greatest mysteries in contemporary physics. One of the most strongly motivated possibilities from a particle physics perspective is the *axion* [1], which is the pseudo-Goldstone boson associated with a $U(1)$ symmetry that solves the strong CP problem of QCD. The axion is a spin-0 particle with a very small mass and extremely weak self-interactions. Axions can be produced in the early universe by a vacuum misalignment mechanism [1]. The resulting axions are extremely nonrelativistic and have huge occupations numbers, so they are naturally described by a classical field. Sikivie and collaborators have pointed out that gravitational interactions can continually rethermalize axion dark matter [2, 3]. They can therefore form a Bose-Einstein condensate (BEC) that remains locally in the lowest-energy accessible state.

A stable gravitationally bound configuration of axions is called an *axion star*. In the ground state of an axion star, the axions are in a BEC. In the known solutions for axion stars, the attractive forces from gravity and from the 4-axion interaction are balanced by the kinetic pressure. We refer to them as *dilute axion stars*, because the number density remains small enough that 6-axion and higher interactions are negligible. There is a critical mass for the dilute axion star beyond which the kinetic pressure is unable to balance the attractive forces. The fate of an axion star that exceeds the critical mass and begins to collapse has not been established. The possibilities for the remnant after the collapse include

- *black hole*. Axion self-interactions may be too weak to prevent the collapse of the axion star to a configuration smaller than its Schwarzschild radius.

- *dilute axion star* with smaller mass. The decrease in mass could come from radiation of axion waves or from a *bosenova*, in which inelastic axion reactions amplified by the increasing number density during the collapse produce a burst of relativistic axions. (An analogous phenomenon has been observed in the collapse of a BEC of ultracold atoms [4].)

- *dense axion star*, in which the number density becomes so high that multi-axion interactions affect the balance of forces (provided such solutions exist).

In this paper, we study axion stars using a simple approximation to the effective potential of the nonrelativistic effective field theory for axions. We find a new branch of dense axion stars in which the attractive force of gravity is balanced by the mean-field pressure of the axion BEC. The mass of a dense axion star can be many orders of magnitude smaller or larger than the critical mass of a dilute axion star.

Relativistic axion field theory. Axions can be described by a relativistic quantum field theory with a real scalar field $\phi(x)$. The Hamiltonian has the form

$$\mathcal{H} = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}\nabla\phi \cdot \nabla\phi + \mathcal{V}(\phi). \tag{1}$$

The potential $\mathcal{V}$ is produced by nonperturbative QCD effects, and depends on quark mass ratios [6]. It is commonly approximated by

$$\mathcal{V}(\phi) = m^2 f^2 \left[1 - \cos(\phi/f)\right], \tag{2}$$

where m is the mass of the axion and f is the axion decay constant. In this paper, we assume that $\mathcal{V}$ in Eq. (2) is an adequate approximation for the relativistic potential.

The product mf of the mass of the axion and its decay constant is $(8\times10^7$ eV$)^2$ [1]. Astrophysical and cosmological constraints restrict f to the window between about

5×10^{17} eV and about 8×10^{21} eV [1]. The window for m is therefore from about 10^{-6} eV to about 10^{-2} eV.

Nonrelativistic axion effective field theory. Axions whose kinetic energies are much smaller than m can be described by a nonrelativistic effective field theory with a complex scalar field $\psi(\boldsymbol{r}, t)$. We refer to this effective theory as *axion EFT*. The number of axions, $N = \int d^3r\, \psi^*\psi$, is conserved in axion EFT. The effective Hamiltonian has the form

$$\mathcal{H}_{\mathrm{eff}} = \frac{1}{2m}\nabla\psi^* \cdot \nabla\psi + \mathcal{V}_{\mathrm{eff}}(\psi^*\psi). \quad (3)$$

The effective potential $\mathcal{V}_{\mathrm{eff}}$ is the generator of $n \to n$ axion scattering vertices. In the case of an axion BEC, $\mathcal{V}_{\mathrm{eff}}$ also gives the mean-field energy of the condensate as a function of its number density $\psi^*\psi$. The coefficients in the expansion of $\mathcal{V}_{\mathrm{eff}}$ in powers of $\psi^*\psi$ can be derived by matching low-energy scattering amplitudes at tree level in the relativistic theory and in axion EFT [5].

A simple approximation to $\mathcal{V}_{\mathrm{eff}}$ can be obtained by a nonrelativistic reduction of the relativistic Hamiltonian in Eq. (1). After inserting

$$\phi(\boldsymbol{r}, t) = \frac{1}{\sqrt{2m}}\left[\psi(\boldsymbol{r}, t)e^{-imt} + \psi^*(\boldsymbol{r}, t)e^{+imt}\right], \quad (4)$$

dropping terms with a rapidly oscillating phase factor e^{inmt}, where n is a nonzero integer, and also dropping terms with time derivatives of ψ, we obtain the effective Hamiltonian in Eq. (3) with the effective potential

$$\mathcal{V}_{\mathrm{eff}}(\psi^*\psi) = \tfrac{1}{2}m\psi^*\psi + m^2f^2[1 - J_0(\hat{n}^{1/2})], \quad (5)$$

where $\hat{n} = 2\psi^*\psi/mf^2$ and $J_0(z)$ is a Bessel function. The first term arises from the ϕ^2 term in Eq. (1). The second term was derived previously in Ref. [7]. At small $\hat{n}$, the second term approaches $\frac{1}{2}m\psi^*\psi$. At large $\hat{n}$, it approaches m^2f^2, oscillating around that value with a decreasing amplitude. The expansion of $\mathcal{V}_{\mathrm{eff}}$ in Eq. (5) in powers of $\psi^*\psi$ agrees with the exact effective potential for axion EFT through order $(\psi^*\psi)^2$ [5]. In this paper, we assume that $\mathcal{V}_{\mathrm{eff}}$ in Eq. (5) is an adequate approximation for the effective potential.

Axion stars. A *boson star* consists of a large number of bosons that are gravitationally bound. Boson stars consisting of bosons with no self-interactions and with gravitational interactions described by general relativity were first studied numerically by Kauf [8] and by Ruffini and Bonazzola [9]. The critical mass M_* beyond which the boson star is unstable to gravitational collapse was first determined accurately by Breit, Gupta and Zaks [10]: $M_* = 0.633/(Gm)$, where G is Newton's gravitational constant and m is the mass of the boson. If $m = 10^{-4\pm2}$ eV, the critical mass is $8.4 \times 10^{-7\mp2}\ M_\odot$, where $M_\odot$ is the mass of the sun. (Here and below, the upper and lower error bars in an exponent correspond to increasing and decreasing m by two orders of magnitude from 10^{-4} eV.)

An *axion star* is a boson star consisting of axions, a possibility first considered by Tkachev [11]. Axion stars with axion self-interactions given by expanding $\mathcal{V}$ in Eq. (2) to order ϕ^6 and with gravitational interactions described by general relativity were studied numerically by Barranco and Bernal [12]. They found solutions that correspond to dilute axion stars. The axions in a dilute axion star are nonrelativistic and their gravitational interactions are accurately described by Newtonian gravity. The gravitational energy is

$$E_{\mathrm{gravity}} = -\frac{Gm^2}{2}\int d^3r_1\!\int d^3r_2 \frac{\psi^*\psi(\boldsymbol{r}_1)\psi^*\psi(\boldsymbol{r}_2)}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}. \quad (6)$$

Axion stars with axion self-interactions given by expanding $\mathcal{V}$ in Eq. (2) to order ϕ^4 and with gravitational interactions described by Newtonian gravity were studied by Chavanis and Delfini [13]. Their result for the critical mass M_* of the dilute axion star above which there are no stable spherically symmetric solutions is

$$M_* = 10.15\, f/\sqrt{Gm^2}. \quad (7)$$

If $m = 10^{-4\pm2}$ eV, the critical mass is $6 \times 10^{-14\mp4}\ M_\odot$, which corresponds to $7 \times 10^{56\mp6}$ axions. The radius of the dilute axion star decreases as M increases. The radius R_{99} that encloses 99% of the axions is $9.9/GMm^2$ for small M, and it decreases to $0.55/(Gm^2f^2)^{1/2}$ at the critical mass [13]. This minimum radius is $3 \times 10^{-4}R_\odot$, where $R_\odot$ is the radius of the sun.

The equations that describe a static BEC of axions can be obtained by considering an axion field $\psi(\boldsymbol{r}, t)$ whose time-dependence is $\psi(\boldsymbol{r})\exp(-i\mu t)$, where μ is the chemical potential. They can be expressed conveniently as coupled equations for $\psi(\boldsymbol{r})$ and the gravitational potential $\phi(\boldsymbol{r})$:

$$\nabla^2\psi = -2m\big[\mu - (\mathcal{V}'_{\mathrm{eff}}(\psi^*\psi) - m) - m\phi\big]\psi, \quad (8a)$$
$$\nabla^2\phi = 4\pi Gm\psi^*\psi. \quad (8b)$$

For spherically symmetric solutions, $\psi(r)$ and $\phi(r)$ are functions of r only. It is convenient to take the central number density $n_0 = |\psi(0)|^2$ as an input parameter. If $|\psi(0)| = n_0^{1/2}$ is specified, the other boundary condition at the origin is $\psi'(0) = 0$ and the boundary conditions as $r \to \infty$ are $\psi(r) \to 0$ and $\phi(r) \to 0$. These boundary conditions determine the eigenvalue μ.

We first consider dilute axion stars, in which the axion number density $\psi^*\psi$ is always small compared to mf^2. The solution of Eqs. (8) gives the same results whether we use the effective potential in Eq. (5) or we truncate it after the $(\psi^*\psi)^2$ term so that $\mathcal{V}'_{\mathrm{eff}} - m = -\psi^*\psi/(8f^2)$. The mass-radius relation for dilute axion stars is shown in the upper left corner of Fig. 1. As the dimensionless central density $\hat{n}_0 = 2|\psi(0)|^2/mf^2$ increases, the radius R_{99} decreases and the mass M increases until it reaches
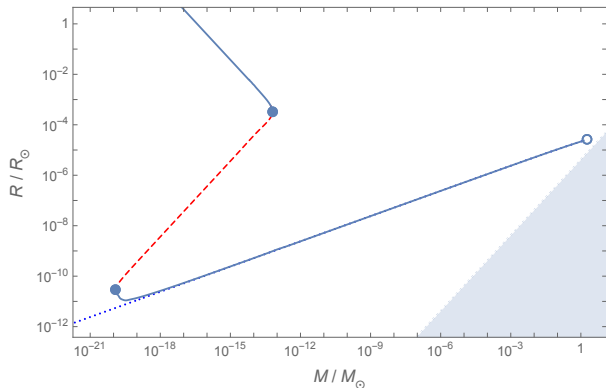
FIG. 1: Radius R_{99} of an axion star (normalized to $R_\odot$) as functions of its mass M (normalized to $M_\odot$) for $m = 10^{-4}$ eV. The curves for the radius are stable dilute axion stars (upper solid line), unstable axion stars (dashed line), stable dense axion stars (lower solid line), and the Thomas-Fermi approximation (dotted line). The shaded region is below the Schwarzchild radius $2GM/c^2$.

the critical mass M_* in Eq. (7) at $\hat{n}_0 = 1.3 \times 10^{-14 \mp 4}$. Beyond this critical point, the solutions are unstable under perturbations that increase the central density [13].

We now consider dense axion stars, in which $\psi^*\psi$ can be comparable to or larger than mf^2. In this case, one must use an approximation to $\mathcal{V}_{\text{eff}}$ that includes all orders in $\psi^*\psi$, such as the effective potential in Eq. (5). As illustrated in Fig. 1, the unstable solutions for a dilute axion star can be continued from the critical point at M_* to smaller values of M where $\mathcal{V}_{\text{eff}}$ can no longer be truncated after the $(\psi^*\psi)^2$ term. The mass decreases to a minimum value M'_* at a second critical point where $\hat{n}_0 = 13$, and then M begins increasing. At both critical points, the slope of the chemical potential μ as a function of M is infinite. By Poincare's theory of linear series of equilibria, the number of unstable modes must change by one at each critical point [14]. We have not excluded the possibility that the number of unstable modes changes to two after the second critical point, but we assume it changes to zero. This assumption is supported by the Thomas-Fermi approximation described below. The solutions beyond the second critical point are then stable dense axion stars, in which the mean-field pressure of the axion BEC balances the force of gravity and the kinetic pressure. For $m = 10^{-4 \pm 2}$ eV, the second critical mass M'_* is $1.2 \times 10^{-20 \mp 6} M_\odot$, which corresponds to $1.3 \times 10^{50 \mp 8}$ axions. The radius R_{99} at the critical point is $2.6 \times 10^{-11 \mp 2} R_\odot$, which is equal to $9.2(\hbar/mc)$.

Thomas-Fermi approximation. Beyond the second critical point, the accurate numerical solution of Eqs. (8) becomes increasingly challenging as n_0 increases further. The mean-field energy also becomes increasingly large compared to the kinetic energy. The solution can therefore be simplified using the *Thomas-Fermi approxima-*

tion, in which the kinetic energy is neglected. This approximation provides an excellent description of BECs of ultracold atoms when the number of atoms is large enough that their interaction energy is comparable to the trapping energy [15]. It has been applied previously by Wang to boson stars in which the interaction between a pair of bosons is repulsive [16]. In the Thomas-Fermi approximation, the $\nabla^2\psi$ term on the left side of Eq. (8a) is neglected. For the effective potential $\mathcal{V}_{\text{eff}}$ in Eq. (5), Eq. (8a) reduces to

$$0 = \left[\mu - m\left(J_1(\hat{n}^{1/2})/\hat{n}^{1/2} - \tfrac{1}{2}\right) - m\phi \right]\psi, \qquad (9)$$

where $J_1(z)$ is a Bessel function. This equation either requires $\psi = 0$ or it determines ϕ in terms of ψ. Inserting the expression for ϕ into Eq. (8b) gives a self-contained second-order differential equation for ψ. Given the central density n_0 and the boundary condition $\psi'(0) = 0$, that differential equation can be integrated to obtain $\psi(r)$ as a function of r. If $\psi(r)$ does not cross zero, there is no solution to Eqs. (8) for that central density. If $\psi(r)$ crosses 0 at some radius R_{TF}, there is a solution. The Thomas-Fermi approximation is $\psi(r)$ for $r < R_{\text{TF}}$ and 0 for $r > R_{\text{TF}}$. We refer to R_{TF} as the *Thomas-Fermi radius*. This approximation is continuous, but its first derivative has a discontinuity at $r = R_{\text{TF}}$. In Fig. 1, the Thomas-Fermi approximation has been used to extend the branch of dense axion stars up to a mass of $1.9\,M_\odot$, beyond which there is no solution within this approximation. The relation between R_{TF} and M is independent of m for fixed mf. It is accurately approximated by a power law: $R_{\text{TF}}/R_\odot = 2.1 \times 10^{-5}(M/M_\odot)^{0.305}$. The Thomas-Fermi approximation has solutions for arbitrarily small M. Its extrapolation to M below its range of validity is shown in Fig. 1. The region where R is smaller than the Schwarzchild radius $2GM/c^2$ of the axion star is shown in Fig. 1 as a shaded region. Since R_{TF} is much larger than $2GM/c^2$ over most of the range of a dense axion star, Newtonian gravity is a good approximation.

Other branches. The existence of dense axion star solutions can be understood by considering the balance of forces in the Thomas-Fermi approximation. The forces are the negatives of the gradients of the energies inside the square brackets in Eq. (8a). The condition for the balance of forces is

$$\mathcal{V}''_{\text{eff}}(\psi^*\psi)\frac{d}{dr}(\psi^*\psi) + m\frac{d}{dr}\phi = 0. \qquad (10)$$

If $\psi^*\psi$ is a positive decreasing function of r and ϕ is a negative increasing function of r, the balance of forces requires $\mathcal{V}''_{\text{eff}}$ to be positive. In a region of r where $\mathcal{V}''_{\text{eff}} > 0$, the repulsive mean-field pressure of the axion BEC can balance the attractive gravitational force. In a region of r where $\mathcal{V}''_{\text{eff}} < 0$, the mean-field pressure is attractive and the two attractive forces must be balanced by the kinetic pressure of the axions. In Fig. 2, $\mathcal{V}''_{\text{eff}}$ is shown as
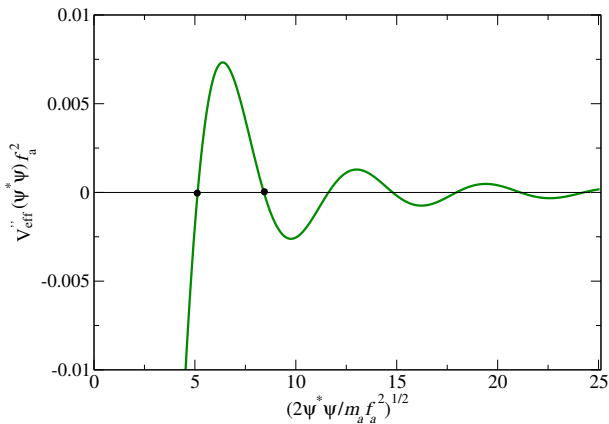
FIG. 2: Second derivative of the effective potential with respect to its argument $\psi^*\psi$ as a function of $|\psi|$. The left and right dots mark values of $|\psi|$ near those at the center of the dense axion star with the smallest mass and the largest mass, respectively.

a function of the dimensionless number density variable $\hat{n}^{1/2}$. The second derivative is positive between the two dots where $\hat{n}^{1/2}$ is 5.14 and 8.42. For most of the range of stable dense axion stars in Fig. 1, $\hat{n}_0^{1/2}$ is in this range. As r increases, $\hat{n}^{1/2}$ decreases steadily, with the repulsive mean-field pressure balancing gravity, until it approaches 5.14. As r continues to increase, a large kinetic pressure is required to balance gravity and the attractive mean-field pressure. The large gradient of ψ results in a rapid decrease of ψ to zero.

In Fig. 2, there are other intervals of $\hat{n}^{1/2}$ in which $\mathcal{V}''_{\mathrm{eff}}$ is positive. For each of these intervals, there may be another branch of stable dense axion stars in which $\hat{n}_0^{1/2}$ is in this interval. As r increases from 0 to ∞, $\hat{n}^{1/2}$ must decrease to 0. It can decrease steadily with r through the values of $\hat{n}^{1/2}$ for which $\mathcal{V}''_{\mathrm{eff}} > 0$, but $\psi(r)$ must decrease precipitously with r through the values of $\hat{n}^{1/2}$ for which $\mathcal{V}''_{\mathrm{eff}} < 0$. The Thomas-Fermi approximation is not applicable, because the kinetic pressure is important in regions where $\mathcal{V}''_{\mathrm{eff}} < 0$.

Discussion. In our solutions for dense axion stars, we used the simple model in Eq. (2) for the relativistic axion potential $\mathcal{V}$ and we approximated the effective potential $\mathcal{V}_{\mathrm{eff}}$ of axion EFT by Eq. (5). To obtain more accurate solutions, it would be necessary to use a more accurate relativistic axion potential [6]. It would also be important to develop a systematically improvable approximation to the effective potential $\mathcal{V}_{\mathrm{eff}}$ of axion EFT [5]. A more accurate approximation for $\mathcal{V}_{\mathrm{eff}}$ would certainly change the quantitative properties of dense axion stars, and it could even affect their existence.

An important issue in axion dark matter is whether the BEC can be coherent over galactic and cosmological distances, as argued by Sikivie and collaborators [2, 3]. Guth, Hertzberg, and Prescod-Weinstein have recently argued that the attractive interaction between a pair of axions prevents the development of long-range correlations in the axion BEC [17]. This suggests that the largest possible coherence length is roughly the radius $10^{-4}R_\odot$ of a dilute axion star with the critical mass. Since dense axion stars have smaller radii, their existence does not modify the conclusions of Ref. [17].

Dense axion stars with sufficiently large masses could be observed through gravitational microlensing. The possibility that most of the dark matter in the Milky Way is made up of massive compact halo objects (such as axion stars) with masses in the range between $2 \times 10^{-9}M_\odot$ and $15M_\odot$ has been excluded [18, 19]. Thus most of axion dark matter must be in the form of gases of axions or dilute axion stars or dense axion stars with mass less than $2 \times 10^{-9}M_\odot$. To obtain quantitative constraints on axions from microlensing, it would be necessary to know the fraction of axions that are bound in axion stars as well as the mass distributions of dilute axion stars and of dense axion stars.

An interesting issue is the ultimate fate of a dilute axion star with the critical mass M_* that accretes additional axions and begins to collapse. Before this work, the most likely possibility for the remnant after the collapse seemed to be a black hole. The alternative possibility was a dilute axion star with a smaller mass and a larger radius, but it is hard to imagine a plausible path to that large dilute axion star from the tiny collapsing axion star. Solutions for stable dense axion stars suggest an alternative possibility for the remnant after the collapse: a dense axion star with mass smaller but of order M_*. The decrease in radius by more than 5 orders of magnitude during the collapse could amplify inelastic axion reactions, such as the scattering of 4 axions from the BEC into 2 relativistic axions, producing a bosenova [4]. The relaxation to a static dense axion star would also involve the radiation of axion waves. It should be possible to determine the fate of a collapsing dilute axion star definitively by solving the time-dependent classical field equations of axion EFT.

If the dark matter consists of axions, the formation of dense axion stars in the early universe may be inevitable. Spacial fluctuations in the vacuum misalignment of the axion field naturally produce gravitationally bound miniclusters of axions with masses comparable to the critical mass M_* of a dilute axion star [20, 21]. Gravitational thermalization will drive a minicluster towards an axion star and ensure that the axion star remains a BEC as it accretes more axions. If accretion of axions increases the mass of the dilute axion star to above M_*, it may collapse into a dense axion star with a smaller mass of order M_*. The resulting mass distribution of dense axion stars could be modified by the subsequent accretion of additional axions. The existence of dense axion stars would open up many new possibilities in the search for axion dark matter.

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[*] Electronic address: braaten@mps.ohio-state.edu

[†] Electronic address: mohapatra.16@buckeyemail.osu.edu

[‡] Electronic address: zhang.5676@osu.edu

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