Evidence for Neutral-Current Diffractive π^{0} Production from Hydrogen in Neutrino Interactions on Hydrocarbon

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Abstract

The MINERvA experiment observes an excess of events containing electromagnetic showers relative to the expectation from Monte Carlo simulations in neutral-current neutrino interactions with mean beam energy of 4.5 GeV on a hydrocarbon target. The excess is characterized and found to be consistent with neutral-current π^0 production with a broad energy distribution peaking at 7 GeV and a total cross section of $0.26 \pm 0.02(stat) \pm 0.08(sys) \times 10^{-39}$ cm^2. The angular distribution, electromagnetic shower energy, and spatial distribution of the energy depositions of the excess are consistent with expectations from neutrino neutral-current diffractive π^0 production from hydrogen in the hydrocarbon target. These data comprise the first direct experimental observation and constraint for a reaction that poses an important background process in neutrino oscillation experiments searching for ν_μ to ν_e oscillations.

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I. INTRODUCTION

Current and future accelerator-based neutrino-oscillation experiments aim to make high precision measurements of oscillation parameters by examining the ν_e and $\overline{\nu}_e$ content of their beams as a function of neutrino energy in the sub-GeV to few-GeV range[1–5]. The signature of a $\nu_e(\overline{\nu}_e)$ charged-current (CC) interaction, the signal in such experiments, is the presence of an $e^-(e^+)$ in the final state that originates from the neutrino interaction vertex. In order to extract the desired parameters, it is necessary to compare the observed signal to a simulation containing all processes that can produce a real single $e^-(e^+)$ in the final state as well as processes that can mimic this signature. Precise estimates of the parameters therefore require accurate and complete models of all potential background processes. Consequently, it is important to characterize and understand any observations of neutrino-induced events in the sub-GeV to many-GeV range that contain electromagnetic showers.

In a separate paper, the MINERvA collaboration reported a measurement of ν_e CC quasielastic and quasielastic-like scattering in the NuMI beam at an average neutrino energy of 3.6 GeV [6]. During the data analysis leading to those results, an unexpectedly large number of events was observed containing electromagnetic showers likely caused by photon conversions. In this Letter, this excess of events is measured relative to the expectation based on a sample of simulated data produced using current state-of-the-art models of neutrino production and interactions. These events are seen to exhibit features expected of so-called neutral current (NC) "diffractive" π^0 production from hydrogen in the hydrocarbon target, thus named because it results from the coherence of particle wavefunctions in analogy with classical diffraction resulting from coherent electromagnetic waves.[7]

These results constitute the first direct experimental observation and characterization of this process. An analogous process that happens exclusively on nuclei heavier than hydrogen, NC coherent π^0 production, has been observed previously[8–15]; however, the contribution from NC diffractive scattering from hydrogen, when present in the target, has been considered only inclusively with the scattering from the heavier nuclei and not examined separately as is done here. This measurement offers an experimental constraint on models of NC diffractive π^0 production and the A-dependence of coherent scattering. It is of general interest in neutrino physics and of particular importance for oscillation experiments using detectors containing water or hydrocarbons or any other material containing hydrogen.

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II. THE MINERVA EXPERIMENT

The MINERvA experiment studies neutrinos produced in the NuMI beamline at Fermi National Accelerator Laboratory. This analysis uses data taken between March 2010 and April 2012 with 3.49×10^{20} protons on target (POT).[16] During this period, the beam consisted predominantly of ν_μ with a peak energy of $3.15\,\mathrm{GeV}$ and a high-energy tail extending up to tens of GeV such that the mean neutrino energy was $4.5\,\mathrm{GeV}$. ν_e and $\overline{\nu}_e$ made up approximately 1.6% of the total neutrino flux. The neutrino beam simulation used by MINERvA is described in Ref. [6] and references therein.

The MINERvA detector [17, 18] consists of a core of scintillator strips arranged in planes and oriented in three views for three-dimensional tracking. The triangular strips (3.4 cm base $\times$ 1.7 cm height) making up the sensitive portion of the detectors are sufficiently fine-grained to ensure reliable detection and characterization of electromagnetic showers at energies of above roughly $0.5\,\mathrm{GeV}$. The scintillator core is augmented by electromagnetic and hadronic calorimetry on both the sides and the downstream end of the detector. The MINERvA detector's response is simulated by tuned GEANT4-based[19, 20] software. The energy scale is set by requiring that the photostatistics and reconstructed energy for energy deposited by momentum-analyzed muons traversing the detector agree in data and simulation. Additional algorithm-specific tuning, including corrections for passive material, is done using the simulation [17].

Simulated neutrino interactions, generated with the GENIE 2.6.2 neutrino event generator [21], are used for comparison to the data and efficiency corrections. Of particular interest in this Letter are processes that contain electromagnetic showers. The dominant source of electromagnetic showers in these neutrino interactions is neutral pion production, which is modeled in the generator via resonant production from nucleons according to the Rein-Sehgal model; via coherent interactions with nuclei according to the PCAC formalism of Rein and Sehgal [23]; and via the hadronization model in non-resonant inelastic production. Further details on other processes simulated by the generator, as well as the external data sets used for tuning the generator, are described briefly in Ref. [6] and references therein.

III. EVENT RECONSTRUCTION AND ANALYSIS

Events of interest were selected as part of the ν_e CC quasielastic scattering analysis[6][22]. Candidate events are created from reconstructed tracks originating in the central scintillator region of the MINERvA detector[17]. To remove the overwhelming background from ν_μ CC events, tracks are not considered if they exit the back of the detector as muons are expected to do. Candidate electromagnetic showers are identified by examining energy depositions within a region that consists of the union of two volumes: a cylinder of radius 50 mm extending from the event vertex along the track direction and a 7.5° cone with an apex at the event vertex (origin of track) and a symmetry axis along the track direction. The full region (referred to below as the 'shower cone') extends in length through the scintillator and electromagnetic calorimeter portions of MINERvA until it reaches a gap of approximately three radiation lengths along the cone where no significant energy is deposited. This shower cone object is evaluated using a multivariate particle identification (PID) algorithm which combines details of the energy deposition pattern both longitudinally (mean dE/dx and the fraction of energy at the downstream end of cone) and transverse to the axis of the cone (mean shower width) using a k-nearest-neighbors (kNN) algorithm[24].

For events deemed by the PID algorithm to be electromagnetic-like, the dE/dx at the front of the shower cone is examined to see if it is more consistent with a single particle, such as that expected from an electron (or positron), or two particles, as would be seen in a photon pair conversion into e^+e^-. Here, the energy in the dE/dx measure is taken to be the minimum energy contained in a 100 mm window along the shower, where the downstream end of the window is allowed to slide up to 500 mm from the vertex. This sliding window technique reduces any potential bias induced by nuclear activity near the interaction point[25]. Figure 1 shows the minimum dE/dx during this process for both the data and simulation. For comparison, the inset of Fig. 1 shows the same variable for simulated samples of single photons or electrons, chosen with a flat energy distribution in the range from 0.5 to 5.0 GeV. Electron showers tend to lie in an interval between 1 and 2 MeV/cm, while the photons populate a somewhat wider range peaking at 3 MeV/cm. The MINERvA modeling of photons and electrons was validated against the data successfully with samples of separated π^0 conversion photons and Michel electrons[17].

The electron region of Fig. 1, peaking at approximately 1.3 MeV/cm in both the data

and the simulation, is well-modeled; in both shape and magnitude, the data and simulation differ by less than 10%. However, the photon peak in the data contains an excess relative to the prediction with 12.5σ statistical-only significance. Systematic uncertainties, particularly those associated with the flux model and the estimate of the other processes predicted in that region, reduce the significance to 3.1σ. (The overall flux prediction and uncertainties, as well as the normalization of the background processes and corresponding uncertainties for the simulation shown in Fig. 1, were constrained by *in situ* measurements in dedicated samples. Both of these and other systematic errors are described in detail in Reference [6].)

Since distributions made using a sideband sample of ν_e events containing Michel electrons agree very well with the simulation, the excess of data events is unlikely to have arisen from the misreconstruction of electrons or errors in the modeling of electromagnetic showers in the simulation. In addition, Fig. 1 shows that the excess is not compatible with an overall normalization offset of the sample. The possibility of the excess arising from mismodeled non-shower activity near the event vertex (i.e., nucleons) was examined by injecting extra protons into simulated electron showers in a fashion consistent with the findings in recent MINERvA muon neutrino scattering results [25] (uniformly from 0-225 MeV in 25% of the simulated showers). These samples did not exhibit an excess in the photon region of the reconstructed dE/dx distribution. Moreover, as will be shown in the following sections, the excess events in the photon region are qualitatively different than any of the event types predicted by the simulation under the photon peak.

IV. CHARACTERIZATION OF THE EXCESS

In order to characterize the excess in Fig. 1, events exhibiting minimum dE/dx between 2.2 and 3.4 MeV/cm were selected in both the data and the simulation. Kinematic distributions of the candidate EM shower in these events were examined after subtracting the simulation from the data bin-by-bin, corresponding to a population of 546 candidates above the prediction. Distributions made in this fashion provide a picture of what is missing in the simulation and thereby characterize the excess.

The excess was compared to single-particle samples of photons and π^0's which were simulated with broad distributions in energy (0-20 GeV) and angle with respect to the longitudinal detector axis (0-$\pi/2$) and processed using the MINERvA reconstruction. A
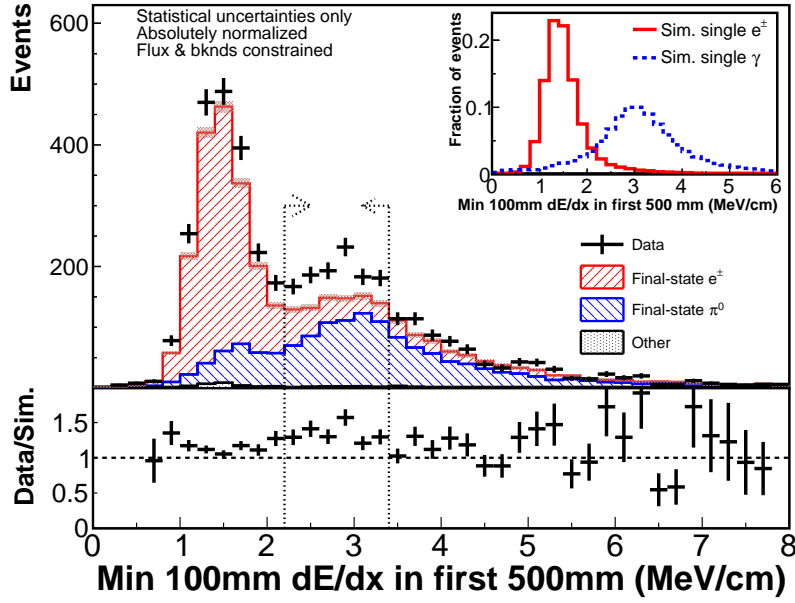
FIG. 1. Measure of the minimum dE/dx near the front of candidate electromagnetic showers for data (crosses) and the simulated neutrino event sample (solid). Simulated events are divided according to the progenitor of the electromagnetic shower. The dashed lines and arrows delineate the excess region discussed in the text. Shown at the bottom is the ratio of data to simulation. The inset shows the same distribution for simulated single-particle samples of electrons and photons.

similar sample of η's was also constructed to investigate the possibility of a heavier state decaying into showering particles. In each of these samples, the events falling into the region of the photon-like excess in dE/dx were generated to have the same two dimensional distribution of energy and angle as in the data excess. Figure 2(a) shows a shape comparison for the "extra energy ratio" variable Ψ, which represents the relative amount of energy outside the cone to that inside the cone, for these single-particle samples compared to the distribution of the excess in the data. Energy depositions within $30\,\mathrm{cm}$ of the interaction vertex were ignored when calculating Ψ to reduce the contribution from low-energy nucleons, which may not be simulated correctly [25]. Here, the data are more consistent with photon or π^0 production than η production. On the other hand, Fig. 2(b) shows the median transverse width of the energy depositions in the cone object ("median shower width") for the single-particle samples and the excess in the data. In this case, the data are less consistent with
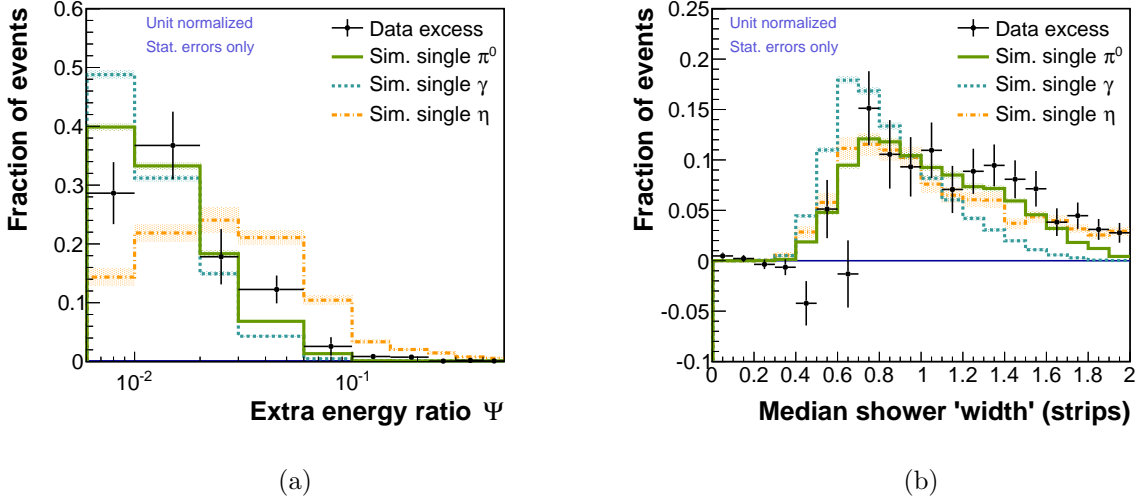
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FIG. 2. Left: ratio of energy outside the shower cone to that inside the shower cone for the data excess (points) compared via shape to samples (histograms) created using different single-particle simulations, weighted to have kinematics similar to the excess events. Right: Distributions of median transverse width of the EM shower for the same samples. Uncertainties are statistical only.

the behavior expected from a single particle than with that from a particle decaying into multiple photons. These single-particle studies along with the Michel and injected proton studies mentioned above, suggest that the showers in the excess are most likely caused by photons from π^0 production and subsequent decay.

The lack of a muon and the fact that the shower exhibits photon-like, rather than electron-like, energy deposition behavior together imply that the process contributing to the excess is a neutral-current (NC) interaction. Other features of the sample can be examined to provide further insight into the nature of the interaction. Figure 3 shows shape comparisons of GENIE NC coherent and incoherent π^0 production with data distributions from the excess in several variables, where the content of each curve is normalized to unity. Figure 3(a) gives the reconstructed energy the electromagnetic shower, E_{shower}, where it can be seen that the data excess has a harder energy spectrum than the NC processes predicted by the model. However, the angular distribution of the shower in the data agrees very well in shape with the expectation from GENIE for NC coherent π^0 production, as demonstrated in the $E_{\text{shower}}\theta^2$ distribution (Fig. 3(b)). The same is true in Ψ, as illustrated in Fig. 3(c), as most of the events have relatively little energy outside the cone. However, the distribution of energy within a cone identical to the one described in Sec. III, except oriented backward along the
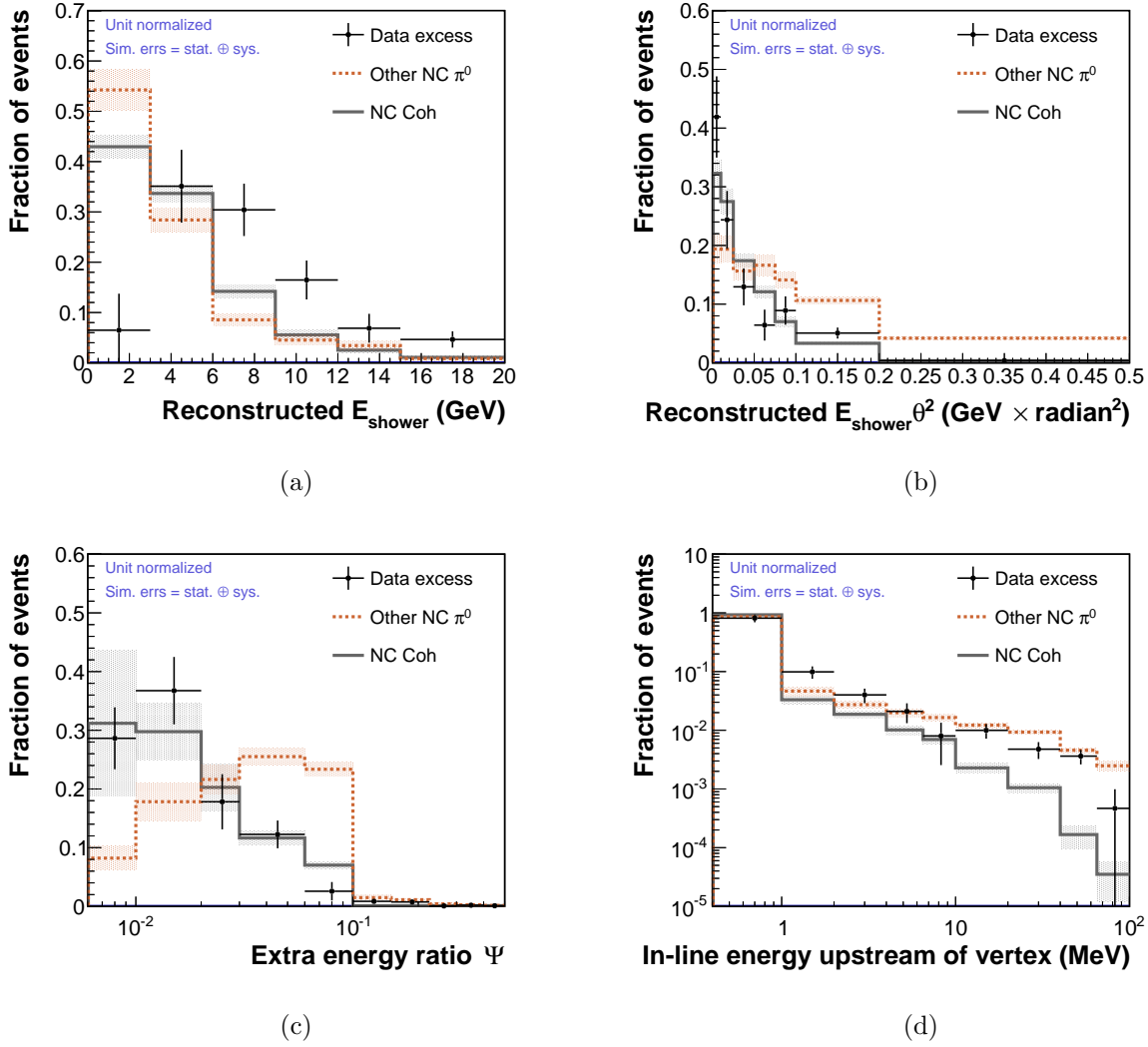
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FIG. 3. The data excess (points) as compared (via shape) to GENIE samples of NC coherent and incoherent π^0 production. The comparisons are made as a function of E_{shower} (upper left), $E_{\mathrm{shower}}\theta^2$ (upper right), Ψ (lower left), and in-line upstream energy (lower right). Data uncertainties are statistcal only; predictions include systematic uncertaintes added in quadrature with statistical.

original cone axis, is different. In this case, illustrated in Fig. 3(d), the data appear to have more in-line upstream energy than the NC coherent process and are more consistent with the NC incoherent process, suggesting a small nuclear recoil from the neutrino interaction. Corroborating this hypothesis, the charge-weighted distance from that energy to the shower vertex was examined in the data sample and seen to fit the exponential decay distance expected for a photon conversion after propagating through the detector from the interaction point defined by the upstream activity.

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The results described above were supplemented by a visual scan of event displays for data in the excess region and a simulated neutrino event sample, as well as simulated single-particle samples. The conclusions from the scan were that the data in this region, relative to the simulated sample, contains a higher fraction of events with a π^0 and more events with in-line upstream energy.

Finally, the difference between the data and the expectation from GENIE between 2.2 and 3.4 MeV/cm in Fig. 1 was used to extract a total cross section for $E_{\mathrm{shower}} > 3\,GeV$ integrated over the MINERvA flux of $0.26 \pm 0.02(stat) \pm 0.08(sys) \times 10^{-39} cm^2/CH$. The phase space for this measurement was limited in E_{shower} to avoid model dependence by ensuring the value reported is in a region where MINERvA has good sensitivity.

V.   DIFFRACTIVE π^0 PRODUCTION

The most plausible source of the excess seen in the data is diffractive NC π^0 production from hydrogen in the scintillator target of MINERvA. Because little momentum is tranferred to the nucleus, this process is expected to be characterized by coherent-like kinematics; but the comparatively small mass of the hydrogen nucleus would result in the proton sometimes being endowed with sufficient kinetic energy to manifest as in-line upstream energy in this analysis. In addition, NC diffractive scattering from hydrogen is not included in the GENIE simulation used by MINERvA.

Though neutral-current excitation of a Δ^+ from a proton within a nucleus produces the same final state after the decay $\Delta^+ \to p + \pi^0$, the latter process is characterized by a strong peak around $1.2\,GeV$ in the invariant mass spectrum of the events. The invariant mass distribution for the excess was computed, using the upstream inline energy distribution to form a rough estimate for the proton kinetic energy, and was found to occupy a broad W spectrum peaking at about $3.5\,GeV$ with FWHM of about $3\,GeV$. Thus a deficiency in the resonant production model in GENIE, which simulates this process, is unlikely to be responsible for the excess, and leaves diffractive scattering as the best hypothesis.

To further test the hypothesis that the observed signal arises from diffractive NC π^0 production, comparisons were made to an early implementation in GENIE of a calculation of the diffractive process based on the work of Rein [26] that is valid for W>2.0 GeV. This model produces events with a similar cross section to the value observed by MINERvA for

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the excess and it contributes events in the region of the excess and very little outside that region. The model qualitatively agrees with the characteristics of the excess in terms of the shower angle, extra energy ratio and in-line upstream energy (Figs. 3(b), 3(c), and 3(d), respectively), but exhibits a somewhat different shape in terms of the energy spectrum of the produced shower. Further details of the comparison of the observed excess and the Rein model can be found in Ref. [22].

VI. CONCLUSIONS

An excess of events containing electromagnetic showers observed by the MINERvA experiment appears to originate from the neutral-current production of neutral pions in a process not predicted by the GENIE neutrino interaction simulation program. Interpretations of the excess as arising from errors in the flux or background predictions, or mismodeling of the electromagnetic shower process, are disfavored based on *in situ* sideband constraints. The observed process resembles coherent production apart from the existence of a small amount of upstream energy, implying that the events likely arise from diffractive pion production from hydrogen. The measured cross section for this process for $E_\pi > 3 \ GeV$, assuming the observed shower to come from photon conversions from the π^0, is comparable to that for NC coherent π^0 production from carbon. These measurements, interpreted as NC diffractive scattering, constitute the first direct experimental observation and characterization of this process. Neutrino oscillation experiments with hydrogen in their targets must account for NC diffractive scattering in order to correctly model backgrounds to ν_e appearance. The data presented above will play an essential role in constraining models for diffractive production, such as the model in Ref. [26]. But because the latter applies only at larger W, this work also highlights the need for models of diffractive scattering which extend to low W and E_π to be developed and incorporated in simulations. Furthermore, these results are useful for understanding the A-dependence of coherent scattering which is important to all oscillation experiments.

ACKNOWLEDGMENTS

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[1] D. S. Ayres *et al.* (NOvA Collaboration), *The NOvA Technical Design Report*, FERMILAB-DESIGN-2007-01 (2007).

[2] K. Abe *et al.* (T2K Collaboration), Nucl. Instru. Methods **659**, 106 (2011), arXiv:1106.1238.

[3] http://www.dunescience.org/.

[4] http://sbn.fnal.gov/.

[5] http://www.hyperk.org/.

[6] J. Wolcott *et al.* (MINERvA Collaboration), Phys. Rev. Lett. **116**, 081802 (2016), arXiv:1509.05729.

[7] G. Alberi and G. Goggi, Phys. Rept. **74**, 1 (1981).

[8] H. Faissner *et al.*, Phys. Lett. **125**B, 230 (1983).

[9] E. Isiksal, D. Rein, J. Morfin, Phys. Rev. Lett. **52**, 1096 (1984).

[10] F. Bergsma *et al.*, Phys. Lett. **157**B, 469 (1985).

[11] H.J. Grabosch *et al.*, Z. Phys. **C31**, 203 (1986).

[12] C. Baltay *et al.*, Phys. Rev. Lett. **57**, 2629 (1986).

[13] A.A. Aguilar-Arevalo *et al.* (MiniBooNE collaboration), Phys. Lett. **B664**, 41 (2008).

[14] C.T. Kullenberg *et al.* (NOMAD collaboration), Phys. Lett. **B682**, 177 (2009).

[15] Y. Kurimoto *et al.* (SciBooNE collaboration), Phys. Rev. D**81**, 111102 (2010).

[16] P. Adamson *et al.*, Nucl. Instr. Meth. Phys. Res. A **806**, 279 (2016).

[17] L. Aliaga *et al.* (MINERvA Collaboration), Nucl. Instru. Methods A**743** 130 (2013).

[18] G.N. Perdue *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **694**, 179 (2012).

[19] S. Agostinelli *et al.*, Nucl. Instru. Methods A**506**, 250 (2003).

[20] J. Allison *et al.*, IEEE Trans. Nucl. Sci. **53**, 270 (2006). Program version 4.9.4p02 with `QGSP_BERT` physics list used here.

[21] C. Andreopoulos, A. Bell, D. Bhattacharya, F. Cavanna, J. Dobson, S. Dytman, H. Gallagher, P Guzowski, R. Hatcher, P. Kehayias, A. Meregaglia, D. Naples, G. Pearce, A. Rubbia, M. Whalley, And T. Yang, Nucl Instrum. Methods **A614**, 87 (2010), program version 2.6.2.

[22] J. Wolcott, Ph.D. Thesis, University of Rochester, 2015, FERMILAB-THESIS-2015-26.

[23] D. Rein and L. M. Sehgal, Nucl.Phys. **B223**, 29 (1983).

[24] T. Hastie, R. Tibshirani, and J. Friedman, **The elements of statistical learning**, Springer, New York, second edition, 2009.

[25] G. A. Fiorentini *et al.* (MINERvA Collaboration), Phys. Rev. Lett. **111**, 022502 (2013).

[26] D. Rein, Nucl. Phys. **B278**, 61 (1986).