Lensing of Fast Radio Bursts as a Probe of Compact Dark Matter

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The possibility that part of the dark matter is made of massive compact halo objects (MACHOs) remains poorly constrained over a wide range of masses, and especially in the $20 - 100 \, M_\odot$ window. We show that strong gravitational lensing of extragalactic fast radio bursts (FRBs) by MACHOs of masses larger than $\sim 20 \, M_\odot$ would result in repeated FRBs with an observable time delay. Strong lensing of an FRB by a lens of mass M_L induces two images, separated by a typical time delay $\sim$ few $\times (M_L/30 \, M_\odot)$ milliseconds. Considering the expected FRB detection rate by upcoming experiments, such as CHIME, of 10^4 FRBs per year, we should observe from tens to hundreds of repeated bursts yearly, if MACHOs in this window make up all the dark matter. A null search for echoes with just 10^4 FRBs, would constrain the fraction $f_{\rm DM}$ of dark matter in MACHOs to $f_{\rm DM} \lesssim 0.08$ for $M_L \gtrsim 20 \, M_\odot$.

Although observations indicate that dark matter accounts for a significant share of the energy density of our Universe [1], we do not know its composition. A long-time candidate to make up the dark matter are massive compact halo objects (MACHOs) [2]. They were originally proposed to be as light as $10^{-7} M_\odot$ and as heavy as the first stars ($\sim 10^3 M_\odot$) [3]. Over the years, different experiments have progressively constrained the fraction $f_{\rm DM}$ of dark matter that can reside in MACHOs with a given mass, placing tight upper bounds over most of the vast range above. High-mass ($\gtrsim 100 \, M_\odot$) MACHOs, for example, are constrained by the fact that they would perturb wide stellar binaries in our Galaxy [4]. Meanwhile, lower-mass ($\lesssim 20 M_\odot$) MACHOs are effectively ruled out as the sole component of Galactic dark matter, as they would create artificial variability in stars, due to gravitational microlensing [5–7].

However, there remains a window of masses between 20 and $100 \, M_\odot$, where the constraints are weaker, and in which arguably all the cosmological dark matter could be in the form of MACHOs [6–10]. This is a particularly interesting window, as it has been recently argued in Ref. [11] that if primordial black holes (PBHs) [12, 13] in the $\sim 30 \, M_\odot$ mass range are the constituents of dark matter, they form binaries in halos, coalesce, and emit observable gravitational waves, with an event rate consistent with the published LIGO detection [14].

In this *Letter* we propose to use strong lensing of fast radio bursts (FRBs) to probe MACHOs of masses $\gtrsim 20 \, M_\odot$, including PBHs, and either confirm that they make up the dark matter, or close this window. FRBs are strong radio bursts with a very short duration, which makes them ideal as microlensing targets. Their temporal width is increased by the dispersion measure (DM), which measures the time delay of photons with different radio frequencies due to scattering by free electrons on their way to Earth. All detected FRBs to date possess high DMs, which yield burst widths of $\sim 1 - 10$ ms

[15–24]. These values of the DM are several times larger than the expected contribution from free electrons within the Milky Way (MW) [25, 26], suggesting their origin is extragalactic (some authors, however, prefer a Galactic origin [27, 28]). Proposed sources of extragalactic FRBs include merging neutron stars [29] or white dwarfs [30], as well as bursts from pulsars [31].

Strong lensing of an FRB by a MACHO will generate two images of the burst. While their angular separation may be too small to be resolved, the time delay between them, on the order of milliseconds for a MACHO lens with mass $M_L \sim 20 - 100 \, M_\odot$, might be large enough to enable a detection of two separate peaks, rather than one, if the time delay is bigger than the pulse width. Fortunately, lensing of FRBs by compact objects is not necessarily an unlikely occurrence. In fact, if all the dark matter is in MACHOs, roughly 1 in 50 FRBs originating at $z = 0.5$ should be lensed. If there are $\sim 10^4$ FRBs on the full sky each day [32], then as many as ~ 20 microlensed FRBs may be reaching Earth daily. Upcoming surveys, like APERTIF [33], UTMOST [34], HIRAX [35], or CHIME [36], which will map a considerable fraction of the sky, may thus see a significant number of lensed FRBs.

Below we calculate the effects of microlensing on a given FRB, and compute the optical depth for strong lensing by compact objects. We then combine those results with different redshift distributions of FRBs, and estimate how many lensed bursts are expected if MACHOs make up all the dark matter. We also estimate the smallest fraction $f_{\rm DM}$ that will give rise to a detectable rate of microlensed events.

A MACHO of mass M_L can be treated as a point lens with an (angular) Einstein radius,

$$\theta_E = 2\sqrt{\frac{G M_L}{c^2} \frac{D_{LS}}{D_S D_L}}, \qquad (1)$$

where D_S, D_L, and D_{LS} are the (angular-diameter) dis-

tances to the source, to the lens, and between the source and the lens [37, 38]. A point lens produces two images, at positions $\theta_\pm = (\beta \pm \sqrt{\beta^2 + 4\theta_E^2})/2$, where β is the (angular) impact parameter. The time delay between these two images is

$$\Delta t = \frac{4\,GM_L}{c^3}(1+z_L)\left[\frac{y}{2}\sqrt{y^2+4} + \log\left(\frac{\sqrt{y^2+4}+y}{\sqrt{y^2+4}-y}\right)\right],\tag{2}$$

where $y \equiv \beta/\theta_E$ is the normalized impact parameter, and z_L is the redshift of the lens. We also define the flux ratio R_f as the absolute value of the ratio of the magnifications μ_+ and μ_- of both images; i.e.,

$$R_f \equiv \left|\frac{\mu_+}{\mu_-}\right| = \frac{y^2 + 2 + y\sqrt{y^2+4}}{y^2 + 2 - y\sqrt{y^2+4}} > 1.\tag{3}$$

To claim that an FRB is strongly lensed we will require three conditions. First, that the brighter image has a signal-to-noise ratio of 10 [19]. Secondly, that the observed time delay is larger than some reference time $\overline{\Delta t}$, which will place a lower bound on the impact parameter $y > y_{\min}(M_L, z_L)$, calculated via Eq. (2). Finally, we demand that the flux ratio R_f is smaller than some critical $\overline{R}_f$ (which we take to be redshift-independent), to ensure that both events are observed (note that for the fainter image the look-elsewhere effect is no longer relevant). This forces the impact parameter to be smaller than $y_{\max} = \left[(1+\overline{R}_f)/\sqrt{\overline{R}_f} - 2\right]^{1/2}$.

We now calculate the probability for an FRB to be lensed. The lensing optical depth of a source at redshift z_S is given by

$$\tau(M_L, z_S) = \int_0^{z_S} d\chi(z_L)(1+z_L)^2 n_L\,\sigma(M_L, z_L),\tag{4}$$

where $\chi(z)$ is the comoving distance at redshift z, n_L is the comoving number density of lenses, and σ is the lensing cross section of a point lens of mass M_L, given by an annulus between the maximum and minimum impact parameters by

$$\sigma(M_L, z_L) = \frac{4\pi GM_L}{c^2}\frac{D_L D_{LS}}{D_S}\left[y_{\max}^2 - y_{\min}^2(M_L, z_L)\right].\tag{5}$$

Eq. (4) can be recast by using the Hubble parameter both at the redshift of the lens, $H(z_L)$, and today, H_0, as

$$\tau(M_L, z_S) = \frac{3}{2}f_{\rm DM}\Omega_c \int_0^{z_S} dz_L \frac{H_0^2}{c\,H(z_L)}\frac{D_L D_{LS}}{D_S}$$
$$\times (1+z_L)^2\left[y_{\max}^2 - y_{\min}^2(M_L, z_L)\right],\tag{6}$$

where $\Omega_c = 0.24$ is the cold-dark-matter density today, and the only remaining dependence on the lens mass M_L is through $y_{\min}$. Lower MACHO masses result in a lower

optical depth, especially at lower source redshifts, due to our minimum time-delay requirement.

To calculate the integrated lensing probability, the optical depth for lensing of a single burst has to be convolved with the redshift distribution of incoming FRBs. We will consider two possible redshift distributions. First, we assume FRBs have a constant comoving number density, in which case the number of FRBs in a shell of width dz at redshift z is proportional to the shell's comoving volume $dV(z) = \left[4\pi\chi^2(z)/H(z)\right]dz$ [39], divided by $(1+z)$ to account for the effect of cosmological time dilation in the rate of bursts. To represent an instrumental signal-to-noise threshold we introduce a Gaussian cutoff at some redshift $z_{\rm cut}$, so the constant-density redshift distribution function would be

$$N_{\rm const}(z) = \mathcal{N}_{\rm const}\frac{\chi^2(z)}{H(z)(1+z)}e^{-d_L^2(z)/[2d_L^2(z_{\rm cut})]},\tag{7}$$

where d_L is the luminosity distance, and $\mathcal{N}_{\rm const}$ is a normalization factor to ensure that $N_{\rm const}(z)$ integrates to unity. Secondly, we consider a scenario in which FRBs follow the star-formation history (SFH) [40], whose density is parametrized as

$$\dot{\rho}_*(z) = h\frac{a + bz}{1 + \left(\frac{z}{c}\right)^d},\tag{8}$$

with $a = 0.0170$, $b = 0.13$, $c = 3.3$, $d = 5.3$, and $h = 0.7$ [41, 42]. In this case, the SFH-based redshift distribution function $N_{\rm SFH}(z)$ is,

$$N_{\rm SFH}(z) = \mathcal{N}_{\rm SFH}\frac{\dot{\rho}_*(z)\,\chi^2(z)}{H(z)(1+z)}e^{-d_L^2(z)/[2d_L^2(z_{\rm cut})]},\tag{9}$$

and the normalization factor $\mathcal{N}_{\rm SFH}$ is chosen to have $N_{\rm SFH}$ integrate to unity.

In Figure 1 we plot a histogram of the estimated redshifts for the current FRB catalog [23], which is well fit by the two FRB distribution functions above, if a cutoff of $z_{\rm cut} = 0.5$ is chosen.

To estimate the total number of FRBs observable in the near future, we consider an experiment like CHIME [36]. In Ref. [32] it was estimated that CHIME will detect $\sim 730 - 15000$ FRBs per year, and so we will take a fiducial, albeit optimistic, value of $N_{\rm FRB} = 10^4$ bursts per year.

Inter-channel dispersion broadens the FRB pulse arrival time to

$$\delta t_{\rm DM} = 0.3\,{\rm ms} \times \frac{\rm DM}{800\,{\rm pc\,cm^{-3}}}\frac{\Delta\nu}{24\,{\rm kHz}}\left(\frac{800\,{\rm MHz}}{\nu}\right)^3,\tag{10}$$

where ν is the frequency, $\Delta\nu$ is the bandwidth, which will be 24 MHz, or smaller, for transient studies with CHIME [36, 43], and DM is the dispersion measure, given by the integrated column density of electrons [40, 44]. The total pulse width of an FRB will have a contribution from its
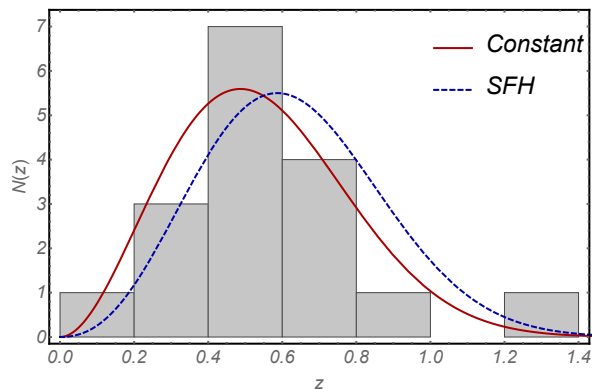
FIG. 1. A histogram of the 17 FRBs observed to date, with inferred redshifts [23]. FRB redshift distributions are plotted assuming a constant comoving density (solid-red), and following the star-formation history (dashed-blue), both with a cutoff at $z_{\mathrm{cut}} = 0.5$, and normalized to match the total number of detected events.
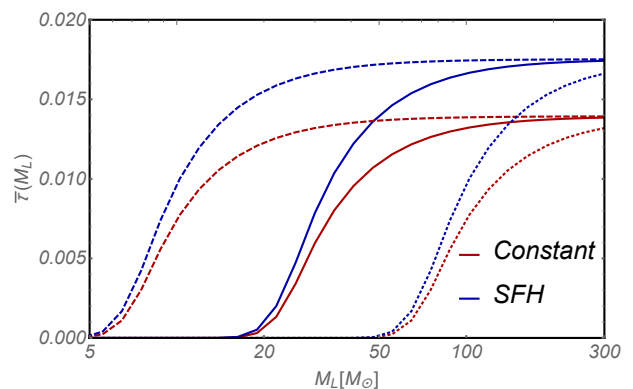

FIG. 2. Integrated optical depth, with weightings corresponding to a population of FRBs with constant comoving density (red) and following the SFH (blue), both with a cutoff at $z_{\mathrm{cut}} = 0.5$. In dashed, solid, and dotted lines we require a time delay $\Delta t > 0.3$ ms, 1 ms, and 3 ms, respectively. In all cases $f_{\mathrm{DM}} = 1$.

(unknown) intrinsic pulse profile, as well as scattering with the intergalactic medium [45, 46], and the lensing time delay has to be bigger than its total width to be easily detectable. To account for this, we will require a lensing time delay longer than $\overline{\Delta t} = 1$ ms as our baseline case. FRBs might have a distribution of intrinsic widths; wider bursts would give rise to more pessimistic results, whereas narrower FRBs might produce more optimistic ones. We will therefore show results for $\overline{\Delta t} = 0.3$ ms and $\overline{\Delta t} = 3$ ms, as well.

Given how little is known about the luminosity function of FRBs [32, 39], we will not attempt to model the lensing magnifications μ_+ and μ_- observable at each source redshift. Instead we will simply require a constant flux ratio, $\overline{R_f} = 5$, as threshold, since this will make the echoed image detectable.

Now, given a distribution function $N(z)$ for FRBs we can calculate their integrated optical depth $\bar{\tau}(M_L)$, due to MACHOs of mass M_L, as

$$\bar{\tau}(M_L) = \int dz\, \tau(z, M_L) N(z). \qquad (11)$$

We show this quantity in Figure 2 for the same two distribution functions discussed above. It is clear that the distribution mimicking the SFH produces a higher optical depth, due to the higher redshift of most sources.

We can finally forecast the number N_{lensed} of lensed FRBs that a fraction f_{DM} of dark matter, in the form of point lenses of different masses, will yield. In all cases we are in the optically thin regime, where the probability to be lensed is just $P_{\mathrm{lens}} = 1 - e^{-\bar{\tau}} \approx \bar{\tau}$. Thus, if we observe a number N_{FRB} of FRBs, $\bar{\tau} N_{\mathrm{FRB}}$ of them should be lensed. Notice that, even if all the dark matter was composed of compact objects of a single mass M_L, the lensing time delays induced on FRBs would not have a unique value, due to the different impact parameters and

redshifts of the lenses.

In Fig. 3 we show the joint probability distribution function (PDF) for a time delay Δt and a flux ratio R_f, assuming a 30 $M_\odot$ lens. This PDF has been calculated by convolving Eqs. (2) and (3), assuming a flat distribution in impact parameters squared up to y_{max}^2, with a population of FRBs following $N_{\mathrm{const}}(z)$, and shows a clear correlation between the lensing time delays and the flux ratios. We also show the probability $P(\Delta t)$ to find a time delay Δt between the two events, calculated by marginalizing the PDF over $R_f < 5$. This time-delay distribution would be broadened further if the MACHOs had some range of masses, instead of a single M_L.
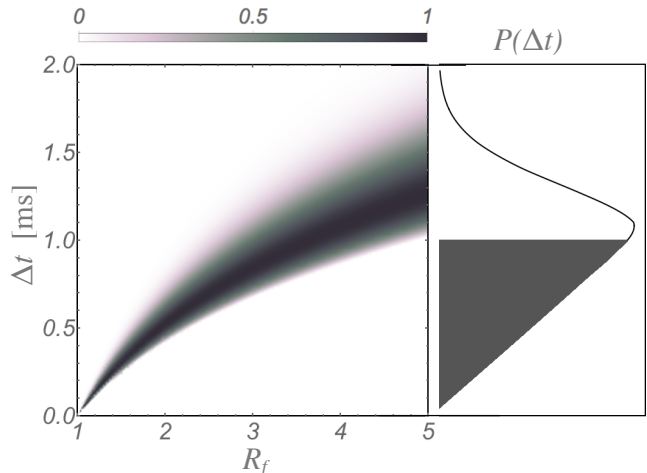

FIG. 3. Joint probability distribution for the flux ratio R_f and time delay Δt between the two peaks of an FRB lensed by a 30 $M_\odot$ MACHO. In the right we marginalize over R_f, and show the probability to find a time delay Δt. The shaded region corresponds to time delays smaller than 1 ms, too short to be detectable.

Considering the most conservative case of a constant-density distribution of FRBs, with cutoff at redshift $z_{\rm cut} = 0.5$ as discussed above, and 10^4 total detected FRBs, corresponding to one year of observation with CHIME, we will see a number $N_{\rm lensed} = 13$ of lensed bursts with a time delay longer than 1 ms, if all the dark matter is in the form of $20\,M_\odot$ MACHOs. If, however, the dark matter is made of $30\,M_\odot$ PBHs, as suggested in Ref. [11], the number of lensed events that will be detected is $N_{\rm lensed} = 60$. For all MACHO masses larger than $M_L = 100\,M_\odot$ the number of lensed events is simply $N_{\rm lensed} = 130$. Here we have required a flux ratio smaller than $\overline{R_f} = 5$ to observe both bursts, although high MACHO masses produce time delays much in excess of the threshold values of $\overline{\Delta t}$, so the cross section annulus in Eq. (6) becomes a circle, and $N_{\rm lensed}$ scales roughly linearly with $\overline{R_f}$.

We can also determine the smallest fraction $f_{\rm DM}$ that will produce at least one lensed event in a survey with 10^4 FRBs. Fig. 4 shows the regions of the $f_{\rm DM}$-M_L parameter space that give rise to at least one such event for lensing time delays longer than 0.3 ms, 1 ms, and 3 ms. We also show the current constraints to $f_{\rm DM}$ from the EROS collaboration [5], the MACHO collaboration [2], and wide-binary disruption [4].

From Figure 4 we see that, if none of the 10^4 upcoming FRBs is lensed, the amount of dark matter in MACHOs will be constrained to $f_{\rm DM} < 0.8\%$ above a cutoff mass of $\sim 100\,M_\odot$, under the assumption that the smallest time delay detectable is 1 ms. This will thus place more stringent constraints over this mass range than those coming from wide-binary disruption [4], by more than an order of magnitude.
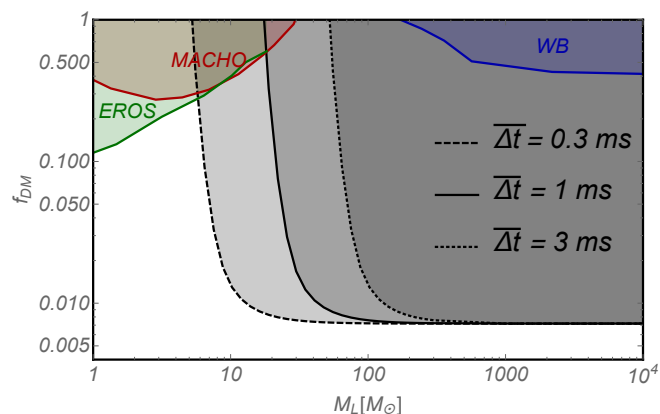

FIG. 4. Fraction $f_{\rm DM}$ of dark matter allowed in the form of point lenses of mass M_L, if no events out of $N_{\rm FRB} = 10^4$ are lensed, where the FRBs have a constant comoving density with a cutoff at $z_{\rm cut} = 0.5$. In dashed, solid, and dotted black we show our constraints when we require a time delay $\Delta t > 0.3$ ms, 1 ms, and 3 ms, respectively. In red we show the current constraints from the MACHO collaboration, in green the ones from the EROS collaboration, and in blue the constraints from galactic wide binaries.

For masses in the $20 - 100\,M_\odot$ window, outside the reach of the Galactic-lensing surveys [2], a dark-matter fraction of $f_{\rm DM} \sim 8\%$, at the lower-mass end of this range, and 0.8% at the higher-mass end, would suffice to detect one lensed FRB, if 10^4 FRBs are observed with a time resolution of 1 ms. As the number of lensed events scales trivially with $N_{\rm FRB}/10^4$, even a smaller number of $\sim 10^3$ FRBs per year should suffice to detect $\sim 1 - 10$ lensed FRBs in the first year of operation of CHIME, if MACHOs in this window made up the dark matter. This conservative number will still allow us to place constraints on $f_{\rm DM}$, comparable in magnitude to all current surveys ($f_{\rm DM} \leq 10\%$), but over the whole mass range $M_L > 20\,M_\odot$, if no lensed events are observed. Interestingly, even with a time resolution of 3 ms one would detect at least one lensed event, if MACHOs of mass $M_L \gtrsim 50\,M_\odot$ were the main component of dark matter.

FRBs might suffer intrinsic repetition. For example, the event FRB 121102 has been observed repeating as quickly as over minutes [24]. Lensing by a MACHO of mass $M_L \sim 10^5 M_\odot$ creates a time delay also on the scale of minutes, which then sets a natural ceiling to the MACHO mass that can be unequivocally probed with lensing of FRBs. In general, the correlation between time delays and flux ratios of the bursts, as shown in Fig. 3, will be of invaluable help to statistically determine whether repetition of FRBs is caused by microlensing.

Throughout this work we have assumed that an upcoming CHIME-like experiment will detect events up to a cutoff redshift of $z_{\rm cut} = 0.5$, as this fits the current FRB data. We can also calculate constraints for an increased cutoff redshift, e.g., $z_{\rm cut} = 0.7$, representing a more optimistic redshift distribution. In that case, for $\overline{\Delta t} = 1$ ms and $\overline{R_f} = 5$, we expect $N_{\rm lensed} = 35$ lensed events out of 10^4 if dark matter is made of MACHOs of mass $M_L = 20\,M_\odot$, $N_{\rm lensed} = 110$ if this mass is $M_L = 30\,M_\odot$, and $N_{\rm lensed} \gtrsim 200$ for masses higher than $75\,M_\odot$. Were none of these 10^4 FRBs to show lensing, however, we could constrain $f_{\rm DM}$ at $30\,M_\odot$ to be smaller than 0.9% (or 0.5% for $\overline{\Delta t} = 0.3$ ms, where this last number would apply to larger masses, and smaller $\overline{\Delta t}$). The increase in the lensing optical depth, due to the higher redshift of the events, leads to either more FRBs being lensed, or better constraints on $f_{\rm DM}$.

It has been argued that we could be preferentially observing strongly-lensed FRBs [47, 48]. If this is the case most observed FRBs will be lensed by intervening objects, such as galactic halos, on their way to Earth. This would create a double image with a time delay on the order of weeks [49]. More importantly, when crossing those galactic halos the probability to be microlensed by a MACHO is close to unity, which would help detect more microlensed FRBs, or improve our constraints on $f_{\rm DM}$.

Note that, due to our requirement that they behave as point lenses, MACHOs need to be smaller than their Einstein radii. This constrains the size of a MACHO of mass

M_L to be more compact than ~ 0.1 pc $\times \sqrt{M_L/30\,M_\odot}$.

An effect similar to femto-/nano-lensing of gamma-ray bursts could be observed in FRBs [50, 51], albeit given the relatively low frequency ($\nu \sim$ GHz) of FRBs, one could only probe lenses with masses higher than $M_L \sim 10^{-5} M_\odot$, since lower masses would create a time delay smaller than $1/\nu$, and not cause interference. An experiment with bandwidth $\Delta\nu \sim 20$ kHz could probe a maximum mass $M_L \sim 0.1\,M_\odot$ with nanolensing (higher masses would cause time delays longer than $1/\Delta\nu$, and interfere within each bandwidth). The unknown FRB frequency spectrum poses a challenge to modelling this effect, so it is left for future work.

Among the FRBs found to date, there is one particular event, FRB 121002, which has been observed with a double peak delayed by 5.1 ms [21]. This delay could have been caused by a MACHO lens of mass $M_L \gtrsim 200 M_\odot$. The second image of FRB 121002 appears brighter, however, which contradicts the usual lensing prediction. In Ref. [52] we will assess how likely it is that this delay is due to lensing, and study further cosmological applications of lensing of FRBs.

To conclude, upcoming interferometers will open up the radio sky, which will allow us to detect FRBs at a staggering pace. By studying whether these FRBs are doubly peaked we can conclude if they have been microlensed or not. Given the existing constraints, compact objects (MACHOs) are allowed to make up a large fraction of dark matter in our Universe (and even all of it in the mass window between 20 and 100 $M_\odot$). We will be able to detect from tens to hundreds of lensed FRBs if the dark matter is indeed composed of these MACHOs. Alternatively, if no FRBs are microlensed, we will place the strongest constraints yet on the fraction of dark matter in the form of compact objects.

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