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Low-energy Spin Dynamics of the Honeycomb Spin Liquid Beyond the Kitaev Limit

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We investigate the generic features of the low energy dynamical spin structure factor of the Kitaev honeycomb quantum spin liquid perturbed away from its exact soluble limit by generic symmetry-allowed exchange couplings. We find that the spin gap persists in the Kitaev-Heisenberg model, but generally vanishes provided more generic symmetry-allowed interactions exist. We formulate the generic expansion of the spin operator in terms of fractionalized Majorana fermion operators according to the symmetry enriched topological order of the Kitaev spin liquid, described by its projective symmetry group. The dynamical spin structure factor displays power-law scaling bounded by Dirac cones in the vicinity of the Γ, K and K' points of the Brillouin zone, rather than the spin gap found for the exactly soluble point.

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Quantum spin liquids (QSLs) have attracted wide attention due to their intriguing highly entangled nature and exotic properties [1, 2]. Amongst the simplest and most interesting QSLs are those with intrinsic topological order, which are of particular interest as potential platforms for quantum computing possessing intrinsic protection from decoherence[3]. A prominent feature of topological phases and QSLs in general is the fractionalization of electrons or spins into other particles, Majorana fermions for example. An unequivocal observation of this fractionalization is a key goal of the QSL field.

Recent theory and experiment have unveiled the exciting prospect of achieving this objective in highly anisotropic spin-1/2 magnets on honeycomb lattices, including Na_2IrO_3, Li_2IrO_3, and α-$RuCl_3$[4–18]. Two key theoretical works presaged this experimental venue. First, a seminal paper by Kitaev[19] introduced a simple near-neighbor spin Hamiltonian on this lattice, possessing a gapless $\mathbb{Z}_2$ QSL phase, the excitations of which are massless relativistic (i.e. linearly dispersing) Majorana fermions and gapped bosonic "fluxes"[20]. Second, Jackeli and Khaliullin[21] showed that Kitaev's anisotropic interactions arises naturally from certain superexchange processes in strongly spin-orbit coupled transition metal compounds. These two developments spurred the search for Kitaev's QSL in this context in the laboratory.

In this paper, we address a key experimental signature of any magnet, the dynamical spin structure factor, for the Kitaev QSL. The dynamic spin response can be measured using conventional experiment techniques such as inelastic neutron scattering and electron spin resonance. It is given, at zero temperature, by

$$S_{ij}^{\mu\nu}(t) = \langle 0|T(\sigma_i^\mu(t)\sigma_j^\nu(0))|0\rangle, \qquad (1)$$

where σ_i^μ is the Pauli operator representing the $\mu^{\rm th}$ component of the spin at site i of the lattice, and the arguments indicate the usual Heisenberg time evolution. Previous studies demonstrated that for Kitaev's exactly soluble model $S_{ij}^{\mu\nu}$ vanishes between all but the neighbor pair of spins [22]. Moreover, the dynamical spin response exhibits a spin gap – a non-zero interval of frequency around zero in which the spectral weight vanishes – despite the existence of gapless excitations [23, 24]. These remarkable properties arise due to the exact integrability of the Kitaev model. Here we ask the important question whether the apparent spin gap persists when moving away from the exactly solvable point in the critical spin liquid phase[25, 26]. We find that the existence of the gap as a robust property of the QSL phase relies critically upon internal symmetries: it is present in the Heisenberg-Kitaev model but *not* in the generic model allowed by physical symmetries in actual materials. In the latter case we obtain *universal* power-law spectral weight at low energies, as shown in Figs. 2, 3. This, rather than the gap found in Ref.23, is the expected behavior should the Kitaev QSL be realized in actual experiments.

To obtain these results, we follow standard arguments of low energy effective field theory. The low energy field theory in the gapless phase of Kitaev model is a single cone of massless Dirac fermions (a convenient formulation for two Majorana cones). Physical operators may be expanded in the primary field and descendents of the field theory, here the Dirac fields, and we expect all terms consistent with symmetry to generically appear in this expansion. Due to the fine-tuned nature of the exactly soluble point, many coefficients in this expansion vanish there, but will become non-zero if symmetry allows. In the case of a QSL phase, the analysis of permitted terms is subtler because the physical symmetries are intertwined with emergent gauge transformations of the non-local fermions. This is described by the mathematical structure of projected symmetry groups (PSGs)[27–31]. Here, we use the PSG analysis of Ref.31 to find the low energy contributions to the microscopic spin operator, and from this deduce the dynamical spin correlations. To complement the PSG analysis, we also study the problem directly by perturbation theory away from the soluble Kitaev point, thereby obtaining the scaling of the prefactors f_j's in Eq. (5) of the symmetry-allowed terms.

We begin by recapitulating Kitaev's model and its solution. It consists of interacting spin-1/2 moments on a honeycomb

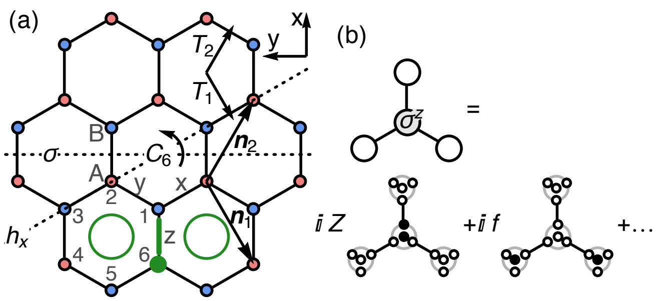

FIG. 1. (a) The honeycomb lattice for Kitaev model. A, B denote sublattice index. A spin $\sigma_6^{\tilde{z}}$ (in green) on the ground state changes the 1-6 bond operator eigenvalue, creating fluxes in two plaquettes sharing that bond. The symmetries T_1, T_2, C_6, σ and h_x are displayed. (x, y) coordinate system is shown on the upper right conner. (b) Spin operators schematically expanded in terms of Majorana fermion operators. Each site contains four Majorana modes (small dots), where the central one is c_i and the surrounding three are c_i^μ's. Each black dot represents a corresponding Majorana operator present.

lattice. Original hamiltonian reads

$$H_0 = J_K \sum_\mu \sum_{\langle ij\rangle^\mu} \sigma_i^\mu \sigma_j^\mu, \quad (2)$$

where $\langle ij\rangle^\mu$ denotes the neighbor sites i, j whose bond direction is labeled by $\mu = x, y, z$ (Fig. 1(a)). There is a local conserved quantity for each plaquette $W_p = \sigma_1^x \sigma_2^z \sigma_3^y \sigma_4^x \sigma_5^z \sigma_6^y$ (Fig. 1(a)), the set of which on all plaquettes is a set of good quantum numbers labeling energy eigensectors. We say a π-flux is present on a plaquette p if $W_p = -1$. The solution to Kitaev model is well known through the mapping to a free fermion Hamiltonian

$$H_0 = J_K \sum_\mu \sum_{\langle ij\rangle^\mu} \mathrm{i} c_i \hat{u}_{\langle ij\rangle^\mu} c_j, \quad (3)$$

if we write each spin as the product of Majorana operators $\sigma_i^a = \mathrm{i} c_i c_i^\mu$ (physical subspace satisfies $c_i c_i^x c_i^y c_i^z = 1$) and define bond operator $\hat{u}_{\langle ij\rangle^\mu} = \mathrm{i} c_i^\mu c_j^\mu$. The bond operators with eigenvalues ± 1 commute with the Hamiltonian and the product of them around a plaquette p is W_p. Identifying the eigenvalue of $\hat{u}_{\langle ij\rangle^\mu}$ as $\mathbb{Z}_2$ gauge field, one can obtain the ground state that exists in the zero-flux subspace by setting $\hat{u}_{\langle ij\rangle^\mu}$'s to 1 and diagonalizing the Hamiltonian. Complementary to the static flux sector, the Majorana fermions c_i's living on-site span the matter fermion (spinon) sector.

From this structure, the short-range and gapped nature of the spin correlations for the soluble model follows directly. When applying a spin operator to the ground state, a spinon excitation is created with two fluxes (illustrated in Fig. 1(a) in green). For $\mathbb{Z}_2$ topological order, there are two bosonic anyons, e and m, corresponding to fluxes, and one fermionic anyon, ε, which is the c_i fermion. In this language, $\sigma \sim \mathsf{e}\mathsf{m}\varepsilon$, which is consistent with general rules since ε carries both "electric" and "magnetic" gauge charge making the combination of three anyons gauge invariant. Thus a spin gap appears since there is a finite energy difference between the ground state and the lowest eigenstate of the two-vison sector. Infinitely massive visons render the spin correlators short-ranged. Yet both properties may be changed if contributions to the spin of the form $\sigma \sim \varepsilon\varepsilon$ arise, which appears natural based on the gauge structure alone. To see how they arise, we first consider the structure of states and operators under perturbations, then turn to a general symmetry based analysis.

Unitary transformation analysis: Perturbation theory defines a unitary mapping $U = e^{\mathrm{i}S}$ from eigenstates of the pure Kitaev model H_0 to those of the perturbed one $H = H_0 + V$, for example the exact ground state $|\psi\rangle = U|\psi_0\rangle$, where $|\psi_0\rangle$ is the unperturbed ground state. One can find U order by order by demanding that the rotated Hamiltonian $\tilde{H} = UHU^{-1}$ has vanishing off-diagonal matrix elements between low-energy eigenstates of H_0, e.g. to first order $\mathrm{i}S = \sum_{n\neq m} \frac{P_n V P_m}{E_n - E_m}$, where P_n is the projection operator onto the nth energy eigenspace. Formally the higher order terms can be found using Baker-Hausdorff formula: they involve more powers of V separated by projection operators and the corresponding energy differences in the denominators. Because the energies of eigenstates of H_0 with non-uniform fluxes are non-trivial, we are not able to evaluate this explicitly. However, we can understand the general structure of the expansion. Moreover, because the Kitaev QSL is a stable phase, U is well-behaved and S defined in this way is a sum of quasi-local operators, at least when restricted to act (on the right) on low energy states.

In general, we can separate any physical operator, including U, into a sum of terms which modify the flux on $2k$ sites: $U = \sum_{k=0}^\infty U_{2k}$, where, since U is physical, only an even number of fluxes can be changed. We may understand U_{2k} as the mixing $2k$ virtual fluxes into the interacting ground state.

Spin operators transform accordingly under this procedure:

$$\sigma_i^\mu \to \tilde{\sigma}_i^\mu \equiv U^\dagger \sigma_i^\mu U = \sum_{k,k'} U_{2k'}^\dagger \sigma_i^\mu U_{2k} = \sum_k \tilde{\sigma}_{i,2k}^\mu. \quad (4)$$

We observe that since σ_i^μ modifies two fluxes, terms with $|k - k'| = 0, 1$ induce contributions to $\tilde{\sigma}_{i,0}^\mu$. The physical picture behind is that the exact ground state of the perturbed system contains terms with two virtual fluxes, which are annihilated by the spin operator. The resulting state is no longer orthogonal to all the exact zero flux eigenstates. Thus the transformed spin operator has an expression of the form (Fig. 1(b))

$$\tilde{\sigma}_i^\mu = \underbrace{\mathrm{i} Z c_i c_i^\mu + \cdots}_{\tilde{\sigma}_{i,2k>0}^\mu} + \underbrace{f_{ijk}^\mu \mathrm{i} c_j c_k + \cdots}_{\tilde{\sigma}_{i,0}^\mu}, \quad (5)$$

where $Z = 1$ for the ideal Kitaev model but is reduced by perturbation. All other terms become non-zero with perturbations. The first set of bracketed terms create fluxes exciting modes above a finite energy threshold. We concern here the latter terms, which consist of matter fermions alone and hence create no fluxes and induce only low-energy excitations.

Generic spin operator form from symmetry and gauge constraints: We first consider the constraints on the terms which may arise, focusing on the maximal physical symmetry group generated by the following operations: translations along two basis directions (T_1, T_2), time reversal($\mathcal{T}$), C_6 (a 6-fold rotation plus mirror reflection across honeycomb plane) and σ symmetry (mirror reflection across the line orthogonal to the z-bond) (Fig. 1(a), c.f. Ref. 31). In addition, we adopt h_x symmetry (a π rotation around the direction of x-bond, equivalent to σC_6). The transformation of Majorana operators should conform to the symmetry enriched topological order of the Kitaev spin liquid, described by PSGs[31].

With these symmetries, we construct combinations of matter fermions compatible with the transformations of spin operators. A basic constraint is gauge invariance imposed on any physical operator. The $\mathbb{Z}_2$ gauge transformation induced by $\eta_i = \pm 1$ takes $u_{\langle ij \rangle} \to \eta_i u_{\langle ij \rangle} \eta_j$ and $c_i \to \eta_i c_i$. Any product of an even number of matter fermions c_i, while not gauge invariant on its own, can be made so by multiplying it by a string operator $\mathcal{U}_{i_1 i_2} = \prod_{j: i_1 \to i_2} \hat{u}_{\langle j_n, j_{n+1} \rangle}$, connecting sites i_1, i_2 of Majorana fermions. It is possible to uniquely restore strings in the low energy Hilbert space with $W_p = +1$, so in the following we adopt a notation with implicit strings (they can be readily restored when desired).

Having understood gauge invariance, we consider the symmetry transformations of matter fermion products. Consider first time-reversal, under which matter fermion operators on the A sublattice are invariant but those on the B sublattice change sign[31]. Since a spin is odd under $\mathcal{T}$, a corresponding product should consist of an even/odd number of matter fermions on each sublattice if the total number of fermions satisfies $N \equiv 2/0 \mod 4$, respectively (note the imaginary unit needed when $N \equiv 2 \mod 4$ to ensure hermiticity).

Thus the smallest appropriate number of matter fermions is two, which must live on the same sublattice. Taking into account the cyclic structure of spin components (which permute under rotations) and the antisymmetry of fermion bilinears, we postulate the form (See Fig. 1(b))

$$\tilde{\sigma}^{\mu}_{i,0} \sim \frac{\mathrm{i}}{2} f \epsilon^{\mu\nu\lambda} c_{i+s_i\hat{\nu}} c_{i+s_i\hat{\lambda}}. \tag{6}$$

Here $\hat{\mu}$ is the vector in the μ direction from the A to B site, and $s_i = +1(-1)$ for the A (B) sublattice. It is straightforward to check(Appendix A [32])that this form is consistent with all the symmetries. Restoring the gauge string connecting sites $i + s_i\hat{\nu}$ and $i + s_i\hat{\lambda}$, we obtain a gauge invariant expression which can be rewritten in terms of bare spin operators, to wit

$$\tilde{\sigma}^{\mu}_{i,0} \sim -f \sigma^{\mu}_i \prod_{\nu \neq \mu} \sigma^{\nu}_{i+s_i\hat{\nu}}. \tag{7}$$

Though this operator involves three spins, it is quadratic in the fermions and is expected to give the largest low energy contribution. Terms with more fermion operators or from further separated sites that give subdominant contributions, are discussed in the Appendix A.

We proceed to check how the form arises in perturbation theory. Consider the effect of two perturbations on the Kitaev model[4, 18, 33, 34], namely the Heisenberg interaction $V_H = J_H \sum_{\langle ij \rangle} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$ and the "cross" term $V_c = J_c \sum_{\mu(\nu\gamma)} \sum_{\langle ij \rangle^\mu} (\sigma^{\nu}_i \sigma^{\gamma}_j + \sigma^{\gamma}_i \sigma^{\nu}_j)$ (ν, γ are the remaining directions). With either *one* of the above terms, one can show accidental symmetries cause the spin gap to remain unbroken (i.e. f=0) – see Appendix B. However, when both are present, we find that the contribution in Eq. (7) is induced at *fourth* order. Specifically, the sequence of two J_H and two J_c perturbations induces the product $\sigma^y_2 \sigma^z_3 \sigma^x_1 \sim (\sigma^x_1 \sigma^x_2)(\sigma^x_1 \sigma^z_2)(\sigma^x_1 \sigma^x_3)(\sigma^x_1 \sigma^y_3)\sigma^x_1$. From this, we estimate that the factor f in Eq. (6) scales as $\frac{J_H^2 J_c^2}{J_K^4}$.

Low-energy weight of the dynamical spin structure factor: With the low-energy component of the spin operator identified in Eq. (6), we can calculate its contribution to the dynamical spin structure factor $S^{\mu\mu}_{ab}(\boldsymbol{q}, \omega) \equiv \frac{1}{N} \sum_{i,j} \int_{-\infty}^{+\infty} \mathrm{d}t S^{\mu\mu}_{ia,jb}(t) e^{\mathrm{i}\omega t - \mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)}$, where N is the number of unit cells in the lattice. We refine the definition of Eq. (1) by introducing the sublattice indices $a, b = A, B$ apart from the unit cell indices i, j. In this section, we will show that the low-energy weight of the spin correlators exhibits power law behavior in frequency. The single-particle spectrum of spinons in the zero-flux sector is $E(\boldsymbol{q}) = |s_{\boldsymbol{q}}|$ where $s_{\boldsymbol{q}} = J_K(1 + e^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{n}_2} + e^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{n}_1})$ ($\boldsymbol{n}_1$, $\boldsymbol{n}_2$ are basis vectors in Fig. 1(a)). There are two Majorana cones located at the K and K' points ($\pm\boldsymbol{q}_0$) at the Brillouin zone corners, which can be combined into a single cone of Dirac fermions. Expanding the matter fermion field around the Dirac point, we can write

$$c_i = \begin{cases} \psi_A(\boldsymbol{r}) e^{\mathrm{i}\boldsymbol{q}_0\cdot\boldsymbol{r}} + \text{h.c.} & i \in A, \\ \psi_B(\boldsymbol{r}) e^{\mathrm{i}\boldsymbol{q}_0\cdot\boldsymbol{r}} + \text{h.c.} & i \in B, \end{cases} \tag{8}$$

where $\psi_a(\boldsymbol{r})$ is a slowly varying Dirac field (we take $\boldsymbol{r}$ to lie at the hexagon center, i.e. $\boldsymbol{r}_i = \boldsymbol{r} + \hat{x}$ for $i \in A$ and $\boldsymbol{r}_i = \boldsymbol{r} - \hat{y}$ for $i \in B$). Inserting this into the Hamiltonian in Eq. (3) and gradient expanding, we obtain the low-energy dynamics of matter fermions described by the action

$$\begin{aligned} S &= \int \mathrm{d}\tau \mathrm{d}^2\boldsymbol{r}\, \psi^\dagger [\partial_\tau - v(\sigma_x \mathrm{i}\partial_x + \sigma_y \mathrm{i}\partial_y)]\psi \\ &= \int \mathrm{d}\omega_n \mathrm{d}^2\boldsymbol{q}\, \psi^\dagger_{\boldsymbol{q},\omega_n} (-\mathrm{i}\omega_n + v\boldsymbol{\sigma}\cdot\boldsymbol{q}) \psi_{\boldsymbol{q},\omega_n}, \end{aligned} \tag{9}$$

where $\psi = (\psi_A, \psi_B)^\intercal$ and the Fermi velocity $v = \sqrt{3} J_K/2$ (The definition of coordinates is indicated in Fig. 1(a)). The field theory is conformally invariant, and the fermion fields scale with length L as $[\psi(\boldsymbol{r}, \tau)] = L^{-1}$. We can decompose the low energy spin operator in Eq. (6) similarly using Eq. (8). With some algebra, we obtain the form

$$\begin{aligned} \sigma^{\mu}_{i \in a} &\sim \hat{M}^{\mu}_a(\boldsymbol{r}) + (\mathrm{i}\hat{N}^{\mu}_a(\boldsymbol{r}) e^{-\mathrm{i}\boldsymbol{q}_0\cdot\boldsymbol{r}} + \text{h.c.}), \\ \hat{M}^{\mu}_a &= \psi^\dagger m_a \psi, \quad \hat{N}^{\mu}_a = \psi^\intercal \boldsymbol{n}^{\mu}_a \cdot \boldsymbol{\nabla}\psi, \end{aligned} \tag{10}$$

where m_a and $\boldsymbol{n}^{\mu}_a$ are two-by-two diagonal matrices, and we used $2\boldsymbol{q}_0 = -\boldsymbol{q}_0$ up to a reciprocal lattice vector. Simple specific forms for these matrices in terms of f and $\boldsymbol{q}_0$ are obtained by starting with Eq. (6), but we expect them to be renormalized generally by higher order terms. Both the simple and general symmetry-allowed forms are given in the Appendix A. Note the presence of the gradient in $\hat{N}^{\mu}_{a}$: this cannot be avoided because time-reversal symmetry requires the sublattice degrees of freedom be in a symmetric state, so that the Pauli exclusion principle for this two-particle creation operator forces the orbital wave function to be odd parity; no such requirement applies to the density operator $\hat{M}^{\mu}_{a}$.

At this point, the scaling of low energy spin correlations is evident. Using the dimension of ψ, the two-point functions of $\hat{M}^{\mu}_{a}$ and $\hat{N}^{\mu}_{a}$ scale as $\frac{1}{L^4}$ and $\frac{1}{L^6}$, respectively. The Fourier transformation in 2+1 dimensions adds 3 powers of $L\sim\frac{1}{\omega}$, so that $S(\boldsymbol{q}\approx 0,\omega)\sim|\omega|$ corresponding to $\hat{M}^{\mu}_{a}$ correlations, and $S(\boldsymbol{q}\approx\pm\boldsymbol{q}_0,\omega)\sim|\omega|^3$ corresponding to $\hat{N}^{\mu}_{a}$ correlations.

Beyond scaling one obtains the exact low-energy forms by calculation in reciprocal space ($\mu\mu$, ab are suppressed):

$$S(\boldsymbol{k},\mathrm{i}\omega_n)\sim\int \mathrm{d}\omega_1 d^2\boldsymbol{k}_1 \mathrm{Tr}[m_a G(\boldsymbol{k}+\boldsymbol{k}_1,\mathrm{i}(\omega_n+\omega_1)) \\ m_b G(\boldsymbol{k}_1,\mathrm{i}\omega_1)], \\ S(\boldsymbol{q}_0+\boldsymbol{k},\mathrm{i}\omega_n)\sim\int \mathrm{d}\omega_1 \mathrm{d}^2\boldsymbol{k}_1 \mathrm{Tr}\{[\boldsymbol{n}^{\mu}_{b}\cdot(2\boldsymbol{k}_1-\boldsymbol{k})]G(\boldsymbol{k}_1,\mathrm{i}\omega_1) \\ [(\boldsymbol{n}^{\mu}_{a})^{\dagger}\cdot(2\boldsymbol{k}_1-\boldsymbol{k})]G^{\mathsf{T}}(\boldsymbol{k}-\boldsymbol{k}_1,\mathrm{i}(\omega_n-\omega_1))\}, \tag{11}$$

where $|\boldsymbol{k}|\ll|\boldsymbol{q}_0|$. The aforementioned scaling follows immediately using $G(\boldsymbol{k},\mathrm{i}\omega_n)\equiv-\langle\psi\psi^{\dagger}\rangle_{\boldsymbol{k},\omega_n}=\frac{1}{\mathrm{i}\omega_n-v\boldsymbol{\sigma}\cdot\boldsymbol{k}}$, which is of dimension $\frac{1}{\omega}$, by rescaling $\boldsymbol{k}_1(\omega_1)\to\frac{\boldsymbol{k}_1}{\omega_n}(\frac{\omega_1}{\omega_n})$. One obtains $S(\boldsymbol{k},\mathrm{i}\omega_n)\sim|\omega_n|\tilde{S}(\frac{v\boldsymbol{k}}{|\omega_n|})$ and $S(\boldsymbol{q}_0+\boldsymbol{k},\mathrm{i}\omega_n)\sim|\omega_n|^3\tilde{S}(\frac{v\boldsymbol{k}}{|\omega_n|})$.

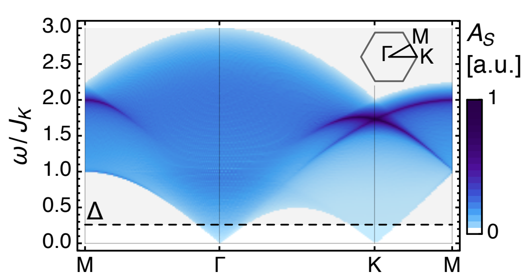

FIG. 2. The spectral function along high symmetry line at the isotropic point. The simplest form as listed in eq (6) is used for calculation performed on a honeycomb lattice with 240×240 unit cells. The dashed line marks the gap $\Delta=0.262J_K$ of flux excitations, above which the leading contribution from the unperturbed Kitaev spin liquid will dominate.

Fig. 2 shows the numerical results of spectral function $A_S(\boldsymbol{q},\omega)$=$-\sum_{\mu}\sum_{a,b}2\mathrm{Im}[S^{\mu\mu}_{ab}(\boldsymbol{q},\omega+\mathrm{i}0_+)]$ calculated based on the simple forms for m_a and $\boldsymbol{n}^{\mu}_{a}$ at the isotropic point. One observes zero low energy spectral weight outside "Dirac cones" centered at Γ and K, i.e. for $\omega<v|\boldsymbol{k}|$. Direct inspection of the frequency dependence in Fig. 3 confirms the expected ω and ω^3 behaviors at Γ and K, respectively. Besides this dominant contribution, there will be additional ones arising from products of more than two matter fermion operators, which give larger powers of frequency since every ψ field adds one ω factor by dimensional analysis. Away form the isotropic limit, i.e. coupling strengths are unequal on bonds for different directions, the Majorana points $\pm\boldsymbol{q}_0$ will be shifted away from the Brillouin zone corners, but the scaling of low energy spin correlations, $S(\boldsymbol{q}\approx 0,\omega)\sim|\omega|$ and $S(\boldsymbol{q}\approx\pm 2\boldsymbol{q}_0,\omega)\sim|\omega|^3$ still hold, since $\mathcal{T}$ still dictates that two-fermion product contains sites on the same sublattice.

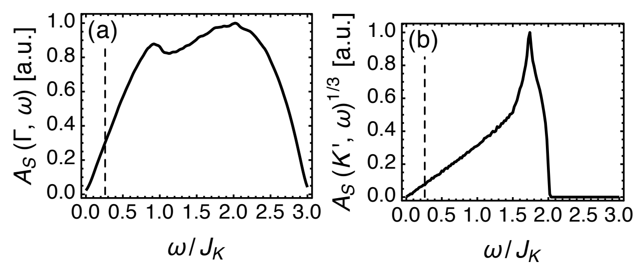

FIG. 3. The spectral function versus frequency at (a) the Γ point $\boldsymbol{q}=0$ and (b) the K' point $\boldsymbol{q}\sim 2\boldsymbol{q}_0$. The cube root of $A_S(K',\omega)$ is plotted in (b) to demonstrate the scaling behavior $A_S(K',\omega)\sim\omega^3$. The dashed line marks the flux gap $\Delta=0.262J_K$.

The spectral function $S(\boldsymbol{q},\omega)$ is of course measured by inelastic neutron scattering. It also describes the longitudinal nuclear spin relaxation rate $\frac{1}{T_1}$ in a nuclear magnetic resonance (NMR) experiment, in which $\frac{1}{T_1}$ is proportional to the local spectral density of spin fluctuations, which is the momentum integral of $S(\boldsymbol{q},\omega)$, with ω at the (low) NMR frequency. This work predicts temperature scaling $\frac{1}{T_1}\sim T^3$, in contrast to the ideal Kitaev spin liquid, where the relaxation rate follows the activated behavior $\sim e^{\frac{-\Delta}{T}}$ due to the spin gap Δ. Finally, our result for the uniform component of the spin operator in Eq. (7) corresponds, if summed over all sites, to the mass term induced by introducing a magnetic field term($V_m=\sum_{\mu,j}h_{\mu}\sigma^{\mu}_{j}$) introduced by Kitaev[19]. In the ideal soluble limit, Kitaev showed that such an external magnetic field induces a gap for the Majorana fermions of order $h_x h_y h_z$, by generating a 2nd neighbor hopping of majorana fermions at 3rd order in perturbation theory. Away from the soluble limit, the same 2nd neighbor Majorana coupling enters directly as a component of the spin operator in the flux-free sector, which couples linearly to the external magnetic field. Therefore, our result implies that, generically, the induced gap is *linear* in the field and exists for any field orientation. Such a gap might be probed by electron spin resonance, which measures $S(\boldsymbol{q}=0,\omega)$ in a field.

Conclusion: We showed that generically the low-energy spectral weight of spin structure factor in the gapless Kitaev QSL phase on honeycomb lattice is non-vanishing, in contrast to the special case of the soluble point. The results illustrate a general effective field theory approach, which could be applied to any gapless QSL based on a PSG analysis. In particular for the Kitaev spin liquid we find spectral weight linear (cubic) in ω near the origin (zone boundary) in momentum space, filling a Dirac-cone-like structure with a sharp spectral edge. These predictions, and not those of the exactly soluble model, describe the proper low energy behavior of any Kitaev spin liquids which might be found experimentally. We note that some previous works[25, 26] have discussed the appearance of power-law correlations upon perturbing the Kitaev model, but our results are distinct and more general.[35] It would be interesting to test these predictions numerically, e.g. by DMRG calculations (One can include a term $H'=\Delta\sum_p W_p$ to raise the flux gap by $\Delta>0$ and open a larger window to observe the low energy weight).

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