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Convert acoustic resonances to orbital angular momentum

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We use acoustic resonances in a planar layer of a half-wavelength thickness to twist wave vectors of an in-coming plane wave into a spiral phase dislocation of an out-going vortex beam with orbital angular momentum (OAM). The mechanism is numerically and experimentally demonstrated by producing an airborne Bessel-like vortex beam. Our acoustic resonance-based OAM production differs from existing means for OAM production by enormous phased spiral sources or by elaborate spiral profiles. Our study can advance the capability of generating phase dislocated wave fields for further applications of acoustic OAM.

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Wave fields with spiral phase dislocations [1] carry orbital angular momentum (OAM) such as for sound fields [2], optical waves [3], and electron beams [4]. A spiral phase $\exp(im\theta)$ linearly proportional to the azimuth angle θ is associated with a null field at the core. The carried OAM is discretized with the integer m (topological charge or order of the beam) to have an OAM-to-energy ratio m/ω (where ω is the radian frequency) for both quantum [3, 4] and classical waves [5, 6]. Transfer of the OAM to matters produces a torque associated with the transfer of wave energy such as in optical [7–9] and acoustic waves [10–14].

Acoustic waves with spiral phases and OAM of useful properties (e.g., [15–19]) were generated by *phased* spiral sources or *physically* spiral sources. A *phased* spiral source consists of an array of individually addressed transducers excited with appropriate screw phases to produce the expected phase profile in acoustic vortex beams [2, 20] or vortice of surface waves [21, 22]. A *physically* spiral source is a passive structure with screw dislocated profiles to produce phase profiles in wave fields, e.g., a helical substrate underneath ferroelectret film for airborne ultrasonic vortex generation [23], an absorbing surface with helix dimension for optoacoustic generation of a helical ultrasonic beam [24], a spiral-shaped object (spiral phase plate) for chiral scattering [25], and spiral gratings for diffracting waves into stable vortex beams [26].

The general principle for producing the spiral phase (denoted by $\phi_{\rm out}$) can be written as,

$$\phi_{\rm out}(\theta) = \phi_{\rm in} + k^{\rm eff} l, \quad (1)$$

where the dependence of $\phi_{\rm out}$ on the azimuth angle θ was produced via a θ-dependent phase $\phi_{\rm in}$ in *phased* spiral sources or via a θ-dependent propagation distance l through *physically* spiral sources.

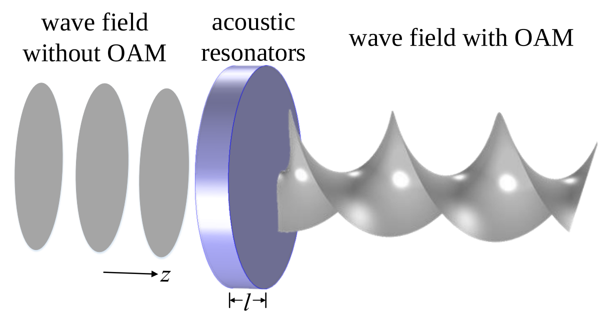

FIG. 1. Illustration of a resonant planar layer (blue) converting an in-coming axisymmetric wave without orbital angular momentum (OAM) to an out-going beam with helical wave front carrying OAM (wave fronts are shown in grey).

The OAM production in this study doesn't have to reply on the θ dependence in either phase $\phi_{\rm in}$ or propagation distance l in Eq. (1). Instead we produce a θ-dependent effective wave number $k^{\rm eff}$. The mechanism is that, regardless of *no* θ-dependence in phase $\phi_{\rm in}$ of an axisymmetric in-coming wave and in propagation distance l of a planar layer [Fig. 1], acoustic resonances are excited in the layer to eventually produce the desired wave number $k^{\rm eff}$ for twisting the wave vectors into a phase dislocation of an vortex beam with OAM. We numerically model and experimentally demonstrate this mechanism by generating a Bessel-like vortex beam.

Model. — We construct the planar layer as an assembly of eight fan-like sections of resonators over the whole azimuth [Fig. 2(a)]. This amount of sections gives a reasonably good resolution for generating a vortex beam with a topological charge $m = 1$, as will

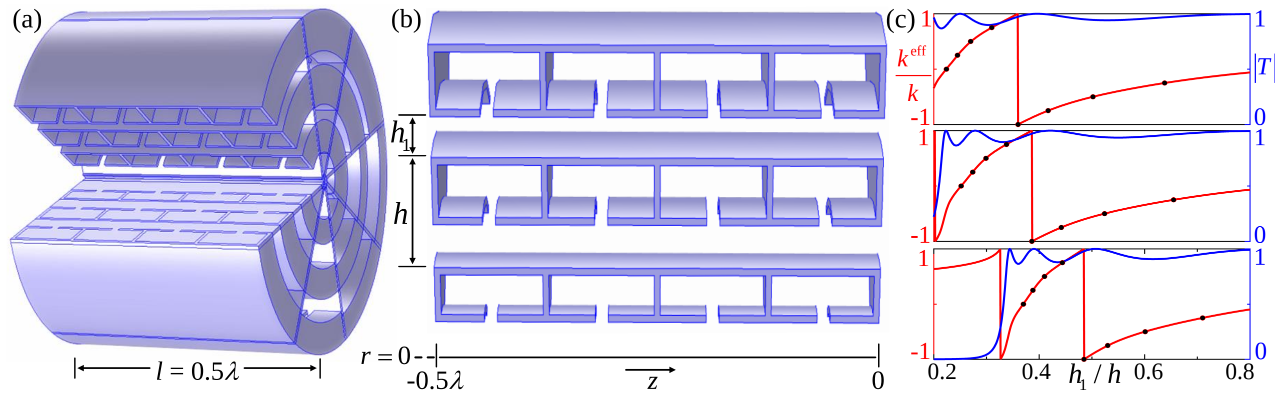

FIG. 2. (a) Schematic of the assembled layer consisting of eight fan-like sections of resonators. (b) An individual section consisting of three rows of resonators in the radial r direction (the radial resolution $h = 0.1\lambda$ with λ being the sound wavelength), sided by pipes of varying height h_1 to produce needed effective wave number [cf. Eq. (1)]; the thickness of the walls is 0.01λ. (c) The effective wave number k^{eff} (red; normalized by $k = 2\pi/\lambda$) and transmission coefficient $|T|$ (blue) as functions of the height ratio h_1/h simulated for the three rows in (b), where in each case the eight black dots are parameters selected for the eight sections in (a) with equally discrete wave numbers to generate a first-order vortex beam. The averaged transmission efficiency of these 8×3 elements is $|T| = 95.1\% \pm 3.8\%$.

be demonstrated via both numerical simulations and experimental measurements. For generating higher-order vortex beams, the amount of sections in the resonant layer can be increased for a finer resolution.

Each individual section [Fig. 2(b)] is configured to compose of three rows of resonators in the radius (more rows can be employed for a larger radius). Each row consists of four Helmholtz cavities and a straight pipe [27]: (I) the series connection of four cavities acting as lumped elements is for a fully flexible manipulation of wave numbers k^{eff} (or phases), (II) the combination of cavities and pipes provides hybrid resonances that overcome the impedance mismatch between the resonators and the surrounding air for a high transmission, and (III) the layer is optimized to have a propagation distance $l = 0.5\lambda$, a value that is small enough to maintain hybrid resonances, but large enough for negligible viscous effects (with the width of each cavity's neck in air fixed at 0.025λ) and for manipulating the wave number k^{eff} over a fully desired range (a reasonably good performance can still be obtained with $l \gtrsim 0.4\lambda$; c.f. Fig. 3 in [27]).

We use the resonant layer to produce desired spiral phases and OAM by manipulating the k^{eff} for an azimuthal dependence via tuning the height of the cavities into an azimuthal dependence. The k^{eff} values are modulated for the eight sections individually, denoted by $k_j^{\text{eff}} (j = 1, 2, ...8)$. Consider generating a first-order $m = 1$ vortex beam, the k_j^{eff} values selected for the eight sections cover the whole range from $-k$ to k with a step of $k/4$ (where $k = 2\pi/\lambda$ is the wave number in the surrounding medium). If the three rows of resonators have an identical transmission, the out-going normalized field from an individual segment is approximated by:

$$p|_{z=0} = \exp(ik_j^{\text{eff}} l - i\omega t) \text{ for } (j-1)\pi/4 < \theta < j\pi/4. \quad (2)$$

The out-going wave propagates in a rigid cylindrical waveguide of radius a as for generating a vortex beam of a Bessel-like profile. The wave can be represented as a sum of cylindrical Bessel modes:

$$p = \sum_m \sum_n A_{m,n} J_m(k_{m,n} r) \exp(ik_z z + im\theta - i\omega t), \quad (3)$$

where $A_{m,n}$ is the modal amplitude, $J_m(k_{m,n} r)$ is the m-order Bessel function, $k_{m,n}$ is the n-th positive root of equation $\partial J_m(k_{m,n} r)/\partial(k_{m,n} r)|_{r=a} = 0$, and the axial wave number is $k_z = \sqrt{k^2 - k_{m,n}^2}$. We restrict the $(m,n) = (1,1)$ mode as the only propagating vortex mode in the waveguide by choosing k to be higher than the critical wave number of the $(1,1)$ mode (i.e. $k_{1,1}$) but lower than that of $(1,2)$ mode (i.e. $k_{1,2}$). Given the condition at $z = 0$ in Eq. (2), the normalized propagating field would be:

$$p = J_1(k_{1,1} r) \exp(ik_z z + i\theta - i\omega t) \quad (4)$$

which is exactly a first-order Bessel-like vortex beam (topological charge $m = 1$) that carries OAM.

Simulations and Measurements. — Now we simulate the conversion of the acoustic resonances to OAM with finite element method based on COMSOL Multiphysics software (with the Pressure Acoustic Interface [28]). The simulations resemble the experimental setup of an in-coming wave at 2287 Hz (with an airborne wavelength of 15 cm), simulated as a plane wave to propagate along an air-filled cylindrical waveguide of a 5-cm radius and transmit through a coaxial resonant layer of a 7.5-cm thickness. The airborne sound wave number $k = 41.9$ rad/s is in between the values of $k_{1,1} = 36.8$ rad/s and $k_{1,2} = 106.4$ rad/s of the waveguide, satisfying the criteria stated in aforementioned model analysis. Solid materials (used in experiments) for the waveguide (PMMA) and for the layer (UV resin) are treated as acoustically rigid in simulations because of the strong contrast of acoustic impedance between these materials and air [29].

Simulations individually for the three rows in Fig. 2(b) (via two-dimensional axisymmetric simulations) indicate an almost unity transmission $|T|$ over a wide range of height ratio h_1/h [Fig. 2(c), blue curves], guaranteeing the efficient conversion of acoustic resonances to OAM. The effective wave number k^{eff}, calculated from $\arg(T)/l$, exhibits the required coverage of full $2k$ range [Fig. 2(c), red curves], where the eight dots give the eight discrete k_j^{eff} values for the eight sections in Fig. 2(a). Their transmissions have an average of 95.1 % ± 3.8 %.

The simulated transmission through the whole layer (via three-dimensional simulations in the air-filled waveguide) exhibits an expected twisted wave front with a screw dislocation along the propagation axis [Fig. 3(a)]. Distribution of phases and sound amplitudes at four cross sections [Fig. 3(b)] illustrates a transition from near to far field. The transition point would be around $a^2/\lambda = 0.11\lambda$, estimated from radiation of circular piston [30]. The distortion in both phases and amplitudes is obvious at the cross section $z = 0.01\lambda$ [left hand side panels in Fig. 3(b)], while at the rest of three cross sections $z \geq 0.11\lambda$ the phase regularly jumps 2π over one annular loop, revealing the expected topological charge $m = 1$. Another representative characteristic of the Bessel-like vortex – null pressure amplitude at the core – is also clearly shown in Fig. 3(b) (bottom), where a small asymmetry over the azimuth is due to the small differences of transmission $|T|$ among the resonators [Fig. 2(c)]. The overall transmission efficiency through the layer is 93.8% when calculated from squared root of out-going to in-coming sound power ratio.

Experiments to verify and demonstrate the OAM production are conducted in a 300-cm long waveguide and using a layer fabricated via 3D printing technology [Fig. 4(a) [31]]. A monochromatic sound, excited by a loudspeaker (4-inch diameter) facing into the waveguide at one end, propagates as a plane wave through the waveguide and illuminate on the layer placed at the middle of the waveguide. Transmitted sound is absorbed by sound absorbing foam at the other end of the waveguide. Three-dimensional sound field scan is conducted by employing two microphones (1/4-inch, Brüel & Kæjr type-4961) with a mobile one scanning the out-going field with OAM and a fixed one detecting the in-coming wave as a reference signal. Phase and amplitude of sound in each scan point are retrieved from cross-spectrum of the two signals.

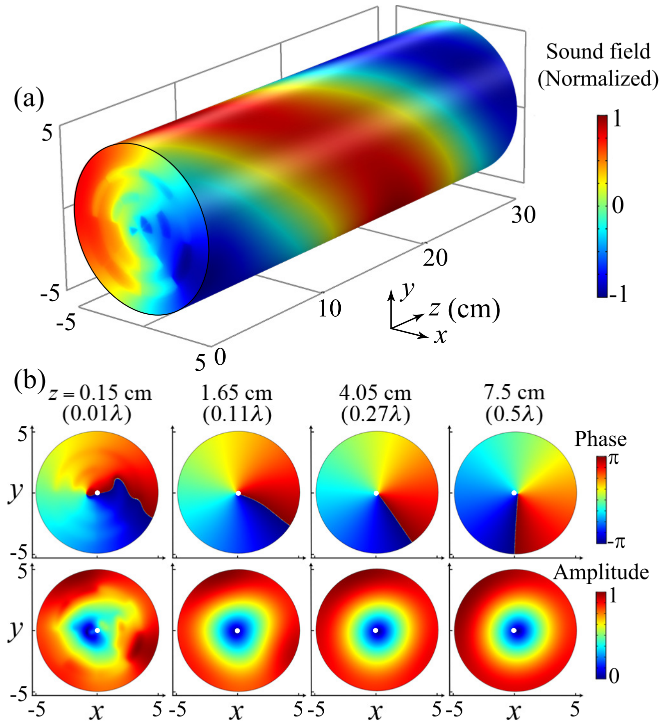

FIG. 3. Numerical simulations. (a) Airborne sound pressure field on outgoing surface of the planar layer (located at $-0.5\lambda \leq z \leq 0$) and inner surface of a cylindrical waveguide of a 5-cm radius. (b) Phase (top) and amplitude (bottom) of the field at four cross sections, illustrating the transition from near- to far-field, where the geometric centers of the cross sections are denoted by the white dots. The simulations are for sound of a frequency 2287 Hz ($\lambda = 15$ cm in air).

The measured phase distributions at far-field cross sections $z = 0.27\lambda$ and 0.5λ [Fig. 4(b)] recover the corresponding simulated results in a good shape [Fig. 3(b)]. The phase profile is measured (via cross correlation) to rotate an angle of 0.217π radians between these two cross sections, revealing an axial wave number $k_z = 19.8$ rad/m (given the known propagation distance of 0.23λ in between), verifying a value of $k_z = \sqrt{k^2 - k_{m,n}^2} = 20$ rad/m calculated from the known frequency and cylinder geometry.

The corresponding sound amplitude measured at the two far-field cross sections is shown in Fig. 4(c) for a direct comparison with simulated results and theoretical profiles $J_1(k_{1,1}r)$ [cf. Eq. (3)]. The results show the primary characteristics of the $m = 1$ Bessel-like vortex in the measured amplitude. The transmission (square root of out-going to in-coming sound power ratio) is measured to be 88.4 % [32], verifying the high efficiency and the effectiveness of the proposed scheme in converting acoustic resonances to OAM.

The underlying mechanism on manipulating k^{eff} allows for adjusting resonant frequencies and selecting the propagating mode via tailoring structural parameters of

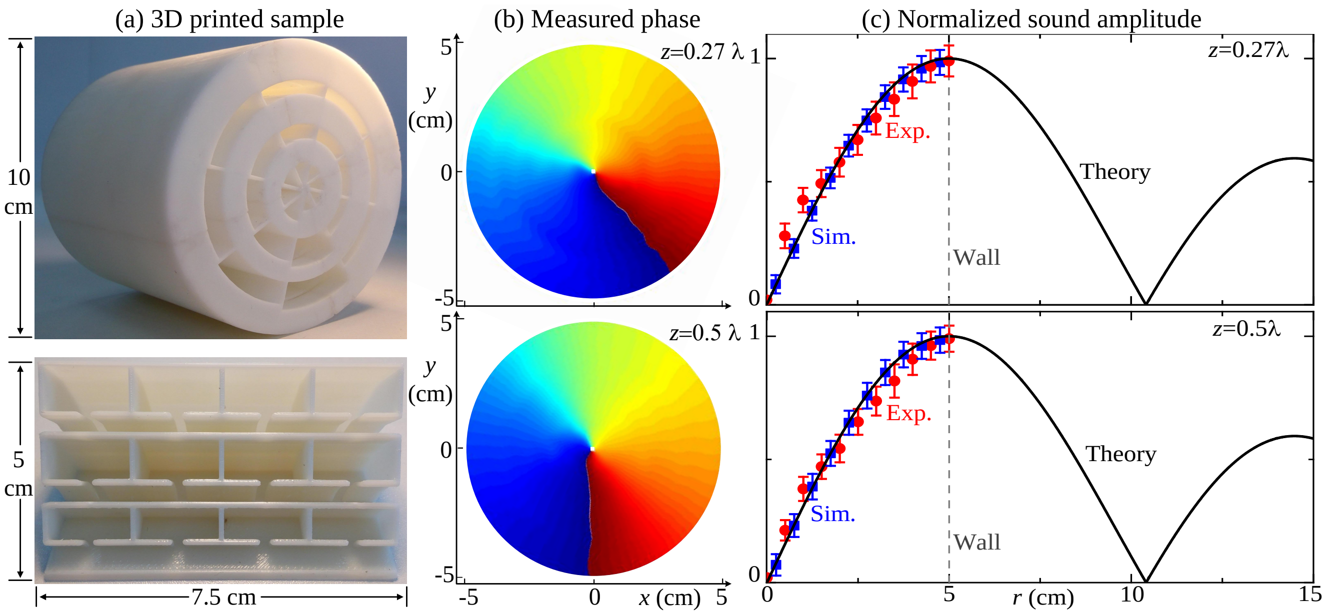

FIG. 4. Experiments. (a) The assembled sample (top) and its individual section (bottom), fabricated by 3D printing technology with UV resin, for generating a first-order Bessel-like vortex beam in experiments. (b)Phase distributions measured at $z = 0.27\lambda$ (top) and 0.5λ (bottom). (c) Measurements of sound pressure amplitudes (red) as functions of radius r at the same z as in (b) are compared with numerical simulations (blue) and theoretical Bessel profiles (black). The measurements are taken at every $\pi/9$ in the azimuthal and every 0.5 cm in the radial. The measurements and simulations are shown in (c) as an average (dots) and uncertainty (error bars) over the azimuthal. The measured transmission at these two cross sections is 88.4%.

the resonators. A proper frequency range ensures the conversion to the desired mode, chosen as the $(1, 1)$ mode in this study, while other modes completely fall in the evanescent regime and are trapped in the near field.

Discussions. — We have used a resonant planar layer to produce a new mechanism for generating acoustic beams with OAM. Performance of the mechanism was demonstrated by employing the resonant layer to produce OAM of a first-order airborne vortex beam of a smooth spiral phase and of a Bessel-like profile. In comparison with existing ways for OAM production by phased spiral sources that need sophisticated electronic control and by physically spiral sources that need screw profiles and may also have a bulky size, our acoustic resonance-based OAM production via manipulating effective wave numbers k^{eff} bears the advantages of high efficiency, compact size and planar profile.

The conversion of OAM from fundamental phenomena of acoustic resonances opens an avenue for producing acoustic vortex beams with OAM and could promote practical applications of the screw wave fields. Given that the resonant planar layer employed here has a small radius, the generated vortex beam would have a strong divergence, but that divergence doesn't occur here for propagation in a waveguide. A layer with a larger radius though, with an increased number of resonators in both the radial and azimuthal directions, can be employed to generate a less diverging beam in free space for applications in long range alignment [2], information transmission [33], etc. The larger layer can even be used to generate a focused vortex beam provided that both the k^{eff} and transmission $|T|$ have a desired dependence on both the azimuth and radius by properly selecting parameters of the resonators [Fig. 2(c)].

We may in principle extend the present mechanism of producing OAM via acoustic resonance-based manipulation of effective wave number k^{eff} to underwater propagation, but that environment requires alternative materials for a sufficient contrast of impedance with that of water or alternative media for resonances in water (e.g., soft media [34]).

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