Complete Construction of Magical, Symmetric, and Homogeneous N=2 Supergravities as Double Copies of Gauge Theories

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Complete construction of magical, symmetric and homogeneous $\mathcal{N}=2$ supergravities as double copies of gauge theories

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We show that scattering amplitudes in magical, symmetric or homogeneous $\mathcal{N}=2$ Maxwell-Einstein supergravities can be obtained as double copies of two gauge theories, using the framework of color/kinematics duality. The left-hand copy is $\mathcal{N}=2$ super-Yang-Mills theory coupled to a hypermultiplet, whereas the right-hand copy is a non-supersymmetric theory that can be identified as the dimensional reduction of a D-dimensional Yang-Mills theory coupled to P fermions. For generic D and P, the double copy gives homogeneous supergravities. For $P=1$ and $D=7,8,10,14$, it gives the magical supergravities. We compute explicit amplitudes, discuss their soft limits and study the UV-behavior at one loop.

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Perturbative calculations in gravity and gauge theory have long been considered to be on fundamentally different footing. Gravity is characterized by a non-polynomial, non-renormalizable action that produces an infinite number of interaction vertices, whereas renormalizable gauge theories only have cubic and quartic interactions. Despite these obvious differences, modern work has clarified that the perturbative expansion of gravity is directly related to that of a pair of gauge theories through a double-copy structure.

It has long been known that the asymptotic states of gravity can be obtained as tensor products of gauge-theory states. That such a simple relationship can be extended to certain interacting theories was first shown 30 years ago by Kawai, Lewellen and Tye [1] using string theory. Modern understanding of this double-copy structure comes from work by Bern, Carrasco and one of the current authors [2, 3], who found a framework that is applicable to loop-level amplitudes and to a broader range of theories. The key observation is that gauge-theory amplitudes can be organized to expose a kinematic Lie algebra which mirrors the gauge-group color structure. Once gauge-theory amplitudes exhibit this duality between color and kinematics, gravity amplitudes are obtained by substituting the color factors with equivalent kinematic objects. This procedure doubles the kinematic structures and thus expresses spin-2 theories as double copies of spin-1 theories [2].

The double-copy construction has proven itself to be a powerful computational tool. It fostered rapid progress in ultraviolet (UV) studies up to four loops in maximal, half-maximal and $\mathcal{N}=5$ supergravities [4–6]. Moreover, a class of black-hole solutions has been shown to exhibit a double-copy structure which relates them to solutions of Maxwell's equations with sources [7–9].

The double copy permits the construction of a broad range of gravitational theories by varying the content of matter (spin $\leq 1/2$) fields and their representations and interactions in the two gauge theories. Pure and matter-coupled gravities, including examples of Maxwell-Einstein and Yang-Mills-Einstein theories, are some of the theories that admit an elegant perturbative formulation in this framework [1–3, 10–17].

A systematic classification of $\mathcal{N}<4$ supergravities that admit double-copy constructions has not yet been obtained. There is a rich space of such theories, and it is not *a priori* obvious that the double copy can reproduce this abundance. Indeed, in this context it is natural to ask whether the double-copy structure can be a general property of gravitational theories.

In this letter we consider $\mathcal{N}=2$ Maxwell-Einstein supergravity (MESG) theories dimensionally reduced from five to four spacetime dimensions. These theories provide a tractable arena in which structures underlying generic gravitational theories can be uncovered. Unlike more supersymmetric theories, they are not uniquely specified by their matter content alone. However, due to their five-dimensional origin, theories in this class can be identified from their three-point interactions [18]. Using this property, we provide a double-copy construction for three complete classes of $\mathcal{N}=2$ MESG theories: magical, symmetric, and homogeneous theories (the latter class containing the former). General homogeneous theories cannot be constructed as truncations of $\mathcal{N}=8$ or matter-coupled $\mathcal{N}=4$ supergravity; their string-theory origin is also unclear. Our construction represents a major advance towards unraveling the double-copy structure of general gravitational theories. Homogeneous supergravities now constitute the largest known family of double-copy-constructible theories.

Homogeneous $\mathcal{N}=2$ MESG theories: While we are ultimately interested in MESG theories in four dimensions, we shall begin our analysis in five dimensions. Unlike $4D$ theories, the full U-duality groups of $5D$, $\mathcal{N}=2$ MESG theories are symmetries of their Lagrangians. Furthermore, $\mathcal{N}=2$ MESG theories that describe low-energy effective theories of compactified M/superstring theory admit uplifts to five dimensions once quantum corrections are neglected [19]. When coupled to n vector multiplets, such five-dimensional theories contain $(n+1)$ abelian vector fields A^I_μ $(I,J=0,\ldots,n)$, n real scalar fields ϕ^x $(x,y=1,\ldots,n)$, and n symplectic-Majorana spinors. Their Lagrangian is [18]:

$$e^{-1}\mathcal{L} \;=\; -\frac{1}{2}R-\frac{1}{4}\overset{\circ}{a}_{IJ}F^I_{\mu\nu}F^{\mu\nu J}-\frac{1}{2}g_{xy}(\partial_\mu\phi^x)(\partial^\mu\phi^y) +\frac{e^{-1}}{6\sqrt{6}}C_{IJK}\varepsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda+\text{fermions}\;, \quad (1)$$

where $F^I_{\mu\nu}$ are abelian field-strengths. A remarkable property of these theories is that the Lagrangian is uniquely determined by the constant symmetric tensor C_{IJK} whose invariance group coincides with the U-duality group. The scalar manifold of $5D$ MESG theories can be interpreted as the hypersurface defined by $\mathcal{V}(\xi)\equiv(2/3)^{3/2}C_{IJK}\xi^I\xi^J\xi^K=1$ in an $(n+1)$-dimensional ambient space with the metric

$$a_{IJ}(\xi)\equiv-\frac{1}{2}\frac{\partial}{\partial\xi^I}\frac{\partial}{\partial\xi^J}\ln\mathcal{V}(\xi)\,. \quad (2)$$

The matrix $\overset{\circ}{a}_{IJ}$ in the kinetic-energy term of the vector fields is the restriction of the ambient-space metric to the constraint surface, while the metric g_{xy} of the scalar manifold is the pullback of the ambient-space metric to the constraint surface.

The given structure is sufficient to calculate the bosonic part of the amplitudes we will discuss in this letter. We refer the reader to [18] for further details and for the fermionic terms. These terms involve a symmetric tensor T_{xyz} which is the pullback of the C-tensor to the constraint surface. The C-tensors of the theories with covariantly-constant T_{xyz} are defined by the cubic norms of Euclidean Jordan algebras of degree three, and their scalar manifolds are symmetric spaces [18]. The four magical MESG theories are defined by simple Jordan algebras of Hermitian 3×3 matrices over real and complex numbers, quaternions, and octonions [20]. They are the only *unified* MESG theories in five dimensions with homogeneous scalar manifolds. The unique unified $5D$ Yang-Mills Einstein theory with homogeneous scalar manifold is obtained by gauging the quaternionic magical MESG theory [21]. The generic Jordan theories are defined by the infinite family of non-simple Jordan algebras of degree three. These two classes exhaust the list of $5D$ MESG theories with symmetric target spaces such that the full isometry group is a symmetry of the Lagrangian. There exists another infinite family of MESG theories with symmetric target manifolds, the so-called generic non-Jordan family [22], where not all the isometries of the target manifolds extend to symmetries of the $5D$ Lagrangian [23].

The most general form of the C-tensor consistent with unitarity was given in ref. [18] and depends on $n(n^2-1)$ parameters. The cubic norms $\mathcal{V}(\xi)$ of MESG theories with homogeneous scalar manifolds and a transitive group of isometries can be brought to the form [23]

$$\mathcal{V}(\xi)=\sqrt{2}\big(\xi^0(\xi^1)^2-\xi^0(\xi^i)^2\big)+\xi^1(\xi^\alpha)^2+\tilde{\Gamma}^i_{\alpha\beta}\xi^i\xi^\alpha\xi^\beta\;, \quad (3)$$

where $i,j=2,3,\ldots,q+2$ and α,β are indices with range r. $\tilde{\Gamma}^i_{\alpha\beta}$ are symmetric gamma matrices forming a real representation of the Clifford algebra $\mathcal{C}(q+1,0)$. $\mathcal{V}(\xi)$ in eq. (3) are generically labeled by two integers $q\geq-1$ and $P\geq0$, except when $q=0,4 \pmod 8$, in which case the extra parameter $\dot{P}\geq0$ is also present.

The corresponding MESG theories give the coupling of $(2+q+r)$ vector multiplets to the gravity multiplet in $5D$, with $r=P\mathcal{D}_q$ or $r=(P+\dot{P})\mathcal{D}_q$. The values for $\mathcal{D}_q$ are listed in table I. The generic Jordan family corresponds to $q=\dot{P}=0$ and P arbitrary and to $P=\dot{P}=0$ and q arbitrary; the magical theories correspond to $P=1$ and $q=1,2,4,8$, while the generic non-Jordan family theories correspond to $q=-1$.

Upon dimensional reduction the Lagrangian (1) can be used to describe four-dimensional MESG theories. The homogeneous $\mathcal{N}=2$ MESG theories in $4D$ were first classified in ref. [24] using the so-called C-map and the known classification of homogeneous quaternionic manifolds [25]. However, the list of ref. [24] is not complete. There exists an additional infinite family of homogeneous theories that descend from the generic non-Jordan family, which under the C-map lead to a novel family of homogeneous quaternionic manifolds [23]. A further infinite family of $4D$ MESG theories can be found with symmetric target manifolds $SU(n,1)/U(n)$, which does not descend directly from $5D$ [26] but that can be obtained by truncation from the generic Jordan family.

The bosonic spectrum of the 4D MESG theory that descends from $5D$ contains the graviton, $(n+2)$ vectors $A^{-1}_\mu,\ldots,A^n_\mu$ and $(n+1)$ complex scalars $z^0,\ldots,z^n$. The $4D$ Lagrangian is associated to the following holomorphic prepotential in a symplectic formulation [24, 27–29],

$$F(Z^A)=-\frac{2}{3\sqrt{3}}\frac{C_{IJK}Z^IZ^JZ^K}{Z^{-1}}\;, \quad (4)$$

where $Z^A(z)$ are holomorphic functions of the scalars z^I. To carry out perturbation theory it is necessary to expand the Lagrangian around some base point, for which a canonical choice is $Z^A=\big(1,\frac{i}{2},\frac{i}{\sqrt{2}},0,\ldots,0\big)$. We then redefine (and dualize) fields to enlarge the manifest symmetry and obtain canonically-normalized quadratic terms. We refer the reader to the supplemental material [33]

q	$\mathcal{D}_q$	$r(q,P,\dot{P})$	conditions	flavor group
-1	1	P	R	$SO(P)$
0	1	$P+\dot{P}$	RW	$SO(P)\times SO(\dot{P})$
1	2	$2P$	R	$SO(P)$
2	4	$4P$	R/W	$U(P)$
3	8	$8P$	PR	$USp(2P)$
4	8	$8P+8\dot{P}$	PRW	$USp(2P)\times USp(2\dot{P})$
5	16	$16P$	PR	$USp(2P)$
6	16	$16P$	R/W	$U(P)$
$k+8$	$16\,\mathcal{D}_k$	$16\,r(k,P,\dot{P})$	as for k	as for k

TABLE I: Parameters in the construction of homogeneous MESGs as double copies. The second column gives the parameter $\mathcal{D}_q$, the third column gives the number r of $4D$ irreducible spinors in the non-supersymmetric gauge theory, which can obey a reality (R), pseudo-reality (PR) or Weyl (W) conditions. The flavor group is listed in the last column.

and to ref. [17] for technical details. Finally, the resulting Lagrangian is used to construct amplitudes which are compared with the ones from the double copy.

Double-copy construction: The m-point amplitudes of Yang-Mills (YM) theories are naturally represented by cubic graphs labeled by their topology, gauge-group representations of internal and external edges, and particle momenta. The i-th graph is associated to the product of the corresponding propagators, to a color factor c_i constructed by dressing each cubic vertex by the Clebsh-Gordan coefficient of the representations of the three fields (structure constants or group generators), and to a kinematic numerator n_i encoding the remaining state dependence. To construct an amplitude which manifestly obeys color/kinematics duality one must find kinematic numerators with the same symmetries and algebraic identities as the color factors [2]. Schematically

$$c_i - c_j = c_k \quad \Leftrightarrow \quad n_i - n_j = n_k \ , \tag{5}$$

where the color factor identities stem from the commutation/Jacobi relations of the gauge group and thus involve three graphs.

The double-copy principle states that, once duality-satisfying numerators are found, the L-loop amplitudes of a supergravity theory are given by

$$\mathcal{M}_m^{(L)} = i^{L+1}\left(\frac{\kappa}{2}\right)^{2L+m-2} \sum_{i\in\text{cubic}} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} s_{\alpha_i}}, \tag{6}$$

where κ is the gravity coupling, S_i are symmetry factors, and $1/s_{\alpha_i}$ are propagator denominators. The $n_i, \tilde{n}_i$ may be identical or distinct gauge-theory numerators. The formula is valid if at least one of the two sets of numerators satisfies manifestly the duality [3, 30]. In our construction, we obtain n_i from a supersymmetric gauge theory, and $\tilde{n}_i$ from a non-supersymmetric one. It is critical that we consider gauge theories with some fields transforming in a generic representation R of the gauge group. Indeed, a judicious choice for the representation R will enable us to capture a larger set of supergravities with our construction.

The gauge-theory copies: The first (left) gauge theory entering the construction is an $\mathcal{N}=2$ super-Yang-Mills (SYM) theory with a single half-hypermultiplet transforming in a pseudo-real representation R of a gauge group G. To be precise, pseudo-real means that there exists a unitary antisymmetric matrix V obeying $VT^{\hat{a}}V^\dagger = -(T^{\hat{a}})^*$, where $T^{\hat{a}}$ are the representation matrices. With this choice, the half-hypermultiplet is CPT self-conjugate and we do not need to include additional fields in the theory. We will see that using the smallest possible multiplet allows to formulate double-copy constructions for larger classes of supergravities, including in particular all the magical supergravity theories. A canonical example for R is the fundamental representation of $USp(2N)$. Note that the matrix V can be used to lower or raise gauge representation indices.

The on-shell spectrum of the supersymmetric gauge-theory factor is

$$\left(A^{\hat{a}}_+, \psi^{\hat{a}}_+, \phi^{\hat{a}}\right)_G \oplus \left(A^{\hat{a}}_-, \psi^{\hat{a}}_-, \bar{\phi}^{\hat{a}}\right)_G \oplus \left(\chi_+, \varphi_1, \varphi_2, \chi_-\right)_R \ ,$$

where $\hat{a}, \hat{b}$ are adjoint indices of G, and indices corresponding to the representation R are suppressed. Amplitudes in this theory can be conveniently organized into superamplitudes with manifest $\mathcal{N}=2$ supersymmetry [33].

The second (right) gauge theory is a non-supersymmetric YM theory with $(q+2)$ scalars and r fermions. Its Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F^{\hat{a}}_{\mu\nu}F^{\hat{a}\mu\nu} + \frac{1}{2}(D_\mu\phi^a)^{\hat{a}}(D^\mu\phi^a)^{\hat{a}} + \frac{i}{2}\overline{\lambda}^\alpha D_\mu\gamma^\mu\lambda_\alpha \\ &+ \frac{g}{2}\phi^{a\hat{a}}\Gamma^a_\alpha{}^\beta\overline{\lambda}^\alpha\gamma_5 T^{\hat{a}}\lambda_\beta - \frac{g^2}{4}f^{\hat{a}\hat{b}\hat{e}}f^{\hat{c}\hat{d}\hat{a}}\phi^{a\hat{a}}\phi^{b\hat{b}}\phi^{a\hat{c}}\phi^{b\hat{d}}.\end{aligned} \tag{7}$$

The scalars transform in the adjoint representation of G, while fermions transform in the pseudo-real representation R. As before $\hat{a}, \hat{b}$ are adjoint gauge-group indices, while $\alpha, \beta = 1, \ldots, r$ and $a, b = 1, \ldots, q+2$ are global-symmetry indices. Spacetime spinor indices and indices associated to the representation R are not displayed. Imposing color/kinematics duality on the numerators of four-point amplitudes [33] gives the following constraint in the two-scalar-two-fermion case:

$$n_u - n_t = n_s \quad \to \quad \{\Gamma^a, \Gamma^b\} = 2\delta^{ab} \ , \tag{8}$$

i.e. that the constant matrices Γ^a appearing in the Yukawa couplings form a $(q+2)$-dimensional Clifford algebra. It is convenient to think of the theory above as the dimensional reduction of a $(q+6)$-dimensional YM theory with fermionic matter to four dimensions. From a higher-dimensional perspective, the spinor λ_α includes

P copies (or flavors) of irreducible $SO(q+5,1)$ spinors, obeying reality (R) or pseudo-reality (PR) conditions:

$$\overline{\lambda} = \lambda^t \mathcal{C}_4 C V \ , \qquad \text{R}: \ C = \mathcal{C}_q \ , \quad \text{PR}: \ C = \mathcal{C}_q \Omega \ , \quad (9)$$

where $\mathcal{C}_q$ and $\mathcal{C}_4$ are the $SO(q+2)$ and $SO(3,1)$ charge-conjugation matrices with $\mathcal{C}_q \Gamma^a \mathcal{C}_q^{-1} = -\zeta(\Gamma^a)^t$, $\mathcal{C}_4 \gamma^\mu \mathcal{C}_4^{-1} = -\zeta(\gamma^\mu)^t$, $\zeta = \pm 1$. Ω is an anti-symmetric real matrix acting on the flavor indices, V is the matrix in the pseudo-reality condition for the gauge representation matrices, and C is unitary. R conditions are appropriate for $q = 0,1,2,6,7$ (mod 8) and generically yield a $SO(P)$ manifest flavor symmetry. PR conditions are imposed for $q = 3,4,5$ (mod 8) and yield a $USp(2P)$ flavor symmetry.

For even q, we can impose Weyl conditions of the form $\Gamma_* \lambda = \pm\lambda$, where Γ_* is the chirality matrix. For $q = 0,4$ (mod 8), Weyl conditions are compatible with R and PR conditions, and the representations with different chiralities are inequivalent. Hence the corresponding theories are parameterized by two distinct integers P and $\dot{P}$ counting the number of representations of each kind. Finally, for $q = 2,6$ (mod 8) one can rewrite the Lagrangian in terms of Weyl spinors, enhancing the manifest flavor symmetry to $U(P)$.

From a double-copy perspective, the resulting $4D$ supergravity theory has one vector multiplet for each $4D$ fermion in the non-supersymmetric gauge theory. The various possibilities are listed in table I, which provides a novel perspective on the results of ref. [23]. In particular, the parameter $\mathcal{D}_q$ introduced in that paper equals the minimal number of $4D$ fermions in the non-supersymmetric gauge theory. Indeed, a large part of the supergravity symmetry is already manifest in this gauge theory. The full U-duality Lie algebras of $4D$ homogeneous supergravity theories decompose as $\mathcal{G} = \mathcal{G}_0 \oplus \mathcal{G}_1 \oplus \mathcal{G}_2$ with

$$\begin{aligned} &\mathcal{G}_0 = so(1,1) \oplus so(q+2,2) \oplus \mathfrak{s}_q(P,\dot{P}) \ , \\ &\mathcal{G}_1 = (1, \text{spinor}, \text{vector}), \quad \mathcal{G}_2 = (2,1,1) \ , \end{aligned} \quad (10)$$

where $\mathfrak{s}_q(P,\dot{P})$ is the flavor group in table I, and the grade 1 and 2 generators are labeled by their grade zero representations. The $4D$ supergravity theories with symmetric target spaces have additional symmetry generators corresponding to the grade -1 and -2 subspaces of the isometry Lie algebras [18].

Amplitudes from the double copy: For the $(\mathcal{N}=2) \otimes (\mathcal{N}=0)$ construction given here, the identification of supergravity states with the double-copy of asymptotic gauge-theory fields is as follows:

$$\begin{aligned} A_-^{-1} &= \bar{\phi} \otimes A_- \ , & h_- &= A_- \otimes A_- \ , \\ A_-^0 &= \phi \otimes A_- \ , & i\bar{z}^0 &= A_+ \otimes A_- \ , \\ A_-^a &= A_- \otimes \phi^a \ , & i\bar{z}^a &= \bar{\phi} \otimes \phi^a \ , \\ A_{\alpha-} &= \chi_- \otimes (U\lambda_-)_\alpha \ , & i\bar{z}_\alpha &= \chi_+ \otimes (U\lambda_-)_\alpha \ , \end{aligned} \quad (11)$$

with similar relations for the CPT-conjugate states. Here U is a unitary rotation of the spinors in the non-supersymmetric theory. The scalar-fermion-fermion amplitude in the non-supersymmetric theory takes the form $\mathcal{A}_3^{(0)}(1\phi^a, 2\lambda_{\alpha-}, 3\lambda_{\beta-}) = -ig/\sqrt{2}\,\langle 23\rangle (\Gamma^a C^{-1})_{\alpha\beta} T^{\hat{a}} V^{-1}$, where C is the matrix in eq. (9). With the identification (11), the double copy (6) of the above and a vector-hyper-hyper amplitude gives, for example, the following vector-vector-scalar amplitude:

$$\mathcal{M}_3^{(0)}(1A_-^a, 2A_{\alpha-}, 3\bar{z}_\beta) = \frac{\kappa}{2\sqrt{2}} \langle 12\rangle^2 (U^t \Gamma^a C^{-1} U)_{\alpha\beta} \quad (12)$$

where global spinor indices have been restricted to the subspaces of appropriate chiralities for $q = 0,4$ (mod 8). The matrices Γ^a are related to the matrices $\tilde{\Gamma}^i$ in the cubic norm (3) as $(U^t \Gamma^a C^{-1} U) = (-\mathbb{1}, i\tilde{\Gamma}^i)$. Explicit expressions for U can be found for each q as explained in the supplemental material [33]. We have verified that three-point amplitudes from the double copy match exactly the ones computed from the supergravity Lagrangian.

It is possible to confirm, without any reference to a Lagrangian, that our construction yields supergravities with scalar manifolds that are locally homogeneous close to the base point. Indeed, a generalization of the arguments of ref. [31] implies that all single soft-scalar limits of amplitudes vanish for scalars parameterizing a homogeneous manifold. It is easy to see that this is so if the soft particle (scalar) transforms under a manifest symmetry. Since all the double-copy scalars except the dilaton-axion pair z^0 transform under the manifest $SO(q+2)$ global symmetry, only the soft dilaton/axion limit requires a detailed analysis. Its vanishing implies that the double-copy theory is invariant under an additional $U(1)$ symmetry. We have verified that this is indeed the case and that the tree-level amplitudes with field configurations with a total non-zero $U(1)$ charge vanish identically at four and five points.

Our construction carries over to loop-level amplitudes. As an example, we give the one-loop divergence for amplitudes between four identical matter vectors:

$$\begin{aligned} \mathcal{M}_4^{(1)}(1A_-^0, 2A_-^0, 3A_+^0, 4A_+^0)\Big|_{\text{div}} &= \frac{b}{\epsilon}\Big(\frac{10}{3} - \frac{q}{6} + \frac{r}{3}\Big) \ , \\ \mathcal{M}_4^{(1)}(1A_-^a, 2A_-^a, 3A_+^a, 4A_+^a)\Big|_{\text{div}} &= \frac{b}{\epsilon}\Big(\frac{10}{3} + \frac{q}{3} + \frac{r}{12}\Big) \ , \end{aligned}$$

with $b = 2i/(4\pi)^2(\kappa/2)^4\langle 12\rangle^2[34]^2$. Interestingly, the two amplitudes have the same divergence when $r = 2q$. This condition is satisfied only by the four magical theories, which are unified, and by the so-called STU model ($q = r = 0$) [32].

In conclusion, we have shown that scattering amplitudes in homogeneous $\mathcal{N}=2$ supergravities – including magical and symmetric theories – can be obtained as double copies of two simple gauge theories using the framework of color/kinematics duality. To date, this is the largest known family of double-copy-constructible theories. Color/kinematics duality naturally requires the Clifford algebra structure that has been instrumental in the classification of homogeneous theories and provides an alternative perspective on these theories; in particular, the homogeneity of their target spaces manifests itself in the amplitudes' vanishing soft limits. We note that it is straightforward to introduce supergravity hypermultiplets in our construction by adding scalars transforming in the representation R to the non-supersymmetric gauge theory. The double-copy approach is particularly well-suited for carrying out loop-level computations. The existence of a double-copy construction for such a large family of theories suggests that the double-copy can play a fundamental role in general gravity theories. Generalizations of our construction to accommodate even larger classes of theories, including supergravities with a lower number of isometries and gauged R-symmetry groups, appears to be within reach.

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[33] Supplemental Material for "Complete construction of magical, symmetric and homogeneous $\mathcal{N} = 2$ supergravities as double copies of gauge theories."