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Event-by-event hydrodynamics + jet energy loss: A solution to the $R_{AA} \otimes v_2$ puzzle

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High $p_T > 10$ GeV elliptic flow, which is experimentally measured via the correlation between soft and hard hadrons, receives competing contributions from event-by-event fluctuations of the low p_T elliptic flow and event plane angle fluctuations in the soft sector. In this paper, a proper account of these event-by-event fluctuations in the soft sector, modeled via viscous hydrodynamics, is combined with a jet energy loss model to reveal that the positive contribution from low p_T v_2 fluctuations overwhelms the negative contributions from event plane fluctuations. This leads to an enhancement of high $p_T > 10$ GeV elliptic flow in comparison to previous calculations and provides a natural solution to the decade long high p_T $R_{AA} \otimes v_2$ puzzle. We also present the first theoretical calculation of high p_T v_3 , which is shown to be compatible with current LHC data. Furthermore, we discuss how short wavelength jet-medium physics can be deconvoluted from the physics of soft, bulk event-by-event flow observables using event shape engineering techniques.

1. Introduction. Inspired by Bjorken's original jet quenching idea [1], the energy loss experienced by fast moving partons in the Quark-Gluon Plasma (QGP) formed in heavy ion collisions has been studied using the nuclear modification factor (at mid-rapidity) $R_{AA}(p_T, \phi) = \frac{dN_{AA}/dp_T d\phi}{N_{\text{coll}} dN_{pp}/dp_T}$, where dN_{AA}/dp_T is the spectrum of the corresponding particle species (e.g., pions) in AA collisions, dN_{pp}/dp_T is the corresponding proton-proton yield, ϕ is the azimuthal angle in the plane transverse to the beam direction, and N_{coll} is the total number of binary collisions [2]. The azimuthally averaged version of this quantity, $R_{AA}(p_T) = \frac{1}{2\pi} \int_0^{2\pi} d\phi R_{AA}(p_T, \phi)$, was predicted [3–6] and later experimentally observed at RHIC [7–11] to strongly depend on geometrical control parameters that involve length scales of the order of the radius of a large nucleus (e.g., ~ 7.5 fm for Au) such as the centrality (multiplicity) of the collisions. This program has been successfully extended to LHC energies [12], whose central collisions produce a hotter, twice as dense QGP than the one formed at RHIC's top energies [13]. The combination of RHIC and LHC data has been instrumental to determine how jets couple with the evolving medium [14, 15] and to motivate new theoretical and phenomenological studies [16–43].

It was recognized early on [44–46] that the azimuthal anisotropy of high p_T hadrons encoded in $R_{AA}(p_T, \phi)$ was a powerful tool to study the energy loss and the path length dependence of hard partons in the QGP. The anisotropic flow coefficients associated with $R_{AA}(p_T, \phi)$ can be computed from its Fourier series

$$\frac{R_{AA}(p_T, \phi)}{R_{AA}(p_T)} = 1 + 2 \sum_{n=1}^{\infty} v_n^{\text{hard}}(p_T) \cos [n\phi - n\psi_n^{\text{hard}}(p_T)] \quad (1)$$

where

$$v_n^{\text{hard}}(p_T) = \frac{\frac{1}{2\pi} \int_0^{2\pi} d\phi \cos [n\phi - n\psi_n^{\text{hard}}(p_T)] R_{AA}(p_T, \phi)}{R_{AA}(p_T)} \quad (2)$$

$$\text{and } \psi_n^{\text{hard}}(p_T) = \frac{1}{n} \tan^{-1} \left(\frac{\int_0^{2\pi} d\phi \sin(n\phi) R_{AA}(p_T, \phi)}{\int_0^{2\pi} d\phi \cos(n\phi) R_{AA}(p_T, \phi)} \right).$$

While the azimuthally averaged $R_{AA}(p_T)$ can be described by many different models, it has proven to be a challenge in the field (see discussion in Refs. [36, 37]) to obtain a simultaneous description of $R_{AA}(p_T)$ and high p_T elliptic flow. Model calculations typically give a small $v_2^{\text{hard}}(p_T)$ that is incompatible with elliptic flow data.

An important detail that has been overlooked so far in model calculations is that the theoretical $v_2^{\text{hard}}(p_T)$ is not the appropriate quantity to be compared with experimental data. In fact, the experimental high $p_T > 10$ GeV flow coefficients $v_n^{\text{exp}}(p_T)$ are measured via the *correlation* between soft and hard hadrons in a given centrality class, emphasized in [47, 48]

$$v_n^{\text{exp}}(p_T) = \frac{\langle v_n^{\text{soft}} v_n^{\text{hard}}(p_T) \cos [n(\psi_n^{\text{soft}} - \psi_n^{\text{hard}}(p_T))] \rangle}{\sqrt{\langle (v_n^{\text{soft}})^2 \rangle}}, \quad (3)$$

where v_n^{soft} , ψ_n^{soft} are the integrated soft flow harmonic and the corresponding event plane angle [49] for all charged particles with $p_T \lesssim 3$ GeV and where $\langle \dots \rangle$ experimentally denotes an average over all events. The idealized limit, $v_2(p_T \gtrsim 10 \text{ GeV}) \sim v_2^{\text{hard}}(p_T)$, considered in previous model calculations, is not realistic since it neglects event-by-event fluctuations of the bulk geometry. Thus, a description of high p_T anisotropic flow $v_n(p_T)$ necessitates modeling of both the soft and hard sectors of heavy ion collisions.

Our understanding of the bulk (soft) properties of the evolving medium has progressed immensely since the first event-by-event hydrodynamic simulations were carried out more than a decade ago [50–53]. It is now understood [54] that the initial spatial anisotropies present in the early stages of nucleus-nucleus collisions are converted to final stage momentum anisotropies (i.e., anisotropic flow) in a way that is consistent with viscous relativistic hydrodynamic simulations performed on an event-by-event basis (see, e.g., the review [47]). Furthermore, once the full information regarding the soft event-by-event v_n distributions became available [55] at the LHC, powerful constraints on the initial conditions of the hydrodynamic modeling of the QGP have been obtained [56–60]. Nevertheless, these recent advances regarding event-by-event medium fluctuations have not yet been incorporated in theoretical studies of high p_T observables.

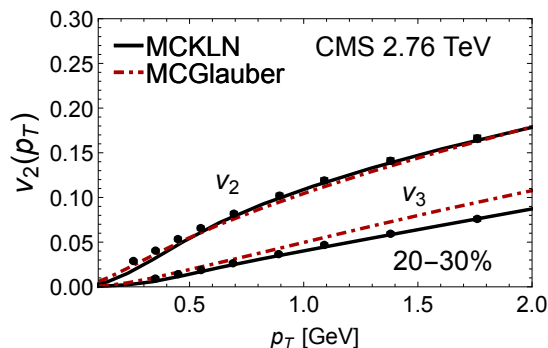


FIG. 1. (Color online) Model comparison to CMS data [82] for low $p_T < 2$ GeV $v_2(p_T)$ and $v_3(p_T)$ of charged particles.

In this paper we show that the long standing high p_T $R_{AA} \otimes v_2$ puzzle can be naturally solved by computing high p_T elliptic flow using its experimental definition in (3) that takes into account the effects of event-by-event fluctuations needed for a realistic description of the QGP. The soft sector is modeled via event-by-event hydrodynamic simulations performed using the v-USPhydro code [61–64] while the hard sector is described using the energy loss framework developed in [36, 65, 66]. We show that the positive contribution from low p_T v_2 fluctuations overwhelms the small, negative contributions from event plane fluctuations, which in turn leads to an overall enhancement of high p_T elliptic flow in comparison to previous calculations. The inclusion of initial state fluctuations followed by viscous hydrodynamics allowed us to perform the first theoretical calculation of high p_T v_3 , which is shown to be compatible with current LHC data.

2. Hydrodynamic evolution. The expanding QGP is modeled through event-by-event simulations performed using the 2+1 (i.e., boost invariant) viscous relativistic hydrodynamics code called v-USPhydro [61–64]. v-USPhydro accurately [68] solves the energy-momentum conservation equations and the equations of motion for

the dissipative currents using the Lagrangian formulation of hydrodynamics encoded in the Smoothed Particle Hydrodynamics (SPH) algorithm [69, 70]. Information about four transport coefficients is required: the temperature-dependent shear and bulk viscosities, η and ζ , and their respective relaxation time coefficients, τ_π and τ_Π (other 2nd order transport coefficients [71] are not yet taken into account). For simplicity, effects from the temperature dependence of η/s in the hadronic [72–74] or in the QGP phase [75] are neglected here and, thus, we set η/s to be a constant. Also, in this first study bulk viscosity [61, 62, 76] is set to zero. The initial time for all the hydrodynamic simulations was 0.6 fm and we used the lattice-based equation of state EOS S95n-v1 [77] and an isothermal Cooper-Frye [78] freezeout with freeze-out temperature $T_F = 120$ MeV for MCKLN and $T_F = 130$ for MCGLauber. Particle decays are included (with hadronic resonances with masses up to 1.7 GeV) via an adapted version of the AZHYDRO code [79].

MCGLauber and MCKLN initial conditions [80] for the mid-central 20 – 30% centrality class of $\sqrt{s} = 2.76$ TeV Pb+Pb collisions at the LHC that provide a good description of this eccentricity data (our results agree with Ref. [55]). Our results for the low p_T v_2 and v_3 for all charged particles (here averaged over 150 events) and the comparison to CMS data [82] ($\eta/s = 0.08$ in MCGLauber and $\eta/s = 0.11$ in MCKLN) determined using the event plane method [49] are shown in Fig. 1. While for $v_2(p_T)$ both types of initial conditions give a reasonable description of the low p_T data, due to $v_3(p_T)$ MCKLN is slightly favored over MCGLauber in our calculations.

3. Energy loss model. The nuclear modification factor and the high p_T azimuthal anisotropies are studied here using the BBMG jet-energy loss model developed in [36, 65, 66] supplemented with the energy density and flow profiles obtained from the event-by-event viscous hydrodynamic simulations for the soft sector described above. In this model, the energy loss per unit length experienced by a fast moving parton in the plasma, dE/dL , is modeled as

$$\frac{dE}{dL} = -\kappa E^a(L) L^z T^c \zeta_q \Gamma_{\text{flow}} \quad (4)$$

where κ is the jet-medium coupling [36], T is the local temperature field along the jet trajectory with $c = 2 + z - a$, ζ_q describes energy loss fluctuations [36], $\Gamma_{\text{flow}} = \Gamma_{\text{f}} = \gamma [1 - v \cos(\phi_{\text{jet}} - \phi_{\text{flow}})]$ is the flow factor defined using the local flow velocities of the medium $\vec{u} = \gamma \vec{v}$ (with $\gamma = 1/\sqrt{1 - \vec{v}^2}$) [83–86], ϕ_{jet} is the angle defined by the propagating jet in the transverse plane, and ϕ_{flow} is the local azimuthal angle of the hydrodynamic flow. In this framework the dependence of the energy loss rate with the jet energy E , path length L , temperature T , and energy loss fluctuations ζ_q is characterized by the parameters (a, z, c, q) .

We focus in this Letter on the “pQCD-scenario” dis-

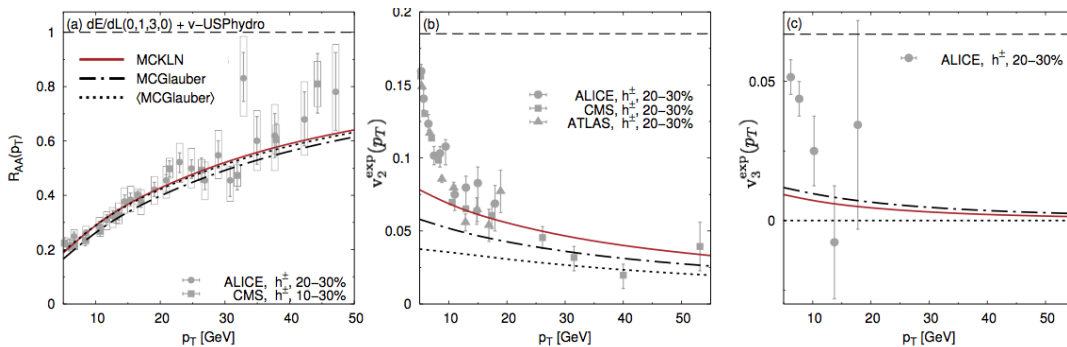


FIG. 2. (Color online) Model calculations for (a) $\pi^0 R_{AA}(p_T)$, (b) $v_2^{exp}(p_T)$, (c) $v_3^{exp}(p_T)$ in mid-central $\sqrt{s} = 2.76$ TeV Pb+Pb collisions at the LHC. MCKLN initial conditions are shown in solid red while the dotted-dashed black line is for MCGlauber. The black dotted line $\langle \text{MCGlauber} \rangle$ corresponds to results obtained neglecting any initial state fluctuations of the soft bulk background by evolving only an event averaged smoothed initial Glauber geometry. The experimental data are taken from Refs. [90–94].

cussed in detail in Ref. [36] where ($a = 0, z = 1, c = 3, q = 0$), i.e., $dE/dL \sim L$. Other dependences on the path length will be discussed elsewhere. The jets are distributed according to the given transverse profile for the medium given by v-USPhydro. The jet path $\vec{x}(L) = \vec{x}_0 + \hat{n}(\phi_{\text{jet}})L$ from a production point \vec{x}_0 is perpendicular to the beam axis and moves in the direction given by ϕ_{jet} . All jet production points with local temperature above 160 MeV are taken into account. In this study we used the KKP pion fragmentation functions [87], which have been tested against RHIC and LHC data [88].

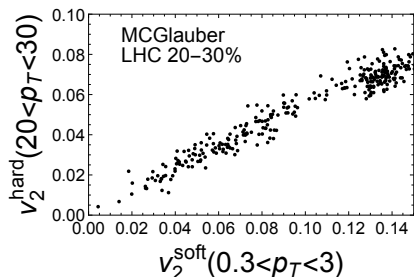


FIG. 3. Event-by-event correlation between v_2^{soft} (computed via viscous hydrodynamics) and v_2^{hard} (defined in Eq. (2)). The approximate linear correlation indicates that initial state fluctuations, which determine v_2^{soft} , also directly affect the 2nd harmonic of $R_{AA}(p_T, \phi)$.

4. Results for R_{AA} and v_n at high p_T . The κ parameter in the BBMG energy loss model is completely fixed (for each type of initial conditions) by matching the computed $\pi^0 R_{AA}(p_T = 10 \text{ GeV})$ to all-charged data in 0-5% most central LHC collisions (initial comparisons between π^0 and all charged for v_2 of high- p_T are very similar [89]). Our results for $\pi^0 R_{AA}(p_T)$ in mid-central Pb+Pb $\sqrt{s} = 2.76$ TeV collisions at the LHC and the experimental data [90, 91] are shown in Fig. 2 (a). The solid red line corresponds to the results computed using the hydro-

dynamic evolution based on the MCKLN initial condition while the dotted-dashed black curve denotes results computed using MCGlauber initial conditions. The black dotted line corresponds to results obtained neglecting any initial state fluctuations of the soft background by evolving only an event averaged, smoothed initial Glauber geometry. While as expected $R_{AA}(p_T)$ is robust with respect to the inclusion of event-by-event fluctuations, the same cannot be said about the anisotropic flow coefficients $v_2(p_T)$ and $v_3(p_T)$, computed using (3), and shown in Fig. 2 (b) and (c). By comparing the dotted and the dashed-dotted curves (both computed using MCGlauber) one can see that the inclusion of event-by-event fluctuations significantly enhances $v_2(p_T)$ and gives a nonzero $v_3(p_T)$. The larger average eccentricity in MCKLN gives larger $v_2(p_T)$ (solid red line) in comparison to MCGlauber results but the opposite is found for $v_3(p_T)$, which is in accordance with the fact that MCKLN gives smaller values for ε_3 in comparison to MCGlauber’s (see, for instance, [63]). One can see in Fig. 2 that a reasonable simultaneous description of R_{AA} and $v_2^{exp}(p_T)$ data can be obtained in this approach. Also, $v_3^{exp}(p_T)$ is in the ballpark of current data uncertainties, which indicates that the initial state fluctuations that generate v_3 in the soft sector [54] are also responsible for triangular flow at high p_T .

There is a simple way to understand why event-by-event fluctuations increase $v_2^{exp}(p_T)$ in comparison to event averaged calculations. First, we observe that the 2nd flow harmonic $v_2^{hard}(p_T)$ defined in (2) fluctuates event-by-event and it is, to a good approximation, *linearly correlated* with its soft counterpart. Indeed, we show in Fig. 3 that the integrated $v_2^{hard}(20 < p_T < 30 \text{ GeV})$ is linearly correlated with $v_2^{soft}(0.3 < p_T < 3 \text{ GeV})$ on an event-by-event basis (which is similar to what is observed in the data [95]). This shows that the initial state geometrical fluctuations responsible for v_2^{soft}

lead to fluctuations in the path length of the jet in the medium. Using this linear correlation and that on average $\psi_2^{hard}(p_T)$ is aligned with ψ_2^{soft} [96], one can see that the effects of *small* fluctuations (kept up to quadratic order) on the soft-hard correlation in (3) are

$$\frac{v_2^{exp}(p_T)}{\langle v_2^{hard}(p_T) \rangle} \simeq 1 + \frac{1}{2} \left\langle \left(\frac{\delta v_2^{soft}}{\langle v_2^{soft} \rangle} \right)^2 \right\rangle - 2 \langle (\delta\psi_2(p_T))^2 \rangle, \quad (5)$$

where $\delta\psi_2(p_T) = \delta(\psi_2^{hard}(p_T) - \psi_2^{soft})$. Event-by-event fluctuations enhance experimental elliptic flow because the positive contribution from fluctuations of the soft δv_2^{soft} overwhelms the small, negative contribution from event plane misalignment. We found that $\langle \cos[2\delta\psi_2] \rangle \sim 0.99$ while $\langle \cos[3\delta\psi_3] \rangle \sim 0.33$ in our simulation. It would be interesting to test if there is an analog of the mapping of the eccentricities onto the final high p_T flow harmonics that also mirrors the soft correlation seen at low p_T [97–99].

The new approach pursued, which combines event-by-event hydrodynamics with jet energy loss, can be used to exploit initial geometrical shape fluctuations as an additional experimental control knob besides centrality by measuring $v_n^{exp}(p_T; C^{soft})$ in different soft bulk multiplicity and eccentricity subclasses, (e.g. $C^{soft} = \{\Delta N_{tracks}/N_{max} = 20 - 30\%, \Delta v_2^{soft}/\langle v_2^{soft} \rangle = 10\%\}$), through Eq. (3). Thus, we propose to couple event shape engineering [95, 100, 101] in the low p_T soft sector with jet quenching observables in subclasses of spatially anisotropic events in the same centrality class. By taking a very narrow window of events near the average $\langle v_2^{soft} \rangle$ of the wide v_2^{soft} distribution, one can systematically reduce the effects of fluctuations in the soft sector to approach the ideal theoretical limit $v_2^{exp}(p_T) \rightarrow v_2^{hard}(p_T)$ at high p_T . This would allow one for the first time to deconvolute the short wavelength jet-medium physics contained in the azimuthal dependence $R_{AA}(p_T, \phi)$ from the physics of soft, bulk event-by-event flow observables. This type of soft-hard event engineering (SHEE) would also allow for novel studies of the path length dependence of energy loss in highly anisotropic media (associated with the events at the tail of the v_2^{soft} distribution). A dedicated study about SHEE will be presented elsewhere.

5. Conclusions. In this paper, event-by-event fluctuations in the soft sector modeled via viscous hydrodynamics were combined with a jet energy loss model to solve the decade long high p_T $R_{AA} \otimes v_2$ puzzle in ultra-relativistic heavy ion collisions. A crucial point to this study was the realization that the experimentally measured high p_T azimuthal coefficients are currently defined via a correlation between soft and hard particles over many events, see (3). Therefore, these observables inherit the well-known geometrical fluctuations of the soft sector (see the linear correlation in Fig. 3) and the fail-

ure of previous model calculations to simultaneously describe $R_{AA}(p_T)$ and $v_2(p_T)$ stem not from the lack of some non-perturbative source but rather from unrealistic assumptions for the evolving medium (event averaged instead of a realistic event-by-event hydrodynamic evolution). We showed that the positive contribution from low p_T v_2^{soft} fluctuations overwhelms the negative contributions from event plane fluctuations and this leads to an overall enhancement of high $p_T > 10$ GeV elliptic flow in comparison to previous calculations. Also, the inclusion of event-by-event fluctuations allowed us to compute for the first time high p_T v_3^{exp} , which displayed a reasonable agreement with data (given current uncertainties).

Our study paves the way for a new era in high p_T physics in which state-of-the-art experimental event shape engineering techniques may be used to deconvolute the (strongly coupled) bulk evolution of the QGP from the perturbative QCD energy loss physics that determines the azimuthal anisotropy of jet quenching phenomena. We encourage experimentalists to check the features of low p_T v_n^{soft} distributions, cumulants, and $\langle p_T \rangle$ of events with triggered high p_T particles in the context of event shape engineering. Future experimental SHEE studies at RHIC and LHC varying both centrality and soft eccentricity subclasses may help to discriminate better between other alternative combinations of soft-hard dynamical models of high energy nuclear collisions.

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