Probing the Small-x Gluon Tomography in Correlated Hard Diffractive Dijet Production in Deep Inelastic Scattering

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Phys. Rev. Lett. **116**, 202301 — Published 20 May 2016

DOI: 10.1103/PhysRevLett.116.202301

Probing the Small-x Gluon Tomography in Correlated Hard Diffractive Dijet Production in DIS

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We investigate the close connection between the quantum phase space Wigner distribution of small-x gluons and the color dipole scattering amplitude, and propose to study it experimentally in the hard diffractive dijet production at the planned electron-ion collider. The angular correlation between the nucleon recoiled momentum and the dijet transverse momentum will probe the non-trivial correlation in the phase space Wigner distribution. This experimental study will not only provide us with three-dimensional tomographic pictures of gluons inside high energy proton, but also give a unique and interesting signal for the small-x dynamics with QCD evolution effects.

Introduction. There have been strong interests in hadron physics community [1–3] to explore the partonic structure of the nucleon, in particular, aiming at a tomography picture from which we can image the partons in three-dimensional fashion. This can provide fruitful and detailed information on the sub-atomic structure of the baryonic building blocks of the universe, and deepen our understanding of the strong interaction facts in constructing the fundamental particles. Among these tomography distributions, the so-called quantum phase space Wigner distributions [4, 5] of partons have been reckoned as the mother distributions of all, since they ingeniously encode all quantum information of how partons are distributed inside hadrons.

The key question now is to find experimental probes to measure these distributions. The goal of this paper is to pioneer this direction, by pointing out that we can have access to the gluon Wigner distributions at small-x. The proposed new observables will stimulate further developments from both experiment and theory sides for the planned electron-ion colliders (EIC). In general, the parton Wigner distributions are not directly measurable in high energy scatterings. Due to the uncertainty principle, they are not positive definite, but only quasi-probabilistic. As we will demonstrate later in this Letter, one can use the diffractive dijet production (or more complicated processes), which has been a subject of study in the small-x physics and the generalized parton distribution approach [6–14], to directly probe the Fourier transform of the gluon Wigner distribution at the EIC.

The phase space distributions [15] of quarks and gluons are often used in small-x literatures, and they are believed to be possibly related to Wigner distributions [4], although the exact connection was not known. We will show that the gluon Wigner distributions at small-x can be simplified and written as the Fourier transform of well-known impact parameter dependent dipole amplitudes, which helps us to build intimate connections to small-x factorization framework developed in the last few decades. This will not only provide the motivation to pursue the gluon Wigner distributions in the future EIC, but also prompt further studies to investigate non-trivial correlations in the small-x dipole scattering amplitude. The latter has become one of the most important elements of the phenomenological studies in heavy ion collisions and deep inelastic scatterings [16, 17].

One of the nontrivial phenomena is the angular correlation between the traverse momentum of the produced dijet and the recoiled momentum of the nucleon, which provides vital information on the gluon Wigner distributions. It is important to emphasize that this correlation can help us test and measure the unique feature of angular correlations between impact parameter and dipole size predicted by small-x evolutions.

The rest of the paper is organized as follows. We first introduce the gluon Wigner distributions and take the small-x limit, which can be connected to the dipole scattering amplitudes. We then apply these results to demonstrate that we will be able to observe these novel correlations in the future EIC. Last, we explore the small-x dynamics by invoking the analytical solution to the BFKL equation [18] to show there exist nontrivial correlation in these gluon Wigner distributions. Finally, we summarize our paper in the end.

Gluon Wigner Distributions at Small-x. The parton Wigner distributions are introduced to describe the quantum phase space distributions of partons inside the nucleon. They unify the two common languages of transverse momentum dependent and the generalized parton distributions in parton distributions framework.

We focus on the gluon Wigner distributions. The gluon Wigner distributions are defined through the following matrix elements,

$$xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ixP^+\xi^- - iq_\perp\cdot\xi_\perp} \left\langle P+\frac{\Delta_\perp}{2} \left| F^{+i}\left(\vec{b}_\perp+\frac{\xi}{2}\right) F^{+i}\left(\vec{b}_\perp-\frac{\xi}{2}\right) \right| P-\frac{\Delta_\perp}{2} \right\rangle , \quad (1)$$

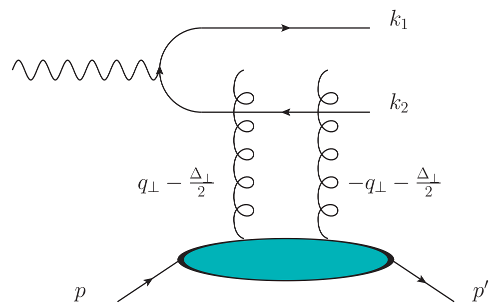

FIG. 1. Diffractive dijet production in electron-ion collisions. Here we assume that the incoming virtual photon has only the longitudinal momentum. The signature of the diffractive process is the rapidity gap between the produced dijet and the target hadron which remains intact.

where $F^{\mu\nu}$ represents the field strength tensor, x and $q_\perp$ for the longitudinal momentum fraction and the transverse momentum for the gluon, $\vec{b}_\perp$ for the coordinate space variable. The Fourier transform of the Wigner distribution w.r.t. the impact parameter $b_\perp$ is also referred as the generalized transverse momentum dependent (GTMD) gluon distribution [19, 20]. The gauge links associated with the gluon fields have been omitted in the above equation for simplicity (see discussions below).

In Ref. [21, 22], it has been demonstrated that TMD gluon distributions are related to small-x unintegrated gluon distributions. The Weizsäcker-Williams (WW) and the dipole gluon distribution used in small-x formalism correspond to two gauge invariant but topologically different operator definitions. In order to pursue deeper connections between Wigner distributions and small-x impact parameter dependent gluon distributions, we first use the dipole gluon distribution as an example, and we will comment on the case of the WW gluon distribution in the end. Following the convention in Ref. [23], we write down the GTMD dipole gluon distribution as

$$xG_{\rm DP}(x, q_\perp, \Delta_\perp) = 2\int \frac{{\rm d}\xi^- {\rm d}^2\xi_\perp e^{-iq_\perp\cdot\xi_\perp - ixP^+\xi^-}}{(2\pi)^3 P^+} \left\langle P + \frac{\Delta_\perp}{2} \middle| {\rm Tr}\left[F^{+i}(\xi/2)\mathcal{U}^{[-]\dagger}F^{+i}(-\xi/2)\mathcal{U}^{[+]}\right] \middle| P - \frac{\Delta_\perp}{2}\right\rangle , \quad (2)$$

where $\mathcal{U}^{[\pm]}$ are the future/past-pointing U-shaped Wilson lines which make the operator gauge invariant. Its Fourier transform $\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp\cdot b_\perp} xG_{\rm DP}(x, q_\perp, \Delta_\perp)$ can be identified as the Wigner distribution $xW_g^T(x, q_\perp, b_\perp)$. Following similar derivation used in Ref [22–24] in the small-x limit which allows us to approximately write $e^{-xP^+\xi^-} \simeq 1$, one can show that Eq. (2) reduces to

$$xG_{\rm DP}(x, q_\perp, \Delta_\perp) = \frac{2N_c}{\alpha_s}\int \frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4} e^{iq_\perp\cdot(R_\perp - R'_\perp) + i\frac{\Delta_\perp}{2}\cdot(R_\perp + R'_\perp)} \left(\nabla_{R_\perp}\cdot\nabla_{R'_\perp}\right) \frac{1}{N_c}\left\langle {\rm Tr}\left[U(R_\perp)U^\dagger(R'_\perp)\right]\right\rangle_x , \quad (3)$$

where we can recognize the impact parameter dependent dipole amplitude. Let us define its double Fourier transform

$$\frac{1}{N_c}{\rm Tr}\left[U\left(b_\perp + \frac{r_\perp}{2}\right)U^\dagger\left(b_\perp - \frac{r_\perp}{2}\right)\right] \equiv \int d^2q_\perp d^2\Delta_\perp e^{-iq_\perp\cdot r_\perp - i\Delta_\perp\cdot b_\perp}\mathcal{F}_x(q_\perp, \Delta_\perp) . \quad (4)$$

Then we can succinctly write $xG_{\rm DP}(x, q_\perp, \Delta_\perp) = (q_\perp^2 - \Delta_\perp^2/4)\frac{2N_c}{\alpha_s}\mathcal{F}_x(q_\perp, \Delta_\perp)$. Setting $r_\perp = 0$ in the above expression, we obtain the normalization condition for $\mathcal{F}_x(q_\perp, \Delta_\perp)$ as $\int d^2q_\perp d^2\Delta_\perp e^{-i\Delta_\perp\cdot b_\perp}\mathcal{F}_x(q_\perp, \Delta_\perp) = 1$.

Correlated Hard Diffractive Dijet Production in DIS. Now let us discuss diffractive dijet production in electron-ion collisions, which has been studied quite recently in Ref. [33], and demonstrate that it directly probes the dipole gluon GTMD in the small-x limit where the quark contribution is negligible. Diffractive events imply that a color neutral exchange must occur in the t-channel between the virtual photon and the target hadron over several units in rapidity. Following the same framework developed in Ref [22], by requiring that the final state quark-antiquark pair forms a color singlet state, we can write the cross section for diffractive dijet production as illustrated in Fig. 1 as follows

$$\frac{d\sigma^{\gamma_T^* A\to q\bar{q}X}}{dy_1 d^2k_{1\perp} dy_2 d^2k_{2\perp}} = 2N_c\alpha_{em}e_q^2\delta(x_{\gamma^*} - 1)z(1-z)[z^2 + (1-z)^2]\int d^2q_\perp d^2q'_\perp \mathcal{F}_x(q_\perp, \Delta_\perp)\mathcal{F}_x(q'_\perp, \Delta_\perp)$$
$$\times\left[\frac{P_\perp}{P_\perp^2 + \epsilon_f^2} - \frac{P_\perp - q_\perp}{(P_\perp - q_\perp)^2 + \epsilon_f^2}\right]\cdot\left[\frac{P_\perp}{P_\perp^2 + \epsilon_f^2} - \frac{P_\perp - q'_\perp}{(P_\perp - q'_\perp)^2 + \epsilon_f^2}\right] , \quad (5)$$

for the transversely polarized photon. A similar cross section formula can be written for the longitudinally polarized photon. In Eq. (5), $y_{1,2}$ and $k_{1,2\perp}$ are rapidities and transverse momenta of the final state quark and antiquark jets, respectively, defined in the center of mass frame of the incoming photon and nucleon. $\vec{P}_\perp \equiv \frac{1}{2}(\vec{k}_{2\perp} - \vec{k}_{1\perp})$ represents the typical dijet transverse momentum and $\Delta_\perp$ is the nucleon recoiled momentum. We are interested in the back-to-back kinematic region for the two final state jets where $|P_\perp| \gg |\vec{k}_{1\perp} + \vec{k}_{2\perp}|$. Suppose $\epsilon_f^2 \equiv z(1-z)Q^2$ is not too large as compared to $P_\perp^2$. Then we expect that the above $q_\perp$ integrals are dominated by the region $q_\perp \sim P_\perp$ and the cross sections are roughly proportional to $\mathcal{F}_x^2(P_\perp, \Delta_\perp)$ for back-to-back dijet configurations. Thus, the diffractive dijet production will be sensitive to the correlations between $\vec{P}_\perp$ and $\vec{\Delta}_\perp$ as mentioned in Ref. [33], and our analysis shows that such a measurement gives experimental access to the gluon Wigner distribution.

Of particular interest is the angular correlation of the form $\cos 2(\phi_{P_\perp} - \phi_{\Delta_\perp})$. This originates from the $\cos 2\phi$ correlation in the GTMD and the Wigner distribution

$$xG_{\rm DP}(x, \vec{q}_\perp, \vec{\Delta}_\perp) = x\mathcal{G}_{\rm DP}(x, |\vec{q}_\perp|, |\vec{\Delta}_\perp|) + x\mathcal{G}^\epsilon_{\rm DP}(x, |\vec{q}_\perp|, |\vec{\Delta}_\perp|)\cos 2(\phi_{q_\perp} - \phi_{\Delta_\perp}) + \cdots , \tag{6}$$

$$xW_g^T(x, \vec{q}_\perp; \vec{b}_\perp) = x\mathcal{W}_g^T(x, |\vec{q}_\perp|, |\vec{b}_\perp|) + x\mathcal{W}_g^\epsilon(x, |\vec{q}_\perp|, |\vec{b}_\perp|)\cos 2(\phi_{q_\perp} - \phi_{b_\perp}) + \cdots . \tag{7}$$

The first terms in the above two equations represent the azimuthally symmetric distributions, whereas the rest of the terms stand for the azimuthally asymmetric distributions. From symmetry considerations (cf. Ref. [20]), one sees that only even harmonics $\cos 2n\phi$ are allowed. We expect that the dominant component is the elliptic ($n = 1$) one as shown above, and we call it the Elliptic Gluon Wigner Distribution, or in short, elliptic gluon distribution. With the detector capability at the future EIC [3], we will be able to identify both $\vec{P}_\perp$ and $\vec{\Delta}_\perp$ and measure the angular correlation between them. In particular, the elliptic angular correlation $\langle \cos 2\,(\phi_{P_\perp} - \phi_{\Delta_\perp})\rangle$ can be observed in this process. This is similar to the elliptic flow phenomena observed in heavy ion collisions.

It is interesting to note that the early studies of diffractive dijet production in DIS have focused on the $\cos 2\phi$ angular correlation between the lepton plane and the jet plane, which has been demonstrated as an important feature of small-x calculations [7–9]. This $\cos 2\phi$ correlation will remain in our formalism too. The combined analyses of both angular correlations of $\cos 2(\phi_{P_\perp} - \phi_{\Delta_\perp})$ and $\cos 2\phi$ will provide a unique opportunity to study the gluon tomography and test the saturation formalism. It has been also pointed out in Ref. [10] that the emission of an additional gluon can diminish the signal, and this could pose a challenge in the data analysis at HERA [37] for events of large diffractive masses (M_X) when $M_X^2 \gg Q^2$. While we propose to study a different type of angular correlation at the EIC, a similar problem may arise and need to be investigated. Nevertheless, we believe that the case becomes simpler if we focus on relatively low mass diffractive events with large Q^2 and $Q^2 \simeq M_X^2$.

Gluon Tomography Induced by Small-x Dynamics. In order to gain analytical insights into the distribution $xG_{\rm DP}(x, \vec{q}_\perp, \vec{\Delta}_\perp)$ and illustrate how the angular correlation arises, let us evaluate it in the BFKL approximation. Consider the dipole scattering amplitude off a dipole $x_\perp$ (quark at $\vec{x}_\perp/2$, antiquark at $-\vec{x}_\perp/2$) evolved up to rapidity $Y = \ln 1/x$. Define the dipole T-matrix in impact parameter space as $\frac{1}{N_c}\left\langle {\rm tr}\, U\left(b_\perp + \frac{r_\perp}{2}\right) U^\dagger\left(b_\perp - \frac{r_\perp}{2}\right)\right\rangle_x = 1 - T(r_\perp, b_\perp, Y)$. In the BFKL approximation and in the regime $b_\perp, r_\perp \gg x_\perp$, T is given by [25–27]

$$T(r_\perp, b_\perp, Y) \approx \frac{\alpha_s^2|\rho|}{\sqrt{\pi}} \frac{\ln\frac{16}{|\rho|}}{\left(\frac{7}{2}\bar{\alpha}_s\zeta(3)Y\right)^{3/2}} \exp\left(4\bar{\alpha}_s Y \ln 2 - \frac{\ln^2\frac{16}{|\rho|}}{14\bar{\alpha}_s\zeta(3)Y}\right) , \tag{8}$$

where

$$|\rho|^2 \equiv \frac{x_\perp^2 r_\perp^2}{\left(b_\perp + \frac{r_\perp}{2} - \frac{x_\perp}{2}\right)^2\left(b_\perp - \frac{r_\perp}{2} + \frac{x_\perp}{2}\right)^2} \approx \frac{x_\perp^2 r_\perp^2}{b_\perp^4 + \frac{r_\perp^4}{16} - \frac{b_\perp^2 r_\perp^2}{2}\cos 2\phi_{br}} . \tag{9}$$

Clearly, one sees that there is nontrivial angular correlation between $\vec{b}_\perp$ and $\vec{r}_\perp$. When $\vec{b}_\perp$ is parallel to $\vec{r}_\perp$, the scattering is stronger than the case when $\vec{b}_\perp$ is perpendicular to $\vec{r}_\perp$. This is a known phenomenon, see for example Ref. [32]. Such a correlation is expected to survive near the nonlinear saturated regime. Indeed, away from the BFKL saddle point, the saturation momentum Q_s is defined by the condition $T(r_\perp = 1/Q_s, b_\perp) = const.$ This leads to

$$\frac{1}{|\rho|^2} \approx \left.\frac{b_\perp^4 + \frac{r_\perp^4}{16} - \frac{b_\perp^2 r_\perp^2}{2}\cos 2\phi_{br}}{x_\perp^2 r_\perp^2}\right|_{r_\perp = 1/Q_s} \sim e^{\frac{\chi(\gamma_s)}{\gamma_s}Y} , \tag{10}$$

where $\chi(\gamma) \equiv \frac{\alpha_s N_c}{\pi}\left[2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\right]$ and $\gamma_s \approx 0.628$. If we look for a solution in the regime $b_\perp \gg r_\perp \simeq 1/Q_s$, we find

$$Q_s^2 \sim \frac{x_\perp^2}{b_\perp^4} e^{\frac{\chi(\gamma_s)}{\gamma_s}Y} + \frac{\cos 2\phi_{br}}{2b_\perp^2}. \tag{11}$$

This is consistent with the numerical study of the nonlinear small-x evolution (e.g., the Balitsky-Kovchegov evolution [28, 29]) in Ref. [30, 31] where it was observed that the angular correlation exists even when $b_\perp$ and $r_\perp$ are of the same order. These features should be a guiding principle when building saturation models with angular correlations.

The elliptic ($\sim \cos 2\phi$) angular correlation can be seen also in the momentum space. After averaging over the angular orientation of the target dipole $x_\perp$, we find that the Fourier transform of $T(r_\perp, b_\perp, Y)$ w.r.t. $\vec{b}_\perp$ and $\vec{r}_\perp$ is

$$\mathcal{T}(q_\perp, \Delta_\perp, Y) = \frac{\alpha_s^2 x_\perp}{(2\pi)^2 \Delta_\perp^3} \frac{e^{4\bar{\alpha}_s Y \ln 2}}{\left(\frac{7}{2}\bar{\alpha}_s \zeta(3) Y \pi\right)^{3/2}} \int_0^{\pi/2} d\theta J_0\left(\frac{\sin\theta \Delta_\perp x_\perp}{2}\right) K_0\left(\frac{\cos\theta \Delta_\perp x_\perp}{2}\right)$$
$$\times \int_0^1 \frac{d\alpha}{\alpha^2(1-\alpha)^2} \, {}_2F_1\left(\frac{3}{2}, \frac{3}{2}, 1, -\frac{|\vec{q}_\perp + (1/2-\alpha)\vec{\Delta}_\perp|^2}{\Delta_\perp^2 \alpha(1-\alpha)}\right), \tag{12}$$

in the high energy limit. Depending on the relative size of $q_\perp$ and $\Delta_\perp$, $\mathcal{T}(q_\perp, \Delta_\perp, Y)$ can have sizable angular correlations with only even harmonics. (It is not hard to show that all odd harmonics vanish.) Note that in the BFKL approximation, we have $xG_{\mathrm{DP}}(x, q_\perp, \Delta_\perp) = -(q_\perp^2 - \Delta_\perp^2/4)\frac{2N_c}{\alpha_s}\mathcal{T}(q_\perp, \Delta_\perp, Y)$ for the case with finite momentum transfer.

In the saturation regime, one can estimate the strength of the angular correlation from numerical studies of the Balitsky-Kovchegov equation with impact parameter dependence [30, 31]. We find that it will lead to a few percent $\langle \cos 2(\phi_{P_\perp} - \phi_{\Delta_\perp})\rangle$ asymmetries in the typical EIC kinematics. More sophistic calculations shall follow to generalize the saturation models [14, 34–36] to incorporate this particular angular correlation feature. We leave that for a future study. Comparing the theoretical computations with the future experimental data will provide us much more insights on the experimental signature of small-x dynamics.

Summary and Discussions. To conclude, let us make some further but brief comments on the consequence of this work, while we will leave the detailed discussion for a future publication.

- Let us comment on the WW gluon distribution case. Following the same technique used above for the dipole gluon Wigner distribution, we generalize the WW gluon distribution at small-x as follows

$$xG_{\mathrm{WW}}(x, q_\perp, \Delta_\perp) = 2\int \frac{\mathrm{d}\xi^- \mathrm{d}^2\xi_\perp e^{-iq_\perp\cdot\xi_\perp - ixP^+\xi^-}}{(2\pi)^3 P^+} \left\langle P + \frac{\Delta_\perp}{2}\right| \mathrm{Tr}\left[F\left(\frac{\xi}{2}\right)\mathcal{U}^{[+]\dagger}F\left(-\frac{\xi}{2}\right)\mathcal{U}^{[+]}\right]\left| P - \frac{\Delta_\perp}{2}\right\rangle, \tag{13}$$

  which allows us to find

$$xG_{\mathrm{WW}}(x, q_\perp, \Delta_\perp) = \frac{2N_c}{\alpha_S}\int \frac{d^2R_\perp}{(2\pi)^2}\frac{d^2R'_\perp}{(2\pi)^2} e^{iq_\perp\cdot(R_\perp - R'_\perp) + i\frac{\Delta_\perp}{2}\cdot(R_\perp + R'_\perp)}$$
$$\times \frac{1}{N_c}\left\langle \mathrm{Tr}\left[i\partial_i U(R_\perp)\right] U^\dagger(R'_\perp)\left[i\partial_i U(R'_\perp)\right] U^\dagger(R_\perp)\right\rangle_x. \tag{14}$$

  Due to the known connection between the WW gluon distribution and color quadrupoles at small-x [22], it is expected that one needs to generate a color quadrupole at the amplitude level in order to probe the WW Wigner distribution. This requires two incoming photons at once which produce four-jet diffractive events in the final states. It seems to be very challenging to measure this type of events at EIC. Nevertheless, it is more probable to perform such measurement in ultra-peripheral diffractive AA collisions at the LHC where photons are much more abundant in the wavefunction of colliding nuclei.

- It is also interesting to note that one can generalize the above derivation to obtain the linearly polarized part [38–45] of the WW and dipole gluon Wigner distribution when the indices of derivatives are off-diagonal, instead of diagonal as in Eqs. (1,2). The cross sections for dijet and four-jet productions depend on both the unpolarized and linearly polarized gluon distributions, which are related in the small-x formalism [41, 42].

  In addition, when integrating over $q_\perp$ in Eqs. (1,2) with off-diagonal indices, the gluon Wigner distributions will reduce to the so-called helicity flip gluon GPDs (also called gluon transveristy), which have been extensively discussed in the collinear GPD framework [46–49]. The nontrivial correlations between $q_\perp$ and $\Delta_\perp$ play important roles in the integral to obtain the helicity flip gluon GPDs.

- The extension to the quark Wigner distribution can be done accordingly. Their contributions will be dominant in the large and moderate x range, where the small-x approximation breaks down. Therefore, the differential cross sections will be much more involved in terms of the quark Wigner distributions, as compared to the simple form of Eq. (5). Nevertheless, it is worthwhile to pursue further studies along the direction of this paper for the diffractive dijet production at large/moderate x.

The parton Wigner distributions, which contain the most complete information, are the cornerstones of all parton distributions. We demonstrate that gluon Wigner distributions are closely related to the impact parameter dependent dipole and quadrupole scattering amplitudes, and point out that they can be measured in diffractive type events at EIC and the LHC. In particular, the correlated hard diffractive dijet production in DIS is one of the golden channels to explore the gluon Wigner distribution. The nontrivial correlation encoded in this distribution could be potentially linked to many observables in high energy hadronic and nuclear collisions. Further theoretical and phenomenological studies shall follow along the direction of this paper.

Acknowledgments: This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under contract number DE-AC02-05CH11231, and by the U.S. National Science Foundation under Grant No. PHY-0855561 and PHY-1417326, as well as by the NSFC under Grant No. 11575070. B.X. acknowledges interesting discussion with A. Mueller, and wishes to thank Dr. X.N. Wang and the nuclear theory group at LBNL for hospitality and support during his visit when this work is finalized.

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