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Did LIGO detect dark matter?

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We consider the possibility that the black-hole (BH) binary detected by LIGO may be a signature of dark matter. Interestingly enough, there remains a window for masses $20 M_\odot \lesssim M_{bh} \lesssim 100 M_\odot$ where primordial black holes (PBHs) may constitute the dark matter. If two BHs in a galactic halo pass sufficiently close, they radiate enough energy in gravitational waves to become gravitationally bound. The bound BHs will rapidly spiral inward due to emission of gravitational radiation and ultimately merge. Uncertainties in the rate for such events arise from our imprecise knowledge of the phase-space structure of galactic halos on the smallest scales. Still, reasonable estimates span a range that overlaps the $2 - 53 \text{ Gpc}^{-3} \text{ yr}^{-1}$ rate estimated from GW150914, thus raising the possibility that LIGO has detected PBH dark matter. PBH mergers are likely to be distributed spatially more like dark matter than luminous matter and have no optical nor neutrino counterparts. They may be distinguished from mergers of BHs from more traditional astrophysical sources through the observed mass spectrum, their high ellipticities, or their stochastic gravitational wave background. Next generation experiments will be invaluable in performing these tests.

The nature of the dark matter (DM) is one of the most longstanding and puzzling questions in physics. Cosmological measurements have now determined with exquisite precision the abundance of DM [1, 2], and from both observations and numerical simulations we know quite a bit about its distribution in Galactic halos. Still, the nature of the DM remains a mystery. Given the efficacy with which weakly-interacting massive particles— for many years the favored particle-theory explanation—have eluded detection, it may be warranted to consider other possibilities for DM. Primordial black holes (PBHs) are one such possibility [3–6].

Here we consider whether the two $\sim 30 M_\odot$ black holes detected by LIGO [7] could plausibly be PBHs. There is a window for PBHs to be DM if the BH mass is in the range $20 M_\odot \lesssim M \lesssim 100 M_\odot$ [8, 9]. Lower masses are excluded by microlensing surveys [10–12]. Higher masses would disrupt wide binaries [9, 13, 14]. It has been argued that PBHs in this mass range are excluded by CMB constraints [15, 16]. However, these constraints require modeling of several complex physical processes, including the accretion of gas onto a moving BH, the conversion of the accreted mass to a luminosity, the self-consistent feedback of the BH radiation on the accretion process, and the deposition of the radiated energy as heat in the photon-baryon plasma. A significant (and difficult to quantify) uncertainty should therefore be associated with this upper limit [17], and it seems worthwhile to examine whether PBHs in this mass range could have other observational consequences.

In this Letter, we show that if DM consists of $\sim 30 M_\odot$ BHs, then the rate for mergers of such PBHs falls within the merger rate inferred from GW150914. In any galactic halo, there is a chance two BHs will undergo a hard scatter, lose energy to a soft gravitational wave (GW) burst and become gravitationally bound. This BH binary will merge via emission of GWs in less than a Hubble time. Below we first estimate roughly the rate of such mergers and then present the results of more detailed calculations. We discuss uncertainties in the calculation and some possible ways to distinguish PBHs from BH binaries from more traditional astrophysical sources. Consider two PBHs approaching each other on a hyperbolic orbit with some impact parameter and relative velocity $v_{pbh}$. As the PBHs near each other, they produce a time-varying quadrupole moment and thus GW emission. The PBH pair becomes gravitationally bound if the GW emission exceeds the initial kinetic energy. The cross section for this process is [19, 20],

$$\sigma = \pi \left(\frac{85 \pi}{3}\right)^{2/7} R^2 \left(\frac{v_{pbh}}{c}\right)^{-18/7}$$

$$= 1.37 \times 10^{-14} M_{pbh}^2 v_{pbh/200}^{18/7} \text{pc}^2,$$  (1)

where $M_{pbh}$ is the PBH mass, and $M_{30}$ the PBH mass in units of $30 M_\odot$, $R = 2GM_{pbh}/c^2$ is its Schwarzschild radius, $v_{pbh}$ is the relative velocity of two PBHs, and $v_{pbh/200}$ is this velocity in units of $200 \text{ km sec}^{-1}$.

We begin with a rough but simple and illustrative estimate of the rate per unit volume of such mergers. Suppose that all DM in the Universe resided in Milky-Way like halos of mass $M = M_{12} 10^{12} M_\odot$ and uniform mass density $\rho = 0.002 \rho_{0.002} M_\odot \text{ pc}^{-3}$ with $\rho_{0.002} \sim 1$. Assuming a uniform-density halo of volume $V = M/\rho$, the rate of mergers per halo would be

$$N \approx (1/2) V (\rho/M_{pbh})^2 \sigma v$$

$$\approx 3.10 \times 10^{-12} M_{12} \rho_{0.002} v_{pbh/200}^{11/7} \text{ yr}^{-1}.$$  (2)

1 In our analysis, PBH binaries are formed inside halos at $z = 0$. Ref. [18] considered instead binaries which form at early times and merge over a Hubble time.
The relative velocity $v_{\text{pbh-200}}$ is specified by a characteristic halo velocity. The mean cosmic DM mass density is $\rho_{\text{dm}} \approx 3.6 \times 10^{10} M_\odot \text{Mpc}^{-3}$, and so the spatial density of halos is $n \approx 0.036 M_\odot^{-1} \text{Mpc}^{-3}$. The rate per unit comoving volume in the Universe is thus

$$\Gamma \approx 1.1 \times 10^{-4} \rho_{0.002} v_{\text{pbh-200}}^{-11/7} \text{Gpc}^{-3} \text{yr}^{-1}. \quad (3)$$

The normalized halo mass $M_{12}$ drops out, as it should. The merger rate per unit volume also does not depend on the PBH mass, as the capture cross section scales like $M_{12}^5$.

This rate is small compared with the $2\ldots3$ Gpc$^{-3}$ yr$^{-1}$ estimated by LIGO for a population of $\sim 30 M_\odot \ldots 30 M_\odot$ mergers [21], but it is a very conservative estimate. As Eq. (3) indicates, the merger rate is higher in higher-density regions and in regions of lower DM velocity dispersion. The DM in Milky-Way like halos is known from simulations [22] and analytic models [23] to have substructure, regions of higher density and lower velocity dispersion. DM halos also have a broad mass spectrum, extending to very low masses where the densities can become far higher, and velocity dispersion far lower, than in the Milky Way. To get a very rough estimate of the conceivable increase in the PBH merger rate due to these smaller-scale structures, we can replace $\rho$ and $v$ in Eq. (3) by the values they would have had in the earliest generation of collapsed objects, where the DM densities were largest and velocity dispersions smallest. If the primordial power spectrum is nearly scale invariant, then gravitational bound halos of mass $M_c \sim 500 M_\odot$, for example, will form at redshift $z_c \approx 28 - \log_{10}(M_c/500 M_\odot)$. These objects will have virial velocities $v \approx 0.2 \text{ km sec}^{-1}$ and densities $\rho \approx 0.24 M_\odot \text{pc}^{-3}$ [24]. Using these values in Eq. (3) increases the merger rate per unit volume to

$$\Gamma \approx 700 \text{Gpc}^{-3} \text{yr}^{-1}. \quad (4)$$

This would be the merger rate if all the DM resided in the smallest haloes. Clearly, this is not true by the present day; substructures are at least partially stripped as they merge to form larger objects, and so Eq. (4) should be viewed as a conservative upper limit.

Having demonstrated that rough estimates contain the merger-rate range $2\ldots5$ Gpc$^{-3}$ yr$^{-1}$ suggested by LIGO, we now turn to more careful estimates of the PBH merger rate. As Eq. (3) suggests, the merger rate will depend on a density-weighted average, over the entire cosmic DM distribution, of $\rho_{0.002} v_{\text{pbh-200}}^{-11/7}$. To perform this average, we will (a) assume that DM is distributed within galactic halos with a Navarro-Frenk-White (NFW) profile [25] with concentration parameters inferred from simulations; and (b) try several halo mass functions taken from the literature for the distribution of halos.

The PBH merger rate $R$ within each halo can be computed using

$$R = 4\pi \int_0^{R_{\text{vir}}} r^2 \frac{1}{2} \left( \frac{\rho_{\text{nfw}}(r)}{M_{\text{pbh}}} \right) \langle \sigma v_{\text{pbh}} \rangle \, dr \quad (5)$$

where $\rho_{\text{nfw}}(r) = \rho_s [r/r_s(1 + r/r_s)^2]^{-1}$ is the NFW density profile with characteristic radius $r_s$ and characteristic density $\rho_s$. $R_{\text{vir}}$ is the virial radius at which the NFW profile reaches a value 200 times the comoving mean cosmic density and is cutoff. The angle brackets denote an average over the PBH relative velocity distribution in the halo. The merger cross section $\sigma$ is given by Eq. (1). We define the concentration parameter $C = R_{\text{vir}}/r_s$. To determine the profile of each halo, we require $C$ as a function of halo mass $M$. We will use the concentration-mass relations fit to DM N-body simulations by both Ref. [26] and Ref. [27].

We now turn to the average of the cross section times relative velocity. The one-dimensional velocity dispersion of a halo is defined in terms of the escape velocity at radius $R_{\text{max}} = 2.1626 R_s$, the radius of the maximum circular velocity of the halo. i.e.,

$$v_{\text{dm}} = \sqrt{GM(r < r_{\text{max}})/r_{\text{max}}} = \frac{v_{\text{vir}}}{\sqrt{2}} \sqrt{\frac{C}{C_m}} \frac{g(C_m)}{g(C)}, \quad (6)$$

where $g(C) = \ln(1+C) - C/(1+C)$, and $C_m = 2.1626 = R_{\text{max}}/R_s$. We approximate the relative velocity distribution of PBHs within a halo as a Maxwell-Boltzmann (MB) distribution with a cutoff at the virial velocity. i.e.,

$$P(v_{\text{pbh}}) = F_0 \left[ \exp \left( -\frac{v_{\text{pbh}}}{v_{\text{dm}}}^2 \right) - \exp \left( -\frac{v_{\text{vir}}^2}{v_{\text{dm}}^2} \right) \right], \quad (7)$$

where $F_0$ is chosen so that $4\pi \int_0^{v_{\text{vir}}} P(v)v^2 dv = 1$. This model provides a reasonable match to N-body simulations, at least for the velocities substantially less than the virial velocity which dominate the merger rate (e.g., Ref. [28]). Since the cross-section is independent of radius, we can integrate the NFW profile to find the merger rate in any halo:

$$R = \left( \frac{85\pi}{12\sqrt{2}} \right)^{2/7} \frac{9G^2 M_{\text{vir}}}{cR_s^7} \left( 1 - \frac{1}{(1+C)^3} \right) \frac{D(v_{\text{dm}})}{g(C)^2}, \quad (8)$$

where

$$D(v_{\text{dm}}) = \int_0^{v_{\text{vir}}} P(v) v^2 dv \left( \frac{2v}{c} \right)^{3/7} dv, \quad (9)$$

comes from Eq. (7).

Eq. (1) gives the cross section for two PBHs to form a binary. However, if the binary is to produce an observable GW signal, these two PBHs must orbit and inspiral; a direct collision, lacking an inspiral phase, is unlikely.
to be detectable by LIGO. This requirement imposes a minimum impact parameter of roughly the Schwarzschild radius. The fraction of BHs direct mergers is $\sim v_{\text{pbh}}^{2/7}$ and reaches a maximum of $\sim 3\%$ for $v_{\text{pbh}} = 2000$ km s$^{-1}$. Thus, direct mergers are negligible. We also require that once the binary is formed, the time until it merges (which can be obtained from Ref. [29]) is less than a Hubble time. The characteristic time it takes for a binary BH to merge varies as a function of halo velocity dispersion. It can be hours for $M_{\text{vir}} \approx 10^{12} M_\odot$ or kyr for $M_{\text{vir}} \approx 10^{6} M_\odot$, and is thus instantaneous on cosmological timescales. Given the small size of the binary, and rapid time to merger, we can neglect disruption of the binary by a third PBH once formed. BH binaries can also form through non-dissipative three-body encounters. The rate of these binary captures is non-negligible in small halos [19, 30], but they generically lead to the formation of wide binaries that will not be able to harden and merge within a Hubble time. This formation mechanism should not affect our LIGO rates. The merger rate is therefore equal to the rate of binary BH formation, Eq. (8).

Fig. 1 shows the contribution to the merger rate, Eq. (8), for two concentration-mass relations. As can be seen, both concentration-mass relations give similar results. An increase in halo mass produces an increased PBH merger rate. However, less massive halos have a higher concentration (since they are more likely to have virialized earlier), so that the merger rate per unit mass increases significantly as the halo mass is decreased.

To compute the expected LIGO event rate, we convolve the merger rate $R$ per halo with the mass function $dn/dM$. Since the redshifts ($z \lesssim 0.3$) detectable by LIGO are relatively low we will neglect redshift evolution in the halo mass function. The total merger rate per unit volume is then,

$$\nu = \int (dn/dM)(M) R(M) dM. \quad (10)$$

Given the exponential falloff of $dn/dM$ at high masses, despite the increased merger rate per halo suggested in Fig. 1, the precise value of the upper limit of the integrand does not affect the final result.

At the lower limit, discreteness in the DM particles becomes important, and the NFW profile is no longer a good description of the halo profile. Furthermore, the smallest halos will evaporate due to periodic ejection of objects by dynamical relaxation processes. The evaporation timescale is [33]

$$t_{\text{evap}} \approx (14 \mathcal{N}/\ln \mathcal{N}) [R_{\text{vir}}/(C v_{\text{dm}})], \quad (11)$$

where $\mathcal{N}$ is the number of individual BHs in the halo, and we assumed that the PBH mass is $30 M_\odot$. For a halo of mass $400 M_\odot$, the velocity dispersion is 0.15 km sec$^{-1}$, and the evaporation timescale is $\sim 3$ Gyr. In practice, during matter domination, halos which have already formed will grow continuously through mergers or accretion. Evaporation will thus be compensated by the addition of new material, and as halos grow new halos will form from mergers of smaller objects. However, during dark-energy domination at $z \lesssim 0.3$, 3 Gyr ago, this process slows down. Thus, we will neglect the signal from halos with an evaporation timescale less than 3 Gyr, corresponding to $M < 400 M_\odot$. This is in any case 13 PBHs, and close to the point where the NFW profile is no longer valid.

The halo mass function $dn/dM$ is computed using both semi-analytic fits to N-body simulations and with analytic approximations. Computing the merger rate in the small halos discussed above requires us to extrapo-
late both the halo mass function and the concentration-mass relation around six orders of magnitude in mass beyond the smallest halos present in the calibration simulations. High-resolution simulations of $10^{-4} M_\odot$ cold dark matter micro-halos [31, 32] suggest that our assumed concentration-mass relations underestimate the internal density of these halos, making our rates conservative.

The mass functions depend on the halo mass through the perturbation amplitude $\sigma(R_{\text{vir}})$ at the virial radius $R_{\text{vir}}$ of a given halo. Due to the scale invariance of the window functions on small scales, $\sigma(R_{\text{vir}})$ varies only by a factor of two between $M_{\text{vir}} = 10^9 M_\odot$ and $10^3 M_\odot$. Thus the extrapolation in the mass function is less severe than it looks. We also note that the scale-invariant nature of the initial conditions suggests that the shape of the halo mass function should not evolve unduly until it reaches the scale of the PBH mass, or evaporation cutoff.

To quantify the uncertainty induced by the $dn/dM$ extrapolation, we obtained results with two different mass functions: the classic analytic Press-Schechter calculation [34] and one calibrated to numerical simulations from Tinker et al. [35]. The agreement between the two small-scale behaviors suggests that extrapolating the mass functions is not as blind as it might otherwise seem. We also include a third mass function, due to Jenkins et. al. [36], that includes an artificial small-scale mass cutoff at a halo mass $M_{\text{vir}} \sim 10^6 M_\odot$. This cutoff is inserted to roughly model the mass function arising if there is no power on scales smaller than those currently probed observationally. We include it to provide a very conservative lower limit to the merger rate if, for some reason, small-scale power were suppressed. We do not, however, consider it likely that this mass function accurately represents the distribution of halo masses in our Universe.

Fig. 2 shows the merger rate per logarithmic interval in halo mass. In all cases, halos with $M_{\text{vir}} \lesssim 10^9 M_\odot$ dominate the signal, due to the increase in concentration and decrease in velocity dispersion with smaller halo masses. The Tinker mass function, which asymptotes to a constant number density for small masses, produces the most mergers. Press-Schechter has $\sim 50\%$ fewer events in small halos, while the Jenkins mass function results in merger rates nearly four orders of magnitude smaller (and in rough agreement with Eq. (3)).

We integrate the curves in Fig. 2 to compute the total merger rate $V$. All mass functions give a similar result, $\sim (3 \pm 1) \times 10^{-4}$ Gpc$^{-3}$ yr$^{-1}$, from halos of masses $\gtrsim 10^9 M_\odot$, representing for the Tinker and Press-Schechter mass function a small fraction of the events. When we include all halos with $M_{\text{vir}} > 400 M_\odot$, the number of events increases dramatically, and depends strongly on the lower cutoff mass $M_c$ for the halo mass. Both the Press-Schechter and Tinker mass functions are for small halos linear in the integrated perturbation amplitude $\propto 1/\sigma(R_{\text{vir}})$ at the virial radius $R_{\text{vir}}$ of the collapsing halo. In small halos, $1/\sigma(R_{\text{vir}})$ is roughly constant. Thus for a mass function $MF(\sigma)$, we have

$$dn/dM \sim (C \log \sigma/dM)[MF(\sigma)/M_{\text{vir}}] \sim M_{\text{vir}}^{-2}. \quad (12)$$

The concentration is also a function of $1/\sigma(R_{\text{vir}})$ and it too becomes roughly constant for small masses. Assuming a constant concentration, the merger rate per halo scales as $R \sim M^{10/21}$. Thus, Eq. (10) suggests that $V \sim M_c^{-11/21}$. This compares well to numerical differentiation of Fig. 2, which yields $V \sim M_c^{-0.51}$.

The integrated merger rate is thus

$$V = 2f(M_c/400 M_\odot)^{-11/21} \text{Gpc}^{-3} \text{yr}^{-1}, \quad (13)$$

with $f \approx 1$ for the Tinker mass function, and $f \approx 0.6$ for the Press-Schechter mass function (the Jenkins mass function results in an event rate $V \approx 0.02$ Gpc$^{-3}$ yr$^{-1}$, independent of $M_c \lesssim 10^9 M_\odot$).

A variety of astrophysical processes may alter the mass function in some halos, especially within the dwarf galaxy range, $10^5 - 10^{10} M_\odot$. However, halos with $M_{\text{vir}} \lesssim 10^9 M_\odot$ are too small to form stars against the thermal pressure of the ionized intergalactic medium [37] and are thus unlikely to be affected by these astrophysical processes. Inclusion of galactic substructure, which our calculation neglects, should boost the results. However, since the event rate is dominated by the smallest halos, which should have little substructure, we expect this to make negligible difference to our final result.

There is also the issue of the NFW density profile assumed. The results are fairly insensitive to the detailed density profile as long as the slope of the density profile varies no more rapidly than $r^{-1}$ as $r \rightarrow 0$. For example, suppose we replace the NFW profile with the Einasto profile [38],

$$\rho(R) = \rho_0 \exp \left( -\frac{2}{\alpha} \left( \frac{R}{R_s} \right)^\alpha - 1 \right) \quad (14)$$

with $\alpha = 0.18$, which has a core as $r \rightarrow 0$. The reduction in the merger rate as $r \rightarrow 0$ is more than compensated by an increased merger rate at larger radii leading to a total merger rate that is raised by $50\%$ relative to NFW, to $\sim 3$ Gpc$^{-3}$ yr$^{-1}$.

Our assumption of an isotropic MB-like velocity distribution in the halo may also underestimate the correct answer, as any other velocity distribution would have lower entropy and thus larger averaged $v^{-11/7}$. Finally, the discreteness of PBH DM will provide some Poisson enhancement of power on $\sim 400 M_\odot$ scales. More small-scale power would probably lead to an enhancement of the event rate beyond Eq. (13).

The recent LIGO detection of two merging $\sim 30 M_\odot$ black holes suggests a $90\%$ C.L. event rate [21] of $2 - 53$ Gpc$^{-3}$ yr$^{-1}$ if all mergers have the masses and emitted energy of GW150914. It is interesting that—although there are theoretical uncertainties—our best estimates of
the merger rate for 30 $M_\odot$ PBHs, obtained with canonical models for the DM distribution, fall in the LIGO window.

The possibility that LIGO has seen DM thus cannot be immediately excluded. Even if the predicted merger rates turn out, with more precise treatments of the small-scale galactic phase-space distribution, to be smaller, conservative lower estimates of the merger rate for PBH DM suggests that the LIGO/VIRGO network should see a considerable number of PBH mergers over its lifetime.

We have assumed a population of PBHs with the same mass. The basic results obtained here should, however, remain unaltered if there is some small spread of PBH masses, as expected from PBH-formation scenarios, around the nominal value of 30 $M_\odot$.

PBH mergers may also be interesting for LIGO/VIRGO even if PBHs make up only a fraction $f_{pbh}$ of the DM, as implied by CMB limits from Refs. [15, 16] or the limits in Ref. [8]. In this case, the number density of PBHs will be reduced by $f_{pbh}$. The cutoff mass will increase as $M_\odot \sim f_{pbh}^{-1}$ if we continue to require $>13$ PBHs in each halo to avoid halo evaporation. The overall event rate will be $\nu \sim f_{pbh}^{23/21}$ Gpc$^{-3}$ yr$^{-1}$. Advanced LIGO will reach design sensitivity in 2019 [39, 40], and will probe $z < 0.75$, an increase in volume to $\approx 80$ Gpc$^3$ (comoving). Thus over the six planned years of aLIGO operation, while we should expect to detect $\sim 1000$ events with $f_{pbh} = 1$, we will expect at least one event if $f_{pbh} > 0.08$.

Distinguishing whether any individual GW event, or even some population of events, are from PBH DM or more traditional astrophysical sources will be daunting. Still, there are some prospects. Most apparently, PBH mergers will be distributed more like small-scale DM halos and are thus less likely to be found in or near luminous galaxies than BH mergers resulting from stellar evolution. Moreover, PBH mergers are expected to have no electromagnetic/neutrino counterparts whatsoever. A DM component could conceivably show up in the BH mass spectrum as an excess of events with BH masses near 30 $M_\odot$ over a more broadly distributed mass spectrum from astrophysical sources [e.g. 41].

Since the binary is formed on a very elongated orbit, the GW waveforms will initially have high ellipticity, exhibited by higher frequency harmonics in the GW signal [29]. We have verified that the ellipticities become observably small by the time the inspiral enters the LIGO band, but they may be detectable in future experiments [42]. Multiply-lensed quasars [43, 44], pulsar timing arrays [45], and FRB lensing searches [46] may also allow probes of the $\sim 30 M_\odot$ PBH mass range.

Another potential source of information is the stochastic GW background. Models for the stochastic background due to BH mergers usually entail a mass distribution that extends to smaller BH masses and a redshift distribution that is somehow related to the star-formation history. Given microlensing limits, the PBH mass function cannot extend much below 30 $M_\odot$. Moreover, the PBH merger rate per unit comoving volume is likely higher for PBHs than for traditional BHs at high redshifts. Together, these suggest a stochastic background for PBHs that has more weight at low frequencies and less at higher ones than that from traditional BH sources.

The results of this work provide additional motivation for more sensitive next-generation GW experiments such as the Einstein Telescope [47], DECIGO [48] and BBO [49], which will continuously extend the aLIGO frequency range downwards. These may enable the tests described above for excesses in the BH mass spectrum, high ellipticity and low-frequency stochastic background that are required to determine if LIGO has detected dark matter.

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