Entanglement Equilibrium and the Einstein Equation

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Entanglement equilibrium and the Einstein equation

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A link between the semiclassical Einstein equation and a maximal vacuum entanglement hypothesis is established. The hypothesis asserts that entanglement entropy in small geodesic balls is maximized at fixed volume in a locally maximally symmetric vacuum state of geometry and quantum fields. A qualitative argument suggests that the Einstein equation implies validity of the hypothesis. A more precise argument shows that, for first order variations of the local vacuum state of conformal quantum fields, the vacuum entanglement is stationary if and only if the Einstein equation holds. For nonconformal fields, the same conclusion follows modulo a conjecture about the variation of entanglement entropy.

I. INTRODUCTION

When restricted to one side of a spatial partition, the vacuum state of a quantum field has entropy because the two sides are entangled. The entanglement entropy of the restricted state is dominated by the ultraviolet (UV) field degrees of freedom near the interface, and hence scales with the area. This is similar to the Bekenstein-Hawking black hole entropy, $A/4L_p^2$, where A is the horizon area and $L_p = (\hbar G/c^3)^{1/2}$ is the Planck length [1–3]. The similarity of these two "area laws" is striking, and has led to the idea that black hole entropy is just a special case of vacuum entanglement entropy [4–9]. To match the Bekenstein-Hawking entropy, vacuum entanglement entropy should be cut off at the Planck scale. Considering the gravitational back-reaction of vacuum fluctuations, such a cutoff appears natural [7, 10], but it lies deep in the regime of poorly understood quantum gravity effects.

Bekenstein defined the *generalized entropy* $S_{\rm gen}$ as the sum of the horizon entropy and the ordinary entropy in the exterior. If the horizon entropy is indeed entanglement entropy, then the (fine-grained) generalized entropy is nothing but the total von Neumann entropy of the quantum state outside the horizon [9, 11, 12]. Bekenstein proposed the generalized second law (GSL) stating that $S_{\rm gen}$ never decreases [2]. The GSL has been shown to hold in various regimes [13], the proofs having been recently strengthened to apply to rapid changes and arbitrary horizon slices [14, 15]. The validity of the law depends on the Einstein equation, which relates the curvature — and therefore the focussing of light rays that determines the change of horizon area — to the local energy-momentum density of matter.

The GSL thus points to a deep link between vacuum entanglement and the Einstein equation. The aim of this paper is to better understand the nature of this link. Motivated by the notion of vacuum as an equilibrium state, I formulate a maximal vacuum entanglement hypothesis (MVEH):

> *When the geometry and quantum fields are simultaneously varied from maximal symmetry, the entanglement entropy in a small geodesic ball is maximal at fixed volume.*

This is formulated in the context of semiclassical gravity, i.e. quantum fields on a classical spacetime. As such, it is predicated on the following assumption:

> *The area density of vacuum entanglement entropy η is finite and universal.*

This assumption is supported by the evidence that horizon entropy can indeed be identified with entanglement entropy (see, e.g. [9, 16, 17], and references therein). However, it involves UV aspects of quantum gravity that are not currently understood, so it remains an assumption.

I will argue that Einstein equation supports the MVEH and, conversely, that the MVEH implies the Einstein equation for first order variations of the local vacuum state for conformal fields. For nonconformal fields the result holds modulo a conjecture about the variation of entanglement entropy to be explained below. It is well-known that diffeomorphism invariance selects the Einstein equation, at second order in derivatives, as the unique gravitational field equation in a metric theory. Since the MVEH is formulated in a diffeomorphim-invariant fashion, it is therefore not surprising that the Einstein equation would arise. Nevertheless, entropy maximization is quite different from Hamilton's principle of stationary action, so something new is learned here. Moreover, the Newton constant that appears in the derived Einstein equation—which is *not* fixed by diffeomorphism invariance—has precisely the value required in order for η to correspond to the Bekenstein-Hawking value, $1/4\hbar G$. This is a nontrivial and essential consistency property of the derivation.

Two lines of evidence motivated this paper. First, the Einstein equation can be derived as a thermodynamic equation of state of the vacuum outside a local causal horizon [18]. That derivation assumes that the entropy change of an otherwise stationary horizon is given by $\delta Q/T$ when a local boost energy δQ crosses the horizon, $T = \hbar/2\pi$ being the Unruh temperature. Second, recent work invokes AdS/CFT (Anti-de Sitter/conformal field theory) duality, and the thermal nature of CFT vacuum entanglement entropy, to derive the linearized Einstein equation for perturbations of AdS spacetime [19–21]. This approach treats the entropy statistically, rather than thermodynamically, and it concerns entropy of a compact region in the CFT at one time, rather than following the change of horizon entropy. The present work combines the local spacetime setting of the equation of state approach, with the statisti-

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cal, compact region setting of the holographic analysis, but it proceeds directly in spacetime, making no use of holography.

II. AREA DEFICIT AND GENERAL RELATIVITY

Einstein's field equation,

$$G_{ab} = 8\pi G\, T_{ab}, \tag{1}$$

relates the Einstein curvature tensor G_{ab} to the energy-momentum tensor of matter, T_{ab}. Central to our story is the equivalence of (1) to the statement that the surface area deficit of any small, spacelike geodesic ball of fixed volume is proportional to the energy density in the ball [22]. We begin by demonstrating this lovely relation.

At any point o in a spacetime of dimension d, choose an arbitrary timelike unit vector u^a, and generate a $(d-1)$-dimensional spacelike ball Σ by sending out geodesics of length ℓ from o in all directions orthogonal to u^a. The point o is the center of the ball, and the boundary $\partial\Sigma$ is the surface (see the grey region of Fig. 1). Choose a Riemann normal coordinate (RNC) system based at o, launched from an orthonormal basis formed by u^a and $d-1$ spacelike vectors tangent to Σ. Let the timelike coordinate be x^0, and let the spacelike ones be $\{x^i\}$. The signature of the spacetime metric is taken here to be $(-{+}{+}{+})$, and units are chosen with $c = 1$.

We will assume the radius of the ball is much smaller than the local curvature length,

$$\ell \ll L_{\text{curvature}} \tag{2}$$

and work to lowest nontrivial order in their ratio. The volume variation at fixed radius, relative to flat space, is then given by

$$\delta V|_\ell = -\frac{\Omega_{d-2}\ell^{d+1}}{6(d-1)(d+1)}\mathcal{R}, \tag{3}$$

where $\mathcal{R} = R_{ik}{}^{ik}$ is the spatial Ricci scalar at o [23], and the area variation of $\partial\Sigma$ is given by $d\delta V/d\ell$, i.e.

$$\delta A|_\ell = -\frac{\Omega_{d-2}\ell^{d}}{6(d-1)}\mathcal{R}. \tag{4}$$

We will also be interested in the area variation at fixed volume, rather than at fixed geodesic radius. When the radius of the ball varies, the volume and area variations have the additional contributions $\delta_r V = \ell^{d-2}\int \delta r\, d\Omega$ and $\delta_r A = (d-2)\ell^{d-3}\int \delta r\, d\Omega$. Choosing $\int \delta r\, d\Omega$ so that the total volume variation vanishes, we obtain the area variation at fixed volume,

$$\delta A|_V = \delta A - \frac{d-2}{\ell}\delta V = -\frac{\Omega_{d-2}\ell^{d}}{2(d^2-1)}\mathcal{R}. \tag{5}$$

This is smaller by the factor $3/(d+1)$ than the variation at fixed radius (4).

To connect now with spacetime and the Einstein equation, note that the spatial Ricci scalar at o is equal to twice the RNC 00-component of the spacetime Einstein tensor:

$$\mathcal{R} = R_{ik}{}^{ik} = R - 2R_0{}^0 = 2(R_{00} - \tfrac{1}{2}Rg_{00}) = 2G_{00}. \tag{6}$$

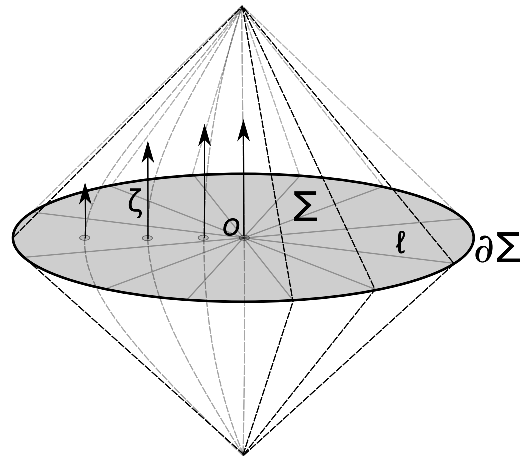

FIG. 1. Causal diamond, in a maximally symmetric spacetime, for a geodesic ball Σ of radius ℓ with center o and boundary $\partial\Sigma$. The dashed curves are flow lines of ζ, the conformal Killing vector field, whose flow preserves the diamond and which vanishes at the top and bottom vertices and on $\partial\Sigma$. The vectors show ζ at four points of Σ.

The area deficit (5) can thus also be expressed as

$$\delta A|_V = -\frac{\Omega_{d-2}\ell^{d}}{d^2-1}G_{00}. \tag{7}$$

Then, using the Einstein equation (1), we see that the area deficit is proportional to the energy density,

$$\delta A|_V = -\frac{8\pi G\Omega_{d-2}\ell^{d}}{d^2-1}T_{00}. \tag{8}$$

Conversely, this simple geometrical relation contains the full content of Einstein's equation, if it holds at all spacetime points and for all timelike unit vectors.

The evidence that the Einstein equation implies maximal vacuum entanglement can now be stated in a qualitative, intuitive fashion. Suppose the ball has a Bekenstein-Hawking entropy $A/4\hbar G$, arising from vacuum entanglement, and we try to increase the entropy by placing an entangled qbit in the ball. To localize the qbit within a region of size ℓ we must give it an energy of at least $\hbar/\ell$ which, according to (8), will contribute an area deficit of order $\hbar G$, hence a surface entropy decrease of order unity, offsetting the added qbit. It would not help to use a "highly entropic object" with many internal states, because the existence of such objects makes its mark in the vacuum as well, diluting the entropic effect of adding the object to the ball. Indeed, in the context of the Rindler wedge, it was argued that $\delta S \leq \delta E/T$, where T is the Unruh temperature $\hbar/2\pi$ and δE is the change of boost Killing energy, since a thermal state maximizes entropy at fixed energy [24, 25]. We now proceed to make this link between the Einstein equation and maximal vacuum entanglement more precise.

III. CAUSAL DIAMOND AND CONFORMAL ISOMETRY

To evaluate the variation of the entanglement entropy in a spacelike geodesic ball Σ it is helpful to consider the spacetime region causally determined by Σ, called the *causal diamond* $D(\Sigma)$. In a maximally symmetric spacetime, $D(\Sigma)$ is the intersection of the future of a past vertex and the past of a future vertex, and has a conformal isometry and rotational symmetry in the rest frame defined by these vertices (see Fig. 1).

The Minkowski line element $ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$ takes the form $ds^2 = -du\,dv + r^2 d\Omega^2$ with null coordinates $u = t-r$ and $v = t+r$. The Minkowski diamond centered on the origin consists of the intersection of the regions $u > -\ell$ and $v < \ell$. The unique conformal isometry that preserves the diamond, and is spherically symmetric, is generated by the conformal Killing vector

$$\zeta = \frac{1}{2\ell}\left[(\ell^2 - u^2)\partial_u + (\ell^2 - v^2)\partial_v\right] \tag{9}$$

(for a derivation see [23]). Expressed in t and r coordinates, ζ is given by

$$\zeta = \frac{1}{2\ell}\left[(\ell^2 - r^2 - t^2)\partial_t - 2rt\,\partial_r\right]. \tag{10}$$

The Lie derivative of the Minkowski metric along ζ is

$$\mathcal{L}_\zeta \eta_{ab} = -(2t/\ell)\,\eta_{ab}. \tag{11}$$

The vector ζ is tangent to the null generators on the past and future null boundaries of the diamond, so those boundaries are conformal Killing horizons [26]. They meet at the ball boundary $\partial\Sigma$, where ζ vanishes, so $\partial\Sigma$ is a bifurcation surface. The surface gravity κ of a conformal Killing horizon is well-defined by the equation $\nabla_a \zeta^2 = -2\kappa\zeta_a$ [27], and with the normalization of ζ in (9) it is equal to unity.

IV. ENTANGLEMENT ENTROPY OF A DIAMOND

The entanglement entropy in a diamond $D(\Sigma)$ is the same as that in Σ. Under a simultaneous variation of the geometry and the state of the quantum fields, $(\delta g_{ab}, \delta|\psi\rangle)$, the diamond entanglement entropy variation will consist of two contributions, a state-independent UV part $\delta S_{\rm UV}$ from the area change induced by δg_{ab}, and a state-dependent IR part $\delta S_{\rm IR}$ from $\delta|\psi\rangle$.

As mentioned above, we are assuming that, as a result of the UV physics, the entanglement entropy in a spatial region is finite in any state, with a leading term ηA. Here A is the area of the boundary of the region and η is a universal constant with dimensions [length]$^{2-d}$. The scaling with area is natural in any theory with a large density of states at short distances. The assumption that η is universal is motivated by the idea that the UV structure of the vacuum is common to all states in the class being considered.[1] Under this assumption, when the geometry is varied, the contribution to the entanglement entropy in Σ from the UV degrees of freedom near the boundary $\partial\Sigma$ changes by an amount

$$\delta S_{\rm UV} = \eta\,\delta A. \tag{12}$$

The total entropy variation will thus be given by

$$\delta S_{\rm tot} = \eta\,\delta A + \delta S_{\rm IR} \tag{13}$$

If $\eta = 1/4\hbar G$, (12) coincides with the variation of Bekenstein's generalized entropy, here interpreted as simply the total entropy in the diamond. The MVEH implies that the total entropy variation (13) is zero at first order, and negative for finite variations, when comparing to a maximally symmetric spacetime with the volume of Σ held fixed.

To motivate this equilibrium condition, we first recall that, for ordinary thermodynamic systems in equilibrium, the Helmholtz free energy $F = E - TS$ is minimized at fixed volume. The MVEH is analogous, but with the additional feature that the energy vanishes. That the free energy of a diamond has no energy term can be motivated by comparison with de Sitter spacetime (dS), and the restriction to fixed volume arises from the fact that the diamond has a conformal Killing vector rather than a true Killing vector [23].

Our next step is to evaluate $\delta S_{\rm IR}$. The vacuum state of any QFT, restricted to the diamond, can be expressed (formally) as a thermal density matrix,

$$\rho = Z^{-1}\exp(-K/T), \quad T = \hbar/2\pi, \tag{14}$$

where K is the "modular Hamiltonian". The temperature $T = \hbar/2\pi$ is factored out here so that K will be the generator of Lorentz boosts, i.e. hyperbolic angle shifts, at the edge of the diamond. For an infinite diamond that coincides with the Rindler wedge in Minkowski space, T is the Unruh temperature [29, 30].

Since ρ in (14) has the form of a thermal state, it minimizes the modular free energy,

$$F_K = \langle K\rangle - TS, \tag{15}$$

where the brackets denote quantum expectation value, and $S = -\operatorname{Tr}\rho\ln\rho$ is the von Neumann entropy. The variation δF_K must therefore vanish for any small variation $\delta\rho$ of the state, i.e.

$$\delta S = \frac{2\pi}{\hbar}\delta\langle K\rangle. \tag{16}$$

This is just the usual Clausius relation for a "thermal" state (14).

In general K in (14) is not a local operator, and does not generate a geometric flow. For a CFT, however, K is equal to H_ζ, the Hamiltonian generating the flow of the conformal boost Killing vector (9) [31]. (This result is conformally related to the better known version that holds for any Poincaré invariant QFT restricted to the Rindler wedge [32].) That is, H_ζ is given by the integral

$$H_\zeta = \int_\Sigma T^{ab}\zeta_b\,d\Sigma_a. \tag{17}$$

[1] This involves an implicit choice of "conformal frame" [28] for the metric, namely, the one for which η is constant in spacetime. This metric turns out to satisfy the Einstein equation, so this frame is the so-called "Einstein frame".

If the quantum field state is varied away from the vacuum, with an excitation length scale much longer than the diamond size,

$$\ell \ll L_{\text{excitation}}, \tag{18}$$

then $\langle T_{ab}\rangle$ can be treated as constant, and using the Killing field (9) we find

$$\delta\langle H_\zeta\rangle = \frac{\Omega_{d-2}\ell^d}{d^2-1}\,\delta\langle T_{00}\rangle. \tag{19}$$

If the matter field is *not* conformal, K is not given by (17), and we cannot directly use (19). However, suppose that the matter is described by a QFT with a UV fixed point, so it is asymptotically conformal at short distances, and that, in addition to (18), the diamond is much smaller than any length scale in the QFT,

$$\ell \ll L_{\text{QFT}}. \tag{20}$$

Then we conjecture—and we shall assume—that $\delta\langle K\rangle$ has the form of (19) with an additional term δX that is a spacetime scalar,

$$\delta\langle K\rangle = \frac{\Omega_{d-2}\ell^d}{d^2-1}\,\big(\delta\langle T_{00}\rangle + \delta X\big). \tag{21}$$

(The common coefficient is factored out to simplify later expressions.) Calculations [33, 34] indicate that for a class of theories and states, this is the case, although in general δX may carry ℓ dependence and can dominate at small ℓ.[2] Note that the relation (21) refers only to the expectation value, and only to lowest order in the radius of the ball.

V. EQUILIBRIUM AND THE EINSTEIN EQUATION

We now postulate that a small diamond is in equilibrium if the quantum fields are in their vacuum state, and the curvature is that of a maximally symmetric spacetime (MSS) (Minkowski or (Anti)-de Sitter). Any MSS seems an equally good candidate, so we will regard the curvature scale of the MSS as a local state parameter that is effectively constant in a small diamond but may depend on the diamond.

The Einstein tensor in a MSS is $G^{MSS}_{ab} = -\lambda g_{ab}$, with λ a curvature scale. When the metric is varied away from the MSS, the area variation at fixed volume is obtained to lowest order in curvature by replacing G_{00} in (5) with $G_{00} - G^{MSS}_{00}$, which yields

$$\delta A|_{V,\lambda} = -\frac{\Omega_{d-2}\ell^d}{d^2-1}(G_{00}+\lambda g_{00}). \tag{22}$$

[2] In a previous draft of this paper, I had conjectured that $X = -\frac{1}{d}\langle T\rangle$, so that what appeared in $\delta\langle K\rangle$ would be just the tracefree part of $\langle T_{ab}\rangle$.

The variation of the total diamond entropy (13) away from the equilibrium can now be written using (22), (16) , and (21):

$$\delta S_{\text{tot}}|_{V,\lambda} = \eta\,\delta A|_{V,\lambda} + \frac{2\pi}{\hbar}\delta\langle K\rangle = \frac{\Omega_{d-2}\ell^d}{d^2-1}\times \left[-\eta\,(G_{00}+\lambda g_{00}) + \frac{2\pi}{\hbar}\big(\delta\langle T_{00}\rangle + \delta X\big)\right]. \tag{23}$$

The Einstein tensor should presumably be understood here as a quantum expectation value $\langle G_{ab}\rangle$, since the entropy that is maximized is, by definition, an expectation value. In using (21) for the matter entanglement variation, we are neglecting corrections that would come from the curvature of the MSS, since those would be of higher order.

The requirement that the variation (23) vanish at all points and with all timelike unit vectors implies a tensor equation,

$$G_{ab} + \lambda\, g_{ab} = \frac{2\pi}{\hbar\eta}\big(\delta\langle T_{ab}\rangle + \delta X g_{ab}\big). \tag{24}$$

The divergence of this equation, together with the Bianchi identity and local conservation of energy, ties λ to δX via

$$\lambda = \frac{2\pi}{\hbar\eta}\delta X + \Lambda, \tag{25}$$

where Λ is a spacetime constant. Had we not allowed for the MSS curvature scale λ in the equilibrium state, (25) would have implied the unphysical restriction that the scalar term δX be constant. Note also that if δX has ℓ-dependence then so does λ.

When (25) is substituted back into (24) we arrive at

$$G_{ab} + \Lambda\, g_{ab} = \frac{2\pi}{\hbar\eta}\delta\langle T_{ab}\rangle. \tag{26}$$

This is Einstein's equation with an undetermined cosmological constant Λ, which evidently must be independent of ℓ, and with Newton's constant defined by

$$G = \frac{1}{4\hbar\eta}. \tag{27}$$

The area density of entanglement entropy η and Planck's constant thus determine the gravitational coupling strength. Stronger vacuum entanglement implies weaker gravity, i.e. greater spacetime rigidity. Note the crucial consistency: when expressed using G, the surface entropy ηA is the Bekenstein-Hawking entropy $A/4\hbar G$. The coefficient would have been off by the factor $(d+1)/3$ had we used the area variation at fixed radius (4) rather than at fixed volume (5).

VI. DISCUSSION

We have shown, given our assumptions, that the semiclassical Einstein equation holds, for first order variations of the vacuum, if and only if the entropy in small causal diamonds is stationary at constant volume, when varied from a maximally symmetric vacuum state of geometry and quantum fields. We assumed the diamond size ℓ is much smaller than the local curvature length, the wavelength of any excitations of the vacuum, and the scales in the matter field theory, but much larger than the UV scale at which quantum gravity effects become strong. Our entanglement variation assumption for nonconformal matter (21) concerns only standard QFT, and is either true or false.

Strictly speaking the "first order variation" refers to the derivative with respect to a parameter labeling the state, evaluated at the vacuum. To be physically applicable, however, the result should apply to finite but small variations. The example of a coherent state reveals a challenge in this regard [35]: such a state can have nonzero energy density while leaving entanglement entropy unchanged [36, 37]. That is, not all energy registers as a change of entanglement. This is consistent with the hypothesis of maximal vacuum entanglement, although the Einstein equation implies that the entropy has decreased — relative to vacuum — by more than it needs to in order to satisfy the hypothesis. Unless a further consequence of that hypothesis is found, or the hypothesis is refined and strengthened in some way, the Einstein equation does not appear to follow from it in all generality.

We close with some questions and remarks concerning the derivation and its implications.

- Do graviton fluctuations contribute to the entanglement entropy? The UV part of the entanglement entropy $S = \eta A$ is inscrutable at this level, and the IR part does not include gravitons. Since the diamond is taken much smaller than the wavelength of any ambient gravitons, they have no gauge-invariant meaning in the diamond. In the RNC gauge they are absent at first derivative order. Moreover, the full, nonlinear Einstein tensor already appears on the geometric side of the equation, so it would be double-counting to include any graviton energy.

- Can a gravitational field equation with higher curvature corrections be derived along these lines? Maybe. We neglected terms of order $\ell/L_{\rm curv}$ in the geometry calculations, whereas a next higher curvature correction to the field equation might be of order $(\ell_1/L_{\rm curv})^2$, where ℓ_1^2 is the relative coefficient of the curvature squared term in the action. To capture this within our approximation would require $\ell/\ell_1 < \ell_1/L_{\rm curv}$. The right hand side is presumably less than unity, in order for higher curvature terms not to dominate, so the diamond would have to be taken smaller than ℓ_1. If, say, ℓ_1 were the string length, would classical geometry and quantum field theory apply at that scale? Probably not. On the other hand, perhaps with improved accuracy of the geometric analysis, and the inclusion of sub-leading UV terms in the entanglement entropy, one could consistently capture higher curvature corrections using a diamond larger than ℓ_1.

- A derivation of Einstein's equation invoking a quantum limit to measurements of the spacetime geometry of small causal diamonds was given in Ref. [38]. How are the assumptions used there related to those made here?

- According to our derivation the Einstein equation is a property of vacuum equilibrium. Does this suggest how to include non-equilibrium effects?

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[1] J. D. Bekenstein, "Black holes and the second law." Nuovo Cimento Lettere **4**, 737–740 (1972).

[2] J. D. Bekenstein, "Black Holes and Entropy," Phys. Rev. D **7**, 2333–2346 (1973).

[3] S.W. Hawking, "Particle Creation by Black Holes," Commun.Math.Phys. **43**, 199–220 (1975).

[4] Rafael D. Sorkin, "1983 paper on entanglement entropy: 'On the Entropy of the Vacuum outside a Horizon'," (2014), arXiv:1402.3589 [gr-qc].

[5] Gerard 't Hooft, "On the Quantum Structure of a Black Hole," Nucl.Phys. **B256**, 727 (1985).

[6] Luca Bombelli, Rabinder K. Koul, Joohan Lee, and Rafael D. Sorkin, "A Quantum Source of Entropy for Black Holes," Phys.Rev. **D34**, 373–383 (1986).

[7] Valeri P. Frolov and Igor Novikov, "Dynamical origin of the entropy of a black hole," Phys.Rev. **D48**, 4545–4551 (1993), arXiv:gr-qc/9309001 [gr-qc].

[8] Mark Srednicki, "Entropy and area," Phys.Rev.Lett. **71**, 666–669 (1993), arXiv:hep-th/9303048 [hep-th].

[9] Sergey N. Solodukhin, "Entanglement entropy of black holes," Living Rev.Rel. **14**, 8 (2011), arXiv:1104.3712 [hep-th].

[10] Ted Jacobson, "Gravitation and vacuum entanglement entropy," Int.J.Mod.Phys. **D21**, 1242006 (2012), arXiv:1204.6349 [gr-qc].

[11] Rafael D. Sorkin, "Toward a proof of entropy increase in the presence of quantum black holes," Phys.Rev.Lett. **56**, 1885–1888 (1986).

[12] Rafael D. Sorkin, "The statistical mechanics of black hole thermodynamics," (1997), arXiv:gr-qc/9705006 [gr-qc].

[13] Aron C. Wall, "Ten Proofs of the Generalized Second Law," JHEP **0906**, 021 (2009), arXiv:0901.3865 [gr-qc].

[14] Aron C. Wall, "A Proof of the generalized second law for rapidly-evolving Rindler horizons," Phys.Rev. **D82**, 124019 (2010), arXiv:1007.1493 [gr-qc].

[15] Aron C. Wall, "A proof of the generalized second law for rapidly changing fields and arbitrary horizon slices," Phys.Rev. **D85**, 104049 (2012), arXiv:1105.3445 [gr-qc].

[16] Ted Jacobson and Alejandro Satz, "Black hole entanglement entropy and the renormalization group," Phys.Rev. **D87**, 084047 (2013), arXiv:1212.6824.

[17] Joshua H. Cooperman and Markus A. Luty, "Renormalization of Entanglement Entropy and the Gravitational Effective Action," JHEP **1412**, 045 (2014), arXiv:1302.1878 [hep-th].

[18] Ted Jacobson, "Thermodynamics of space-time: The Einstein equation of state," Phys.Rev.Lett. **75**, 1260–1263 (1995), arXiv:gr-qc/9504004 [gr-qc].

[19] Nima Lashkari, Michael B. McDermott, and Mark Van Raamsdonk, "Gravitational dynamics from entanglement 'thermodynamics'," JHEP **1404**, 195 (2014), arXiv:1308.3716 [hep-th].

[20] Thomas Faulkner, Monica Guica, Thomas Hartman, Robert C. Myers, and Mark Van Raamsdonk, "Gravitation from Entanglement in Holographic CFTs," JHEP **1403**, 051 (2014), arXiv:1312.7856 [hep-th].

[21] Brian Swingle and Mark Van Raamsdonk, "Universality of Gravity from Entanglement," (2014), arXiv:1405.2933 [hep-th].

[22] For a related statement by Feynman see Supplemental Material at [url].

[23] See Supplemental Material [url], which includes Refs. [39–48] .

[24] Donald Marolf, Djordje Minic, and Simon F. Ross, "Notes on space-time thermodynamics and the observer dependence of entropy," Phys. Rev. **D69**, 064006 (2004), arXiv:hep-th/0310022 [hep-th].

[25] Donald Marolf, "A Few words on entropy, thermodynamics, and horizons," in *General relativity and gravitation. Proceedings, 17th International Conference, GR17, Dublin, Ireland, July 18-23, 2004* (2004) pp. 83–103, arXiv:hep-th/0410168 [hep-th].

[26] C. C. Dyer and E. Honig, "Conformal Killing horizons," Journal of Mathematical Physics **20**, 409–412 (1979).

[27] Ted Jacobson and Gungwon Kang, "Conformal invariance of black hole temperature," Class.Quant.Grav. **10**, L201–L206 (1993), arXiv:gr-qc/9307002 [gr-qc].

[28] Eanna E. Flanagan, "The Conformal frame freedom in theories of gravitation," Class. Quant. Grav. **21**, 3817 (2004), arXiv:gr-qc/0403063 [gr-qc].

[29] W.G. Unruh, "Notes on black hole evaporation," Phys.Rev. **D14**, 870 (1976).

[30] Geoffrey L. Sewell, "Quantum fields on manifolds: PCT and gravitationally induced thermal states," Annals Phys. **141**, 201–224 (1982).

[31] Peter D. Hislop and Roberto Longo, "Modular Structure of the Local Algebras Associated With the Free Massless Scalar Field Theory," Commun.Math.Phys. **84**, 71 (1982).

[32] J.J Bisognano and E.H. Wichmann, "On the Duality Condition for Quantum Fields," J.Math.Phys. **17**, 303–321 (1976).

[33] Horacio Casini, Damin A. Galante, and Robert C. Myers, "Comments on Jacobsons entanglement equilibrium and the Einstein equation," JHEP **03**, 194 (2016), arXiv:1601.00528 [hep-th].

[34] Antony J. Speranza, "Entanglement entropy of excited states in conformal perturbation theory and the Einstein equation," (2016), arXiv:1602.01380 [hep-th].

[35] Madhavan Varadarajan, "A Note on Entanglement Entropy, Coherent States and Gravity," Gen. Rel. Grav. **48**, 35 (2016), arXiv:1602.00106 [gr-qc].

[36] Thomas M. Fiola, John Preskill, Andrew Strominger, and Sandip P. Trivedi, "Black hole thermodynamics and information loss in two-dimensions," Phys. Rev. **D50**, 3987–4014 (1994), arXiv:hep-th/9403137 [hep-th].

[37] Eric Benedict and So-Young Pi, "Entanglement entropy of nontrivial states," Annals Phys. **245**, 209–224 (1996), arXiv:hep-th/9505121 [hep-th].

[38] Seth Lloyd, "The quantum geometric limit," (2012), arXiv:1206.6559 [gr-qc].

[39] W. Pauli, *Theory of Relativity* (Pergamon Press, 1958).

[40] R.P. Feynman, *Lectures on Gravitation* (Westview Press, 2002).

[41] R.P. Feynman, "The Feynman Lectures on Physics, Vol. 2," `http://www.feynmanlectures.caltech.edu/`.

[42] G.W. Gibbons and S.W. Hawking, "Action Integrals and Partition Functions in Quantum Gravity," Phys.Rev. **D15**, 2752–2756 (1977).

[43] G.W. Gibbons and S.W. Hawking, "Cosmological Event Horizons, Thermodynamics, and Particle Creation," Phys.Rev. **D15**, 2738–2751 (1977).

[44] James M. Bardeen, B. Carter, and S.W. Hawking, "The Four laws of black hole mechanics," Commun.Math.Phys. **31**, 161–170 (1973).

[45] Vivek Iyer and Robert M. Wald, "Some properties of Noether charge and a proposal for dynamical black hole entropy," Phys.Rev. **D50**, 846–864 (1994), arXiv:gr-qc/9403028 [gr-qc].

[46] Vivek Iyer, "Lagrangian perfect fluids and black hole mechanics," Phys.Rev. **D55**, 3411–3426 (1997), arXiv:gr-qc/9610025 [gr-qc].

[47] Stephen R. Green, Joshua S. Schiffrin, and Robert M. Wald, "Dynamic and Thermodynamic Stability of Relativistic, Perfect Fluid Stars," Class.Quant.Grav. **31**, 035023 (2014), arXiv:1309.0177 [gr-qc].

[48] Stefan Hollands and Robert M. Wald, "Stability of Black Holes and Black Branes," Commun.Math.Phys. **321**, 629–680 (2013), arXiv:1201.0463 [gr-qc].