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# Axion Isocurvature and Magnetic Monopoles

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We propose a simple mechanism to suppress axion isocurvature fluctuations using hidden sector magnetic monopoles. This allows for the Peccei-Quinn scale to be of order the unification scale consistently with high scale inflation.

**Introduction.** Cosmic inflation addresses many issues in early universe cosmology [1–3]. An exciting aspect of inflation is that it sources gravitational waves. If inflation occurs at a sufficiently high scale ( $\sim 10^{15}$ – $10^{16}$  GeV), the amplitude of these gravitational waves is large enough to leave a measurable imprint on the polarization of the cosmic microwave background (CMB) [4]. A number of CMB polarization experiments are presently searching for this signal [5], and a positive signal would have important implications for particle physics. In particular, bosonic fields with masses smaller than the inflationary Hubble scale are efficiently produced by inflation and can cause isocurvature perturbations in the CMB [6]. High scale inflation thus leads to interesting constraints on ultra-light bosons, including the QCD axion.

It is widely regarded [7] that a discovery of inflationary gravitational waves would rule out the QCD axion with a decay constant  $f_a \gtrsim 10^{16}$  GeV, a range that is favored by several theoretical considerations [8]. Experiments have also been proposed to search for the QCD axion in this parameter range [9], and it is of great interest to delineate the viable parameter space accessible to these efforts. While the bound discussed here disappears if the QCD axion acquires a large mass during inflation, and models achieving this do exist (see [10] for example), they face the difficulty that the mechanism responsible for generating a large axion mass during inflation has to violate the Peccei-Quinn symmetry while ensuring that this violation remains sufficiently sequestered from the axion after inflation. Other proposals to alleviate the tension between high scale inflation and the QCD axion include a dynamically changing Peccei-Quinn breaking scale [11], which, however, sacrifices some of the theoretical arguments underlying high  $f_a$  axions. There are also attempts that involve transfer of the axion isocurvature from one species to another [12], but these typically deplete the dark matter abundance of the axion, eliminating one of the promising ways to search for them. It might also be possible to relax the constraints by dumping entropy into the universe around the QCD phase transition [13], but these channels are rather constrained [7]. Other relevant attempts include Refs. [14–20].

In this paper, we investigate an alternative possibility: what if the QCD axion acquires a large mass *after* inflation, which subsequently disappears *before* the QCD

phase transition? If this mass is larger than the Hubble scale during a large interval, between the reheating and QCD scales, then the axion field oscillates earlier and the fluctuations in the field will be damped. When this mass (and potential) subsequently disappears, the average axion field takes a value corresponding to the minimum of the potential that generated this large mass. Since this minimum is in general displaced from the QCD minimum, the axion regains its cosmic abundance when it reacquires a mass from QCD, enabling it to be dark matter. The isocurvature perturbations will be small since the initial evolution of the field causes the perturbations to coalesce around the initial minimum, while the subsequent dark matter abundance is generated by homogeneous condensations.

How can we give such a large initial mass that then disappears? We accomplish this by coupling the QCD axion to a new  $U(1)'$  gauge group. If the reheating produces magnetic monopoles under this  $U(1)'$ , the monopole density generates a mass for the axion [21]. This is because topological terms like  $F\tilde{F}$  become physical in the presence of magnetic monopoles due to the Witten effect [22]. Specifically, it gives a free energy density that depends on a background axion field, thus creating an effective mass for the axion. This mass is sufficient to damp isocurvature perturbations in the axion field. After the perturbations have been damped, the monopole density can be efficiently eliminated by breaking  $U(1)'$ . The monopole density forces the axion field to relax into a value chosen by  $CP$  phases in the  $U(1)'$  sector. Since this phase need not be aligned with the QCD minimum, the axion generally acquires a homogeneous cosmic abundance during the QCD phase transition, with suppressed isocurvature perturbations. For large  $f_a \gg 10^{12}$  GeV, this misalignment needs to be small, but this can be environmentally selected [23]. We will show that there is sufficient time for damping axion isocurvature fluctuations so that axion dark matter with a unification scale  $f_a$  is consistent with high scale inflation giving an observable size of the gravitational wave polarization signal.

**Required Damping of Isocurvature Perturbations.** Inflation generally induces quantum fluctuations of order  $H_{\text{inf}}/2\pi$  for any massless field, where  $H_{\text{inf}}$  is the Hubble parameter during inflation. This implies that if  $U(1)_{\text{PQ}}$  is broken before or during inflation, then the

angle  $\theta = a/f_a$  of the axion field  $a$  has fluctuations  $\delta\theta(T_R) \approx H_{\text{inf}}/2\pi f_a$  at reheating temperature  $T_R$ .<sup>1</sup> Since the axion potential is flat during inflation, these fluctuations are of isocurvature type.

There is a tight constraint on the amount of allowed isocurvature perturbations from the Planck data [24], which can be written as (see, e.g., [18])

$$\frac{\Omega_a}{\Omega_{\text{DM}}} \frac{\delta\theta(T_{\text{QCD}})}{\theta_{\text{mis}}} \lesssim 4.8 \times 10^{-6}, \quad (1)$$

where  $\theta_{\text{mis}}$  is the average axion misalignment angle, while  $\delta\theta(T_{\text{QCD}})$  is the angle fluctuation at temperature  $T_{\text{QCD}} \sim 1$  GeV. Here,  $\Omega_a$  and  $\Omega_{\text{DM}} \simeq 0.24$  represent the axion and total dark matter abundances, respectively, and we assume  $\theta_{\text{mis}} > \delta\theta(T)$  throughout. Using the expression for the axion relic density

$$\frac{\Omega_a}{\Omega_{\text{DM}}} \approx 1.0 \times 10^5 \theta_{\text{mis}}^2 \left( \frac{f_a}{10^{16} \text{ GeV}} \right)^{1.19}, \quad (2)$$

we may rewrite Eq. (1) as

$$\delta\theta(T_{\text{QCD}}) \lesssim 1.5 \times 10^{-8} \sqrt{\frac{\Omega_{\text{DM}}}{\Omega_a}} \left( \frac{10^{16} \text{ GeV}}{f_a} \right)^{0.6}. \quad (3)$$

Assuming the standard cosmological history after inflation,  $\delta\theta(T_{\text{QCD}}) \approx \delta\theta(T_R)$ , so that we find

$$H_{\text{inf}} \lesssim 9.4 \times 10^8 \text{ GeV} \sqrt{\frac{\Omega_{\text{DM}}}{\Omega_a}} \left( \frac{f_a}{10^{16} \text{ GeV}} \right)^{0.4}. \quad (4)$$

This severely constrains inflationary models in the presence of a unification scale axion [7]. In particular, unification scale axion dark matter— $\Omega_a = \Omega_{\text{DM}}$  and  $f_a \sim 10^{16}$  GeV—is inconsistent with unification scale inflation— $E_{\text{inf}} \equiv V_{\text{inf}}^{1/4} \sim 10^{16}$  GeV, which leads to  $H_{\text{inf}} = E_{\text{inf}}^2/\sqrt{3}\bar{M}_{\text{Pl}} \sim 10^{13}$  GeV, where  $\bar{M}_{\text{Pl}} \simeq 2.4 \times 10^{18}$  GeV is the reduced Planck scale.

Below, we discuss a scenario in which axion isocurvature fluctuations are damped due to dynamics after inflation. Defining the (inverse) damping factor  $\Delta$  by

$$\Delta = \frac{\delta\theta(T_{\text{QCD}})}{\delta\theta(T_R)}, \quad (5)$$

Eq. (3) yields

$$\Delta \lesssim 1 \times 10^{-4} \sqrt{\frac{\Omega_{\text{DM}}}{\Omega_a}} \left( \frac{f_a}{10^{16} \text{ GeV}} \right)^{0.4} \left( \frac{10^{13} \text{ GeV}}{H_{\text{inf}}} \right), \quad (6)$$

the required amount of damping.

**Basic Mechanism.** Our basic idea is that the axion mass obtains extra contributions beyond that from QCD in the early universe so that it is larger than the Hubble parameter in some period. In this period, damped oscillations of the axion field reduces axion isocurvature perturbations giving  $\Delta < 1$ .

We achieve this by introducing a coupling of the axion to a hidden  $U(1)'$  gauge group<sup>2</sup>

$$\mathcal{L} \sim \frac{1}{f_a} a F'^{\mu\nu} \tilde{F}'_{\mu\nu}. \quad (7)$$

We assume that at some temperature  $T_M$  after inflation ( $T_M \lesssim T_R \approx E_{\text{inf}}$ ), monopoles of  $U(1)'$  are created. This can happen, for example, if a hidden sector  $SU(2)'$  gauge group is broken to  $U(1)'$  at that scale. In the presence of magnetic monopoles, the coupling in Eq. (7) induces an effective mass for the axion [21]:

$$m_a^2(T) = \gamma \frac{n_M(T)}{f_a}, \quad (8)$$

where  $\gamma$  is determined by the structure of the  $U(1)'$  sector.  $n_M(T)$  is the number density of the monopoles; assuming the abundance determined by the Kibble-Zurek mechanism [25], we find

$$n_M(T) \approx \alpha \left( \frac{T}{T_M} \right)^3 H(T_M)^3, \quad (9)$$

where  $H(T)$  is the Hubble parameter at temperature  $T$ , and  $\alpha \gtrsim 1$ .<sup>3</sup> The contribution of Eq. (8) makes the effect of the axion mass dominate over the Hubble friction

$$m_a(T) \gtrsim 3H(T), \quad (10)$$

below some temperature  $T_i$  ( $\leq T_M$ ), so that the axion field is subject to damped oscillations for  $T \lesssim T_i$ .

We assume that  $U(1)'$  is spontaneously broken at some temperature  $T_f$  ( $\ll T_i$ ), so that monopoles quickly disappear. Axion isocurvature fluctuations are then damped efficiently between temperatures  $T_i$  and  $T_f$ . Suppose

$$m_a^2(T) \propto T^n, \quad (11)$$

( $n = 3$  for a constant  $\gamma$ ). Since the axion “number density”  $m_a(T)\delta\theta(T)^2$  scales as  $T^3$  while Eq. (10) is satisfied, we find

$$\delta\theta(T) \propto T^p, \quad p \approx \frac{6-n}{4}, \quad (12)$$

<sup>2</sup> We assume that the relevant anomaly coefficient, with respect to  $U(1)'$ , is large enough that the coefficient in front of Eq. (7) is not much smaller than order unity. If this is not the case, temperatures  $T_i$  and  $T_f$  below become smaller (e.g. because  $\tilde{\gamma}$  in Eq. (14) is smaller than order unity), which may result in extra constraints on the  $U(1)'$  gauge sector.

<sup>3</sup>  $\alpha$  can be much larger than  $O(1)$ , depending on the dynamics of the phase transition; see e.g. [26]. In this case, monopole-antimonopole annihilations at  $T \sim T_M$  may become important.

<sup>1</sup> In this paper we adopt the instant reheating approximation for simplicity. An extension to more general cases is straightforward.

in this period. The final damping factor is thus

$$\Delta \approx \left( \frac{T_f}{T_i} \right)^{\frac{6-n}{4}}, \quad (13)$$

which can be compared with the required amount of damping from observations, Eq. (6).

The average axion field  $\langle \theta \rangle = \langle a \rangle / f_a$ , after this mechanism operates, is determined by the structure of the hidden sector, which in general differs from the minimum of the late-time axion potential,  $\theta_{\text{QCD}}$ . A homogeneous displacement of the axion field from  $\theta_{\text{QCD}}$  determines the late-time axion dark matter abundance. For  $f_a \gg 10^{12}$  GeV, the value of this displacement must be small, but it can be environmentally selected to be consistent with  $\Omega_a \leq \Omega_{\text{DM}}$  [23].

**Minimal Model.** Suppose the  $U(1)'$  sector below  $T_M$  contains only a charged scalar field  $\varphi$ , which breaks  $U(1)'$  at scale  $T_f$  ( $\ll T_M$ ). In this case, the factor  $\gamma$  in the expression for the induced axion mass, Eq. (8), is

$$\gamma \approx \tilde{\gamma} \frac{T_M}{f_a}, \quad (14)$$

where we have used  $T_M \lesssim f_a$ , and  $\tilde{\gamma} \approx O(1)$  assuming an  $O(1)$   $U(1)'$  gauge coupling. The axion mass just after the monopole production is then

$$\frac{m_a(T_M)}{3H(T_M)} \simeq 0.2 \sqrt{\alpha \tilde{\gamma}} g_{*M}^{\frac{1}{4}} \sqrt{\frac{T_M^3}{f_a^2 \bar{M}_{\text{Pl}}}}, \quad (15)$$

where we have used  $H(T_M) = \rho(T_M)^{1/2} / \sqrt{3} \bar{M}_{\text{Pl}}$  and  $\rho(T_M) = (\pi^2/30) g_{*M} T_M^4$  with  $g_{*M}$  being the effective number of relativistic degrees of freedom at temperature  $T_M$ . Assuming that  $T_M$  is not much smaller than the unification scale, this number is not too far from order unity. The axion field thus starts having damped oscillations at  $T \sim T_i$ , within a few orders of magnitude from  $T_M$ . Specifically

$$T_i \simeq 1 \times 10^{11} \text{ GeV } \alpha \tilde{\gamma} \sqrt{\frac{g_{*M}}{100}} \left( \frac{10^{16} \text{ GeV}}{f_a} \right)^2 \left( \frac{T_M}{3 \times 10^{15} \text{ GeV}} \right)^{\frac{1}{2}}, \quad (16)$$

Note that if  $T_i$  in this expression exceeds  $T_M$ , then  $T_i$  must be set to  $T_M$ .

At temperatures below  $T_i$ , axion isocurvature fluctuations are damped. Since Eq. (14) implies  $n = 3$ , so that  $p \approx 3/4$  (see Eq. (12)),

$$\frac{\delta \theta(T)}{\delta \theta(T_i)} \approx \left( \frac{T}{T_i} \right)^{\frac{3}{4}}. \quad (17)$$

Therefore, to avoid the observational constraint of

Eq. (6), we need

$$T_f \lesssim 2 \times 10^5 \text{ GeV } \alpha \tilde{\gamma} \sqrt{\frac{g_{*M}}{100}} \left( \frac{\Omega_{\text{DM}}}{\Omega_a} \right)^{\frac{2}{3}} \times \left( \frac{T_M/E_{\text{inf}}}{0.3} \right)^4 \left( \frac{10^{16} \text{ GeV}}{f_a} \right)^{1.5} \left( \frac{E_{\text{inf}}}{10^{16} \text{ GeV}} \right)^{\frac{4}{3}}, \quad (18)$$

(substituting  $H_{\text{inf}} \approx E_{\text{inf}}^2 / \sqrt{3} \bar{M}_{\text{Pl}}$ ). The required value of  $T_f$  is generated by  $V_{\text{hid}} = \lambda' (|\varphi|^2 - v'^2)^2$ , with  $v' \approx T_f$ . We find that unification scale axion dark matter with unification scale inflation can be made consistent by our mechanism.

Incidentally, ignoring  $U(1)'$  breaking, monopoles dominate the energy density of the universe at

$$T_* \simeq 6 \times 10^6 \text{ GeV } \alpha \sqrt{\frac{g_{*M}}{100}} \left( \frac{T_M}{3 \times 10^{15} \text{ GeV}} \right)^3 \left( \frac{m_M}{3 \times 10^{15} \text{ GeV}} \right), \quad (19)$$

which is slightly above the upper bound in Eq. (18). Here,  $m_M$  is the monopole mass. This implies that the universe may be monopole dominated toward the end of the damped oscillation period,  $T_f \lesssim T \lesssim T_i$ .

**Monopole Annihilations.** After  $U(1)'$  is broken at some temperature  $T_S$  ( $\sim T_f$ ), monopoles and anti-monopoles become connected by strings. For monopole-antimonopole annihilations to occur, the string-monopole system must lose their energies, and there are several processes that can contribute to this energy loss.

We assume the existence of a renormalizable coupling between the  $U(1)'$  and standard model sectors, e.g. a quartic coupling between the  $U(1)'$  breaking and standard model Higgs fields or a kinetic mixing between  $U(1)'$  and  $U(1)$  hypercharge:

$$\mathcal{L} \sim \epsilon \varphi^\dagger \varphi h^\dagger h, \quad \epsilon F'_{\mu\nu} F_Y^{\mu\nu}. \quad (20)$$

We will find that monopoles quickly disappear, well within a Hubble time, unless the coupling  $\epsilon$  is significantly suppressed. Note that cosmic strings formed by  $U(1)'$  breaking are harmless for  $T_S \lesssim 10^{15}$  GeV [27].

(i) Monopole friction. Suppose the correlation length of  $\varphi$  is of order or larger than the average distance between monopoles,  $d(T_S) \sim n_M(T_S)^{-1/3} \sim \bar{M}_{\text{Pl}} / \alpha^{1/3} T_S T_M$ , at  $T \sim T_S$ . In this case, strings will connect monopoles through the shortest possible path, and the energy of a monopole-antimonopole pair to be dissipated is  $E_0 \sim \eta d(T_S) \sim \bar{M}_{\text{Pl}} T_S / \alpha^{1/3} T_M$ , where we have estimated the string tension  $\eta$  to be of order  $T_S^2$ .

If the monopoles scatter with a thermal bath of temperature  $T_S$  through a coupling of strength  $\epsilon$ , as in Eq. (20), then the energy loss rate due to friction is  $\dot{E} \sim -\epsilon^2 T_S^2 v^2$  [28]. Here,  $v$  is the velocity of the monopoles, which is given by  $\sim (T_S^2 d(T_S) / m_M)^{1/2} \sim (T_S \bar{M}_{\text{Pl}} / \alpha^{1/3} T_M^2)^{1/2}$  if  $T_S \ll \alpha^{1/3} T_M^2 / \bar{M}_{\text{Pl}}$  and  $\sim 1$  otherwise. In each case, the annihilation timescale  $\tau_{\text{ann}} \sim$

$|E_0/\dot{E}|$  is given by

$$\tau_{\text{ann}} \sim \begin{cases} \frac{T_M}{\epsilon^2 T_S^2} & \text{for } T_S \ll \frac{\alpha^{1/3} T_M^2}{M_{\text{Pl}}}, \\ \frac{\bar{M}_{\text{Pl}}}{\epsilon^2 \alpha^{1/3} T_S T_M} & \text{for } T_S \gtrsim \frac{\alpha^{1/3} T_M^2}{M_{\text{Pl}}}. \end{cases} \quad (21)$$

We find that in both cases,  $\tau_{\text{ann}}$  is of order or shorter than the Hubble timescale,  $t_S \sim \bar{M}_{\text{Pl}}/T_S^2$ , unless  $\epsilon$  is much smaller than of order unity. For  $T_M \sim 10^{15}$  GeV and  $T_S \sim 10^5$  GeV, this requires  $\epsilon^2 \gtrsim 10^{-3}$  (assuming  $\alpha \gg 10^{-21}$ ).

(ii) Particle production from strings. If the correlation length of  $\varphi$  at  $T_S$ ,  $\xi(T_S)$ , is much smaller than the average monopole distance, then we expect that a string connecting a monopole-antimonopole pair to have a significant number of kinks (from a Brownian formation), and particle production from the string contributes significantly to the dissipation.

Based on the analysis in Ref. [28], we estimate that the power for a string of thickness  $\delta$  to radiate standard model particles is  $P \sim \epsilon^2/\delta \xi(T_S)$  per a portion of a string of length  $\xi(T_S)$ .<sup>4</sup> In the case of Brownian strings, the average string length is given by  $L \sim d(T_S)^2/\xi(T_S)$ , so that the total energy of the string-monopole system to be dissipated is  $E_0 \sim \eta L \sim T_S^2 d(T_S)^2/\xi(T_S)$  and the emission power from it is  $\dot{E} \sim PL/\xi(T_S) \sim \epsilon^2 T_S d(T_S)^2/\xi(T_S)^3$ , where we have used  $\eta \sim T_S^2$  and  $\delta \sim 1/T_S$ . The monopole-antimonopole annihilation timescale is thus

$$\tau_{\text{ann}} = \frac{E_0}{\dot{E}} \sim \frac{1}{\epsilon^2} T_S \xi(T_S)^2 \ll \frac{1}{\epsilon^2} T_S d(T_S)^2 \sim \frac{\bar{M}_{\text{Pl}}^2}{\epsilon^2 \alpha^{2/3} T_S T_M^2}. \quad (22)$$

Again, this is of order or shorter than the Hubble timescale unless  $\epsilon$  is much smaller than order unity. For  $T_M \sim 10^{15}$  GeV and  $T_S \sim 10^5$  GeV, this requires  $\epsilon^2 \gtrsim 10^{-7}/\alpha^{2/3}$ .

Since annihilation is quick regardless of the  $\varphi$  correlation length, possible increase of the correlation length due to interactions of the strings with the thermal bath (which we ignored here but may become important for  $T_S \lesssim T_M^2/\bar{M}_{\text{Pl}}$ ) does not affect our conclusion.

**Technical Naturalness of  $U(1)'$ .** In our minimal model,  $U(1)'$  breaking was achieved by a scalar field  $\varphi$  whose potential contained a scale  $v'$ , which is not radiatively stable. In this section, we discuss extensions/modifications of the minimal model in which the issue of radiative stability does not arise.

(i) Supersymmetric  $U(1)'$  sector. One way to construct a technically natural model is to make the  $U(1)'$  sector supersymmetric. This requires promoting  $\varphi$  to

chiral superfields  $\Phi(+1)$  and  $\bar{\Phi}(-1)$ . The complication arises because the induced axion mass is suppressed in the presence of light fermions charged under  $U(1)'$  [21]. To obtain a significant contribution to the axion mass, we need to have a supersymmetric mass for  $\Phi$  and  $\bar{\Phi}$ :

$$W = M_\Phi \Phi \bar{\Phi}. \quad (23)$$

The breaking of  $U(1)'$  is then caused by supersymmetry-breaking squared masses for  $\Phi$  and  $\bar{\Phi}$  of order  $\tilde{m}^2 \sim T_S^2$ . To maximize the axion mass, we also take  $M_\Phi \sim T_S$ .<sup>5</sup> The coupling between the  $U(1)'$  and the standard model sectors needed for monopole annihilations can be taken as a kinetic mixing between  $U(1)'$  and  $U(1)$  hypercharge:  $\mathcal{L} \sim \epsilon [\mathcal{W}'^\alpha \mathcal{W}_{Y\alpha}]_{\theta^2}$ . This implies that the standard model sector is also supersymmetric above  $\sim (4\pi/\epsilon)\tilde{m}$ .

With this setup, the induced axion mass is given by Eq. (8) with

$$\gamma \approx \frac{M_\Phi}{f_a} \sim \frac{T_S}{f_a}. \quad (24)$$

Plugging this into Eq. (18) with  $T_S \sim T_f$ , we find that  $\alpha$  must be much larger than 1 for the model to work. We thus suppose that the dynamics of the phase transition producing monopoles is such that  $\alpha \gg 1$ . The largest possible abundance of monopoles obtained in this case is determined by the freezeout abundance (instead of Eq. (9)), which is given by [31]

$$n_M(T) \approx \left(\frac{T}{T_M}\right)^3 \frac{\sqrt{g_{*M}} T_M^4}{\bar{M}_{\text{Pl}}}. \quad (25)$$

The axion mass at  $T \sim T_M$  is then

$$\frac{m_a(T_M)}{3H(T_M)} \sim \frac{\sqrt{T_S \bar{M}_{\text{Pl}}}}{g_{*M}^{1/4} f_a}, \quad (26)$$

so that the axion field starts damped oscillations at

$$T_i \sim \frac{T_S T_M \bar{M}_{\text{Pl}}}{\sqrt{g_{*M}} f_a^2}. \quad (27)$$

This gives the damping factor of

$$\Delta \approx \left(\frac{T_f}{T_i}\right)^{\frac{3}{4}} \sim \left(\frac{f_a^2}{T_M \bar{M}_{\text{Pl}}}\right)^{\frac{3}{4}}. \quad (28)$$

We find that the mechanism is not as strong as in the minimal model, but it can still save the scenario with  $f_a$ ,  $T_M$ ,  $E_{\text{inf}}$  as large as  $\sim 10^{15}$  GeV.

(ii) Possibility of unbroken  $U(1)'$ . We finally mention an alternative (and very different) possibility that

<sup>4</sup> The process of energy dissipation may be much faster,  $P \sim \epsilon^2 \eta (\delta/\xi(T_S))^{1/3}$ , if cusps form efficiently [29]. Here we adopt a conservative estimate, which is sufficient to eliminate the monopoles quickly.

<sup>5</sup> The coincidence of the scales  $\tilde{m}$  and  $M_\Phi$  is analogous to the  $\mu$  problem in the minimal supersymmetric standard model, which can be addressed, e.g., as in Ref. [30].

$U(1)'$  monopoles may be efficiently eliminated without breaking  $U(1)'$ . This may happen if the monopole under consideration is in fact a dyon that also carries a charge under a hidden non-Abelian gauge group  $G'$  (to which the axion field does not couple). In this case, if  $G'$  confines at a scale  $\Lambda'$ , then dyons can be subjected to extra strong annihilation processes.

Suppose the  $G'$  sector contains light particles that are electrically charged under  $G'$ . When  $G'$  confines at  $T \sim \Lambda'$ , dyons pick up these light particles, becoming  $G'$  hadrons. The dyon-antidyon annihilation cross section is then expected to become large  $\sim 1/\Lambda'^2$ , as in the analogous situation for a heavy stable colored particle [32]. This will efficiently eliminate dyons if  $\Lambda' \lesssim 100$  TeV, hence giving  $T_f \sim \Lambda'$ . Since this scenario does not require  $U(1)'$  breaking, the  $U(1)'$  sector need not have a light charged scalar or fermion. Further studies of this possibility, including a detailed analysis of whether dyon annihilation is indeed strong enough, are warranted.

**Conclusions.** Because the axion provides a leading solution to the strong  $CP$  problem, it is important to fully study its consistency. If a future CMB experiment discovers inflationary gravitational wave signals, it would exclude naive axion models with the Peccei-Quinn symmetry broken before the end of inflation. Our mechanism makes the QCD axion alive even in such a case, without requiring the Peccei-Quinn symmetry breaking scale to be below the inflationary scale. This is particularly important for a string axion, which has a virtue that explicit breaking of the Peccei-Quinn symmetry (which needs to be extremely small to solve the strong  $CP$  problem [33]) is generated only at a nonperturbative level [8]. Our mechanism allows for a string axion to be a consistent solution to the strong  $CP$  problem even if inflationary gravitational wave signals are discovered, and it would also keep open the possibility that axion dark matter may be discovered by high precision experiments such as those proposed in Ref. [9].

**Note added:** While completing this paper, we received Ref. [34] which discusses a similar idea.

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