# Topological Effects on Quantum Phase Slips in Superfluid Spin Transport

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# Topological Effects on Quantum Phase Slips in Superfluid Spin Transport

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We theoretically investigate effects of quantum fluctuations on superfluid spin transport through easy-plane quantum antiferromagnetic spin chains in the large-spin limit. Quantum fluctuations result in the decaying spin supercurrent by unwinding the magnetic order parameter within the easy plane, which is referred to as phase slips. We show that the topological term in the nonlinear sigma model for the spin chains qualitatively differentiates decaying rate of the spin supercurrent between integer and half-odd-integer spin chains. An experimental setup for a magnetoelectric circuit is proposed, in which the dependence of the decaying rate on constituent spins can be verified by measuring nonlocal magnetoresistance.

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*Introduction.*—One-dimensional quantum magnetism has been a natural hotbed to seek and study exotic states that defy classical descriptions [1, 2]. A prototypical example showing importance of quantum effects is provided by Heisenberg antiferromagnetic spin chains. For isotropic spin-$s$ chains, Haldane suggested in 1983 [3] that integer-$s$ chains have disordered ground states with gapped excitations unlike half-odd-integer-$s$ chains having gapless excitations [4]. The existence of the gap has been experimentally confirmed for $s = 1$ [5].

By considering anisotropic antiferromagnetic spin chains in the large-$s$ limit, Affleck [6] was able to attribute this distinction between integer and half-odd-integer spin chains to the topological term in the O(3) nonlinear sigma model that describes the dynamics of the local Néel order parameter [3, 7, 8]. For sufficiently large $s$, easy-plane spin-$s$ chains are in the gapless XY phase, where order-destroying excitations are vortices of the order parameter in the two-dimensional Euclidean spacetime. It is the skyrmion charge $Q$ of a vortex, quantifying how many times the order parameter wraps the unit sphere, that serves as the topological charge in the nonlinear sigma model. Figure 1 illustrates vortices with minimum nonzero skyrmion charges $Q = \pm 1/2$, which are often referred to as merons [9]. Only for half-odd-integer spin chains, the topological term creates destructive interference between vortices and, thereby, suppresses effects of their quantum fluctuations [1, 10].

Superfluid spin transport, a spin analog of an electrical supercurrent, has been proposed in magnets with easy-plane anisotropy, where the direction of the local magnetic order within the easy plane plays the role of the phase of a superfluid order parameter [11–14]. Spin supercurrent therein is sustained by a spiraling texture of the magnetic order, being proportional to the gradient of the in-plane components of the order parameter. Under the guidance of established theories for resistance in superconducting wires [15], we have recently investigated intrinsic thermal dissipation in one-dimensional superfluid spin transport, which arises via thermally-activated

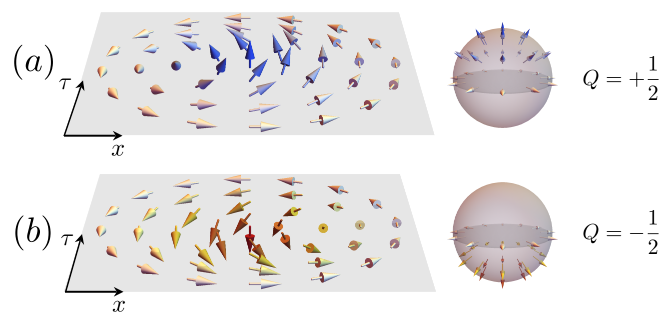


FIG. 1. (color online) Vortex configurations of the local Néel order parameter in the Euclidean spacetime $(x, \tau)$ with skyrmion charges (a) $Q = 1/2$ and (b) $Q = -1/2$.

phase slips [16] (that unwind the phase by lifting the magnetic order out of the easy plane [17]). At sufficiently low temperatures, however, dissipation is mainly induced by quantum fluctuations via quantum phase slips (QPS) [18, 19]. The QPS in superconducting wires correspond to vortices of the phase of the order parameter in the Euclidean spacetime. Likewise, the QPS in one-dimensional spin superfluidity correspond to vortices of the magnetic order parameter. Then, there arises a natural question regrading the role of the topological term for the integer-$s$ and half-odd-integer-$s$ chains in the QPS-induced dissipation of superfluid spin transport.

In this Letter, we theoretically study the QPS in superfluid spin transport through easy-plane quantum antiferromagnetic spin chains. For integer $s$, the topological term is inoperative, and dissipation arises due to the QPS of skyrmion charges $Q = \pm 1/2$ that change winding number by $2\pi$. For half-odd-integer $s$, these QPS are completely suppressed due to destructive interferences. Instead, the QPS of twice-larger skyrmion charges, $Q = \pm 1$, give rise to dissipation by unwinding the phase by $4\pi$. See Fig. 2 for illustrations. Dissipation in superfluid spin transport can be characterized by the spin-current decay rate, $\kappa(I, T)$, which depends on the spin current $I$ and the ambient temperature $T$. One

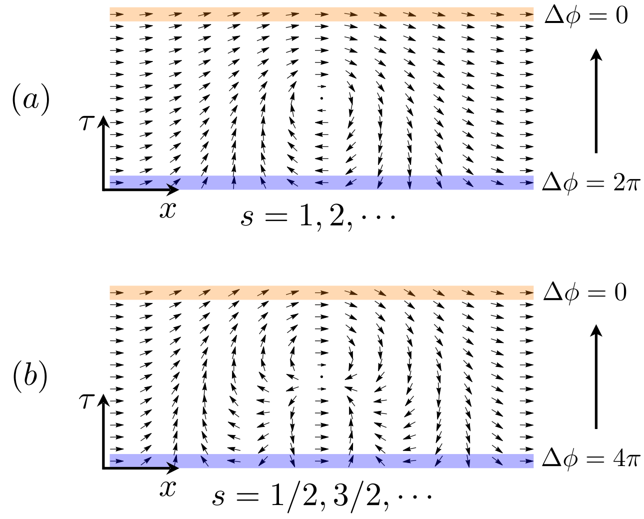


FIG. 2. (color online) Elementary vortices, which control the winding number $\Delta\phi$, with skyrmion charges (a) $Q = 1/2$ and (b) $Q = 1$. For half-odd-integer spin chains, $2\pi$ phase slips are prohibited by destructive interference between vortices with skyrmion charges $Q = \pm 1/2$. See the main text for a detailed discussion.

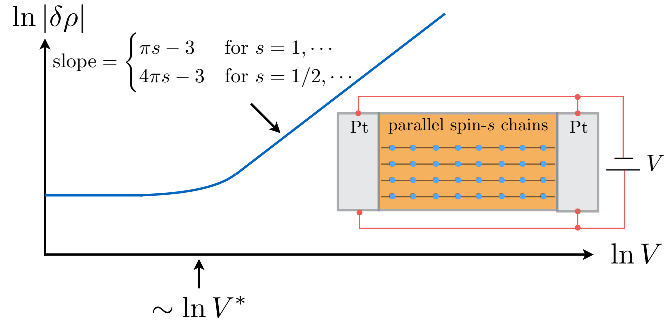


FIG. 3. (color online) A change in electrical resistance $|\delta\rho|$ of the magnetoelectric circuit as a function of an applied voltage $V$ on logarithmic scale. The circuit consists of a quasi one-dimensional antiferromagnet (a 3D stack of parallel spin chains) and two platinum layers. See the main text for a detailed discussion.

of our main findings is a qualitative difference between the decay rates in integer-$s$ and half-odd-integer-$s$ spin chains for large spin $s \gg 1$, which can be summarized as $\kappa(I,T) \propto [\max(I,T)]^{2\mu-3}$ where

$$\mu = \begin{cases} \pi s/2\,, & \text{for integer } s \\ 2\pi s\,, & \text{for half-odd-integer } s \end{cases}. \tag{1}$$

The exponent $\mu$ parametrizes the strength of interaction between the QPS, which is proportional to the square of their skyrmion charges; $\mu$ is thus four times larger for half-odd-integer $s$ than for integer $s$. These spin-dependent transport exponents can be measured through the voltage or temperature dependence of electrical resistance of the magnetoelectric circuit in Ref. [20] (see Fig. 3 for its schematics), which we propose for probing superfluid spin transport, using a quasi-one-dimensional easy-plane antiferromagnetic insulator, e.g., $(CH_3)_4NMnCl_3$ ($s = 5/2$) [21] as a spin transport channel.

*Model.*—We consider an anisotropic Heisenberg antiferromagnetic spin-$s$ chain that can be described by the Hamiltonian

$$H = J \sum_n \left[\mathbf{S}_n \cdot \mathbf{S}_{n+1} - a S^z_n S^z_{n+1} + b (S^z_n)^2\right] \tag{2}$$

with $\mathbf{S}_n^2 = s(s+1)$, where small positive constants $a \ll 1$ and $b \ll 1$ parametrize the anisotropy. In the large-$s$ limit, neighboring spins are mostly antiparallel, $\mathbf{S}_n \approx -\mathbf{S}_{n+1}$ in the low-energy states, and the long-wavelength dynamics of the chain can be understood in terms of the slowly varying unit vector $\mathbf{n} \approx (\mathbf{S}_{2n} - \mathbf{S}_{2n+1})/2s$ parametrizing the direction of the local Néel order parameter. The dynamics of the field $\mathbf{n}$ follows the nonlinear sigma model [3, 6–8] with the Euclidean action $S = i\theta Q + S_0$ (in units of $\hbar$), where $\theta \equiv 2\pi s$ is referred to as the topological angle. Here,

$$Q \equiv \frac{1}{4\pi} \int dx \int_0^{\hbar\beta} d\tau\, \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_\tau \mathbf{n}) \tag{3}$$

is the skyrmion charge of $\mathbf{n}$ that measures how many times $\mathbf{n}(x,\tau)$ wraps the unit sphere as the space and imaginary-time coordinates, $x$ and $\tau$, vary, and is thus topological. The nontopological part of the action is given by

$$S_0 = \frac{1}{2g} \int dx \int_0^{\hbar\beta c} d(c\tau) \left[\frac{(\partial_\tau \mathbf{n})^2}{c^2} + (\partial_x \mathbf{n})^2 + \frac{n_z^2}{\lambda^2}\right], \tag{4}$$

where $c \equiv 2Jsd/\hbar$ serves as a speed of "light" for the theory, $d$ is the lattice constant, and $\lambda \equiv d/\sqrt{2(a+b)}$ is a characteristic length scale (providing the ultraviolet cutoff for our theory) governed by the anisotropy. Here, $g \equiv 2/s$ is the dimensionless coupling constant, which sets the quantum "temperature" governing the magnitude of quantum fluctuations [3].

The corresponding partition function is given by

$$\mathcal{Z} = \int \mathcal{D}\mathbf{n}(x,\tau) \delta(\mathbf{n}^2 - 1) \exp(-i\theta Q - S_0)\,. \tag{5}$$

We consider the fields $\mathbf{n}$ that are periodic in the imaginary time $\tau$, $\mathbf{n}(x,\tau) = \mathbf{n}(x, \tau + \hbar\beta)$. The partition function $\mathcal{Z}$ is then a periodic function of the topological angle $\theta$. For integer and half-odd-integer $s$, therefore, we can effectively set $\theta = 0$ and $\theta = \pi$, respectively [1].

*Spin superfluidity.*—The classical action for $\mathbf{n}(x,t)$ can be obtained from the above quantum action $S_0$ by Wick rotation, $\tau \mapsto it$. Its invariance under spin rotations about the $z$ axis implies conservation of spin angular momentum (polarized along the $z$ axis) and leads us to parametrize $\mathbf{n}$ in spherical coordinates, $\psi$ and $\phi$, defined by $\mathbf{n} = (\sin\psi\cos\phi, \sin\psi\sin\phi, \cos\psi)$. The density and current of spin angular momentum, $\rho \equiv (\hbar^2/4Jd)\sin^2\psi\partial_t\phi$ and $I \equiv -Js^2 d\sin^2\psi\partial_x\phi$, satisfy the continuity equation [11, 22]:

$$\partial_t\rho + \partial_x I = 0 \,. \tag{6}$$

Time-independent stable solutions to the classical equations of motion (which includes the above continuity equation) are given by

$$\psi(x) = \pi/2\,, \quad \phi(x) = \phi_0 + kx \quad \left(|k| < \lambda^{-1}\right)\,, \tag{7}$$

with $\phi_0$ an arbitrary reference angle [11]. The spin current, $I = -Js^2kd$, is sustained by a spiraling texture of $\mathbf{n}$ within the easy plane, which we identity as the spin supercurrent by the analogy to the electrical supercurrent maintained by a gradient of the phase of the superconducting order parameter. The ultraviolet cutoff $\lambda^{-1}$ sets a critical current for stable superfluid spin transport. When the chain is long enough, $L \gg \lambda$, which we assume henceforth, actual boundary conditions at the ends of the chain are not important. Imposing periodic boundary conditions on the order parameter, $\mathbf{n}(x=0,\tau) = \mathbf{n}(x=L,\tau)$, quantizes allowed spin supercurrent, $k_\nu = 2\pi\nu/L$, where $\nu = \Delta\phi/2\pi$ is the winding number of $\mathbf{n}$ in the easy plane.

*QPS in spin superfluidity.*—The spin supercurrent in a closed chain can be indefinitely maintained if there are no fluctuations. Finite dissipation, however, arises due to thermal and quantum fluctuations, which provide transition channels between steady states with different winding numbers $\nu \neq \nu'$ [17]. Such events changing winding numbers are referred to as phase slips. In this Letter, we are interested in the QPS, which dominate over the thermally-activated phase slips at sufficiently low temperatures.

The QPS are vortex configurations of $\mathbf{n}$ in the two-dimensional Euclidean spacetime [15]. For a single vortex centered at the origin, which is a saddle point of the action $S_0$, the azimuthal angle is given by

$$\phi_q(x,\tau) = \phi_0 + q\arctan(c\tau/x)\,, \tag{8}$$

where nonzero integer $q$ is the vortex vorticity. The polar angle is given by a function $\psi(r)$ of the radial distance $r \equiv \sqrt{x^2 + c^2\tau^2}$, which solves $d^2\psi/dr^2 + (1/r)d\psi/dr = -\sin\psi\cos\psi(1/\lambda^2 - q^2/r^2)$ with boundary conditions $\psi(0) = (1-p)\pi/2$ and $\psi(r\to\infty) = \pi/2$ [23]. The order parameter $\mathbf{n}$ is substantially out of the easy plane only within the vortex core $r \lesssim \lambda$. At the vortex center, the order parameter points either toward the north pole, $p = +1$, or the south pole, $p = -1$, which is referred to as the vortex polarity. Vortex vorticity $q$ and polarity $p$ govern the skyrmion charge $Q = pq/2$ [24]. See Fig. 1 for illustrations of vortices with $Q = \pm 1/2$.

Let us now consider a dilute gas of $n$ QPS in the background of a low spin current $k \ll \lambda^{-1}$ [25]. The gas of the QPS must be vorticity-neutral $\sum_i q_i = 0$ to meet the periodic boundary conditions. Substituting a saddle-point solution, $\phi = kx + \sum_i \phi_{q_i}(x - x_i, \tau - \tau_i)$ and the corresponding $\psi(x,\tau;\{p_i\})$, into the action, we find

$$S = i\theta\sum_i p_i q_i/2 + S_0\,, \tag{9}$$

$$S_0 = \sum_i S_{\text{core}}(q_i) - (2\pi/g)\sum_{i<j} q_i q_j \ln(d_{ij}/\lambda) + (2\pi/g)ck\sum_i q_i\tau_i\,, \tag{10}$$

where $d_{ij} = \sqrt{(x_i - x_j)^2 + c^2(\tau_i - \tau_j)^2} \gg \lambda$ is the distance between the QPS [26]. The nontopological part of the action $S_0$ consists of three terms. The first term is the contribution from the vortex cores to the action, which can be estimated as $S_{\text{core}} \sim \pi/g$ (increasing with $q$). The second term is logarithmic interaction between the QPS. The third term couples the QPS to the spin current $\propto k$.

The topological term $i\theta\sum_i p_i q_i/2$ depends on polarities $\{p_i\}$ of the QPS, whereas the nontoplogical term $S_0$ does not. For fixed vorticity configuration $\{q_i\}$, the partition function is summed over two possible polarities for each QPS, $p_i = \pm 1$, which results in

$$\mathcal{Z} \propto \left[\prod_i \cos\left(\frac{\theta q_i}{2}\right)\right] e^{-S_0(\{q_i\})}\,. \tag{11}$$

As pointed out by Affleck [6], the prefactor of the partition function distinguishes integer and half-odd-integer $s$. For integer $s$, the topological angle is zero $\theta = 0$, and thus the prefactor is 1. Half-odd-integer $s$, however, yields $\theta = \pi$, and the prefactor vanishes when any of vorticities $\{q_i\}$ is odd. This destructive interference between the QPS with odd vorticities can be effectively captured by setting an elementary vorticity of the QPS to 2. Let us use the symbol $q_0$ to denote an elementary vorticity; $q_0 = 1$ and $q_0 = 2$ for integer and half-odd-integer $s$, respectively. Low-energy dynamics of the order parameter will be dominated by the QPS with the elementary vorticity. We therefore focus on a gas of such QPS, which is described by the effective action:

$$S_{\text{eff}} = nS_{\text{core}} - 2\mu\sum_{i<j}\tilde{q}_i\tilde{q}_j\ln(d_{ij}/\lambda) + \sigma\sum_i\tilde{q}_i\tau_i\,, \tag{12}$$

where $\mu \equiv \pi q_0^2/g$ [Eq. (1)] is the interaction strength between the effective QPS, $\sigma \equiv 2\pi q_0 ck/g$ is the rescaled spin current, and $\tilde{q}_i \equiv q_i/q_0 = \pm 1$ is the elementary vorticity sign. The effective action $S_{\text{eff}}$ without the last term has been invoked when studying the phase diagram of spin chains, e.g, in Ref. [1].

*Analogy to superconducting wires.*—Owing to the formal equivalence of the action $S_{\text{eff}}$ to the action for a gas of the QPS in a superconducting wire, specifically Eq. (4) in Ref. [19], we can adopt the results for superconductivity to our case of spin superfluidity. First of all, there is a superfluid-to-insulator phase transition at the critical interaction strength $\mu^*$ in the absence of the spin current, $\sigma = 0$. For $\mu > \mu^*$, the QPS of opposite vorticity attract strongly and form bound pairs, keeping spin superfluidity intact. As $\mu$ decreases below $\mu^*$, the QPS proliferate and destroy spin superfluidity, driving the system to the insulating phase. These insulating and superfluid phases are, respectively, the gapped Haldane and the gapless XY phases of anisotropic spin chains [1]. The condition for being in the superfluid phase is $\mu > \mu^* \approx 2$ [19, 27], which corresponds to $s \geq 2$ and $s \geq 1/2$ for integer and half-odd-integer $s$, respectively [28].

Secondly, the QPS rates have been derived for a superconducting wire in Ref. [19] by following the Langer's theory for the decay of metastable states [29]. By adopting the results to the case of spin superfluidity, we can find the average decay rate $\kappa(I,T)$ of the winding number, $\dot{\nu} = -\kappa\nu$, as a function of the spin current $I$ and the ambient temperature $T$ in the deep superfluid regime $\mu \gg 1$:

$$\begin{aligned} \kappa(I,T) &= z^2\omega_0(T/\hbar\omega_0)^{2\mu-2}\mathcal{F}(I/T)\,, \\ \mathcal{F}(\xi) &\equiv C\sinh(\xi/2)\left|\Gamma(\mu-1/2+i\xi/2\pi)\right|^2\,, \end{aligned} \tag{13}$$

where $z \equiv \exp(-S_{\text{core}})$ is the fugacity of the QPS, $\omega_0 \equiv c/\lambda$ is the characteristic frequency of the spin chain ($\hbar\omega_0$ is the gap of the out-of-easy-plane spin wave branch [30]), and $C \equiv 8\pi^{3/2}(2\pi)^{2\mu-2}\Gamma(\mu-1/2)/\Gamma(\mu)\Gamma(2\mu-1)$ is a numerical constant [31]. The expression for $\kappa(I,T)$ can be simplified as [32, 33]

$$\kappa(I,T) \propto \begin{cases} z^2\omega_0(T/\hbar\omega_0)^{2\mu-3}\,, & \text{for } I \ll T \\ z^2\omega_0(I/\hbar\omega_0)^{2\mu-3}\,, & \text{for } T \ll I \end{cases}. \tag{14}$$

To see such quantum effects, we should work at sufficiently low temperatures, where quantum fluctuations dominate over thermal fluctuations. The crossover temperature $T^*$ can be estimated by matching the classical phase-slip energy barrier (divided by $T$) [17] to the action of two noninteracting QPS [19], $2\hbar c/\lambda T^* \approx 2S_{\text{core}}$. Using $S_{\text{core}} \sim \pi/g$ yields $T^* \sim 2\hbar c/\pi s\lambda$.

*Experimental proposal.*—The supercurrent decay rate can be experimentally inferred by measuring electrical resistance of the magnetoelectric circuit that has been proposed for probing superfluid spin transport [20]. The circuit consists of a quasi one-dimensional easy-plane antiferromagnet and two parallel-connected metals with strong spin-orbit coupling (e.g., platinum) sandwiching it. See Fig. 3 for schematics of the setup. With charge current flowing, two interfaces of the antiferromagnet to the metals act as a spin source and drain for spin transport via spin-transfer torque and spin pumping [34]. The spin supercurrent is sustained by a spiraling texture of the local order parameter within the easy plane. The QPS disturb the texture and unwind it by $2\pi$ for integer $s$ and $4\pi$ for half-odd-integer $s$, with the frequency $\kappa$. This unwinding of the phase propagates to the ends of spin chains and induce dynamics of spins at the interfaces. Via spin pumping, spin rotations generate an electromotive force on electrons in the metals, which decreases the effective resistance of the circuit.

Following derivations of Refs. [17, 20], we can calculate the change of the effective resistance: $\rho \to \rho + \delta\rho$, where $\delta\rho = -\vartheta^2\kappa(I,T)LA/2Js^2d$ (treating the QPS as a perturbation to uniform spin-current states). Here, $I$ is the spin current flowing through a single chain of cross section $A$, $\rho$ is the resistivity of the metal, and $\vartheta$ is related to the effective interfacial spin Hall angle $\Theta$ via $\vartheta \equiv (\hbar/2et)\tan\Theta$, with $-e$ being the electric charge of a single electron and $t$ being the thickness of the metals in the direction perpendicular to the interface. Figure 3 schematically depicts the resistance change $\delta\rho$ as a function of a voltage $V$ on logarithmic scale at a fixed temperature. Above the transition voltage $V^*$, at which the spin current is equal to the temperature $I = T$, $\ln|\delta\rho|$ increases linearly as $\ln V$ increases with the slope $2\mu - 3$ that is determined by constituent spins. Below the transition voltage, $\delta\rho$ converges to a constant value that is determined by the ambient temperature.

For quantitative estimates, let us take the following parameters for a quasi-one-dimensional antiferromagnet $(CH_3)_4NMnCl_3$ [21]: $s = 5/2$, $Js^2 = 85$ K, $Js^2(a+b) = 2$ K, $d = 3$ nm, and the interchain distance $d' = 9$ nm (yielding $A = d'^2 = 81$ nm$^2$). The associated continuum parameters are $\lambda = 10$ nm and $c = 3 \times 10^5$ m/s, which yield the critical spin current $I_c = Js^2d/\lambda = 18$ K and the crossover temperature $T^* = 5$ K. For geometry of the materials, we consider the platinum metals with thickness $t = 5$ nm and the antiferromagnet with length $L = 1$ $\mu$m. Using $\Theta = 0.03$ for the interfacial spin Hall angle (measured for Pt|YIG interfaces [35]), the change in the effective resistance is $\delta\rho = -0.1$ $\mu\Omega$ at the spin current of $I = I_c/10$ and temperature $T = 3$ K.

*Discussion*—In certain spin chains, dimerization of sites can occur at low temperatures, e.g., as a result of the spin-Peierls transition [36]. The Hamiltonian then acquires a new term that breaks the sublattice symmetry; $H \to H + \alpha J \sum_i (-1)^i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$. The topological term in the nonlinear sigma model changes as well: $\theta = 2\pi S(1+\alpha)$ [37]. With this change of $\theta$, for half-odd-integer $s$, a pair of the QPS with skyrmion charges $Q = \pm 1/2$ contributes to the partition function with the prefactor $4\sin^2(\pi\alpha/2)$, which would change the elementary vorticity $q_0$ from 2 to 1.

The QPS occur not only in one-dimensional spin chains, but also in two- and three-dimensional easy-plane magnets. We have focused on spin chains in this Letter, in which the effect of the QPS is strong enough to destroy long-range magnetic order at zero temperature. Quantum fluctuations are less important in higher-dimensional systems. For example, the Heisenberg easy-plane antiferromagnet on the square lattice orders at zero temperature [38], which justifies the semiclassical mean-field treatment of superfluid spin transport [14].

We would like to mention that QPS in topological superconductors occur in multiples of $4\pi$ (instead of $2\pi$ in conventional superconductors) [39] as in superfluid spin transport through half-odd-integer spin chains.

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