Model-Independent Determination of the Shear Viscosity of a Trapped Unitary Fermi gas: Application to High-Temperature Data

M. Bluhm and T. Schäfer

Phys. Rev. Lett. **116**, 115301 — Published 18 March 2016
DOI: 10.1103/PhysRevLett.116.115301

Model-independent determination of the shear viscosity of a trapped unitary Fermi gas: Application to high temperature data

M. Bluhm and T. Schäfer
Department of Physics, North Carolina State University, Raleigh, NC 27695

Determinations of the shear viscosity of trapped ultracold gases suffer from systematic, uncontrolled uncertainties related to the treatment of the dilute part of the gas cloud. In this work we present an analysis of expansion experiments based on a new method, anisotropic fluid dynamics, that interpolates between Navier-Stokes fluid dynamics at the center of the cloud and ballistic behavior in the dilute corona. We validate the method using a comparison between anisotropic fluid dynamics and numerical solutions of the Boltzmann equation. We then apply anisotropic fluid dynamics to the expansion data reported by Cao et al. In the high temperature limit we find $\eta = 0.282(mT)^{3/2}$, which agrees within about 5% with the theoretical prediction $\eta = 0.269(mT)^{3/2}$.

Introduction: A number of studies have been devoted to extracting the transport properties of dilute atomic Fermi gases. Quantities of interest include the shear viscosity [1–11], the bulk viscosity [12], and the spin diffusion constant [13–15]. These transport coefficients provide valuable information about the nature of the low energy degrees of freedom. Strongly correlated Fermi gases also contribute important insights into the transport properties of other quantum many-body systems, such as high-T_c superconductors or the quark-gluon plasma [16–18]. Truly model-independent determinations of the transport coefficients of trapped atomic gases have so far been precluded, however, by the fact that there is a transition from fluid dynamical behavior in the dense part of the cloud to weakly collisional kinetic behavior in the dilute corona.

Consider, for example, a unitary Fermi gas expanding after release from a deformed harmonic trap [19]. Fluid dynamics predicts that the difference in pressure gradients along the short and the long axis of the cloud translates into a larger acceleration along the short direction. This implies that the aspect ratio A_R of the cloud, which is initially much smaller than one, quickly grows and eventually exceeds unity, as was first observed by O'Hara et al. [20]. Shear viscosity η slows down the acceleration in the transverse direction, and measurements of $A_R(t)$ for different initial values of T/T_F, where T_F is the Fermi temperature, can be used to constrain the dependence of $\eta(n,T)$ on density n and temperature T. This task is simplified by the scale invariance of the unitary Fermi gas, which implies that the bulk viscosity vanishes, and that $\eta = (mT)^{3/2} f(n/(mT)^{3/2})$, where $f(x)$ is a universal function. Note that we use units $\hbar = k_B = 1$.

The natural tool for extracting $\eta(n,T)$ is the Navier-Stokes (NS) equation. The problem in determining $\eta(n,T)$ is that $A_R(t)$ is a global property of the cloud, and that the NS equation breaks down in the dilute corona, where the mean free path is large compared to the density and the pressure scale heights. Because the total number of particles in the corona is small, one might hope that this does not lead to serious difficulties. Unfortunately, this is not the case: The rate of dissipative heating is $\dot{q} = \frac{\eta}{2}(\sigma_{ij})^2$, where $\sigma_{ij} = \nabla_i u_j + \nabla_j u_i - \frac{2}{3}\delta_{ij}\vec{\nabla}\cdot\vec{u}$ is the strain tensor, and $\vec{u}$ is the fluid velocity. In the dilute limit kinetic theory predicts that the shear viscosity is only a function of temperature, and not of density, $\eta \sim (mT)^{3/2}$ [21, 22]. The square of the strain tensor scales as $(\sigma_{ij})^2 \sim \tau_{exp}^{-2}$, where $\tau_{exp}^{-1} = \vec{\nabla}\cdot\vec{u}$ is the expansion rate of the fluid. This means that the local heating rate is $\dot{q} \sim T^{3/2}\tau_{exp}^{-2} \sim T^3$, independent of density [37]. Thus, integrating the NS equation over volume leads to the prediction that dissipation produces an infinite amount of heat. This result is, of course, an artifact of applying the NS equation in a regime where the mean free path is large. It implies, however, that any attempt to address this problem by imposing a cutoff radius will give results that are very sensitive to the precise nature of the cutoff.

Prior work: Previous analyses have dealt with this issue in a variety of ways. In [2] it was argued that collective mode and expansion experiments primarily constrain the trap integral of the shear viscosity, $\alpha_n \equiv \frac{1}{N}\int d^3x\,\eta(n_0(\vec{x}),T_0)$, where N is the total number of particles, $n_0(\vec{x})$ is the initial density, and T_0 is the initial temperature. The integration volume was restricted to lie within the surface of last scattering, defined using the mean free path computed in kinetic theory. Later, Cao et al. [4] assumed that the local shear viscosity scales as $\eta(\vec{x}) = n(\vec{x})[\eta(0)/n(0)]$, so that $\alpha_n = \eta(0)/n(0)$ is determined by η and n at the trap center. This assumption has a number of nice properties, because for a scaling expansion $\eta(0)/n(0)$ is approximately independent of time. In the more recent work by Joseph et al. [10] the integration volume was restricted to the interior of an ellipsoid. The length of the principle axes was taken to be $R_i = \gamma\langle x_i^2\rangle^{1/2}$, where $\langle x_i^2\rangle^{1/2}$ is the the rms radius, and γ is a temperature-independent coefficient that was fitted in order to reproduce the theoretically computed high-T limit of the shear viscosity, $\eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2}$ [21, 22].

Anisotropic fluid dynamics: These methods are clearly not fully satisfactory, because they involve model assumptions for which the error cannot be quantified. For example, the analysis of Cao et al. gives $\eta = 0.33(mT)^{3/2}$ in the high temperature limit $T \gg T_F$ [4]. This agrees to within 25% with the theoretical prediction, but there is no a priori estimate of the theoretical error related to

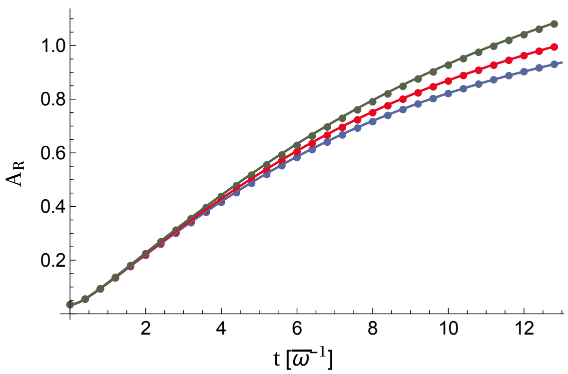

FIG. 1: This figure shows the aspect ratio A_R of an expanding unitary Fermi gas as a function of time t in units of the inverse mean trap frequency $\bar{\omega}^{-1}$. The three curves correspond to three different initial temperatures $T/T_F = 0.79, 1.11, 1.54$ (from top to bottom). The solid lines show results obtained using anisotropic fluid dynamics, and the points are solutions of the Boltzmann equation obtained by Pantel et al. [34]. The aspect ratio is defined using the rms radii, $A_R = [\langle x^2\rangle/\langle z^2\rangle]^{1/2}$.

the assumption $\eta(\vec{x}) = n(\vec{x})[\eta(0)/n(0)]$. Also, there is no reliable method for estimating the systematic uncertainty in the low temperature data obtained by Joseph et al. [10].

A possible approach that resolves the difficulty is to couple a fluid dynamical calculation for the center of the cloud with a kinetic treatment based on the Boltzmann equation for the dilute corona. However, this method is computationally very demanding, and extensive studies would be required to establish that the results are independent of the prescription for switching between thermodynamic variables in fluid dynamics and distribution functions in kinetic theory. A much simpler approach, termed anisotropic fluid dynamics, was recently proposed in [23]. This method has also been studied in connection with relativistic heavy-ion collisions [24, 25]. The idea is to include certain non-hydrodynamic variables in the fluid dynamical description. In the limit of short mean free paths, these variables relax to their equilibrium values on a microscopic time scale, and NS theory is recovered. In the limit of long mean free paths, in contrast, the non-hydrodynamic modes are approximately conserved, and the additional conservation laws ensure a smooth transition to free streaming.

The fluid dynamical variables describing a non-relativistic fluid in the normal phase are the mass density ρ, the momentum density $\vec{\pi} = \rho\vec{u}$, and the energy density $\mathcal{E}$. The conservation laws can be written as

$$D_0\rho \;=\; -\rho\vec{\nabla}\cdot\vec{u}\,, \tag{1}$$

$$D_0 u_i \;=\; -\frac{1}{\rho}\left(\nabla_i P + \nabla_j \delta\Pi_{ij}\right)\,, \tag{2}$$

$$D_0\epsilon \;=\; -\frac{1}{\rho}\nabla_i\left(u_i P + \delta j_i^{\mathcal{E}}\right)\,, \tag{3}$$

where we have introduced the comoving time derivative $D_0 = \partial_0 + \vec{u}\cdot\vec{\nabla}$, the energy per mass $\epsilon = \mathcal{E}/\rho$, and the pressure P. In NS theory the dissipative stress tensor is given by $\delta\Pi_{ij} = -\eta\sigma_{ij}$ and the dissipative energy current is $\delta j_i^{\mathcal{E}} = u_j\delta\Pi_{ij}$. For simplicity, we neglect the effects of heat conduction, which are not important for the physical systems studied in this work [26]. The fluid dynamical equations close once we provide an equation of state $P = P(\mathcal{E}^0)$, where $\mathcal{E}^0 = \mathcal{E} - \frac{1}{2}\rho\vec{u}^2$ is the energy density in the fluid rest frame. The unitary Fermi gas is scale invariant and $P = \frac{2}{3}\mathcal{E}^0$.

In anisotropic fluid dynamics we treat the components of the dissipative stress tensor as additional, independent, fluid dynamical variables. In the present case the stresses remain diagonal and we only have to keep the diagonal components of $\delta\Pi_{ij}$ [38]. We define anisotropic components of the pressure, P_a for $a = 1, 2, 3$, and write $\delta\Pi_{ij} = \sum_a \delta_{ia}\delta_{ja}\Delta P_a$, where $\Delta P_a = P_a - P$. We also define anisotropic components of the energy density $\mathcal{E}_a$ such that $\mathcal{E} = \sum_a \mathcal{E}_a$. The anisotropic components of the energy per mass satisfy the fluid dynamical equations [23]

$$D_0\epsilon_a = -\frac{1}{\rho}\nabla_i\left[\delta_{ia}u_i P + (\delta j_a^{\mathcal{E}})_i\right] - \frac{P}{2\eta\rho}\,\Delta P_a\,, \tag{4}$$

where $\epsilon_a = \mathcal{E}_a/\rho$ and $(\delta j_a^{\mathcal{E}})_i = \delta_{ia}u_j\delta\Pi_{ij}$. To close the fluid dynamical equations we have to provide an equation of state. For a scale invariant fluid we have $P_a(\mathcal{E}_a^0) = 2\,\mathcal{E}_a^0$ with $\mathcal{E}_a^0 = \mathcal{E}_a - \frac{1}{2}\rho u_a^2$. Then $P = \frac{1}{3}\sum_a P_a$ satisfies the isotropic equation of state and equ. (4) reproduces the equation of energy conservation equ. (3) when summed over a. Equations (1)-(4) can be solved using standard techniques in computational fluid dynamics. We have developed a code based on the PPM scheme of Colella and Woodward [23, 26–28].

The precise form of the fluid dynamical equations (1-4) can be derived using moments of the Boltzmann equation [23]. In particular, the new equation (4) arises from taking moments with $p_a^2/(2m)$, where p_a is a Cartesian component of the quasi-particle momentum. Note that physically equ. (4) is a relaxation time equation for the viscous stresses. To demonstrate the relation to the NS equation we solve equ. (4) for ΔP_a order by order in the small parameter $Kn = (\eta/P)\vec{\nabla}\cdot\vec{u}$ [39]. At leading order we find $\delta\Pi_{ij} = -\eta\sigma_{ij}$ and, thus, recover NS theory [23]. This is true for any functional form of the shear viscosity $\eta(n, T)$. In the opposite limit, $Kn \gg 1$, the components of $\mathcal{E}_a$ are independently conserved. This corresponds to the ballistic limit, because without collisions the components of the internal energy corresponding to motion in different directions are individually conserved.

Comparison to solutions of the Boltzmann equation: Anisotropic fluid dynamics can be viewed as a low density regulator for the NS equation. The theory exactly reduces to NS theory in a dense fluid, and the relaxation time equation ensures that in the dilute limit free streaming is recovered. Given that the crossover between these limits is smooth [29–31] we expect that anisotropic

T/T_F	$\omega_x/(2\pi)$	$\omega_y/(2\pi)$	$\omega_z/(2\pi)$	$\omega_z^{mag}/(2\pi)$	N
0.79	5283 Hz	5052 Hz	182.7 Hz	21.5 Hz	$4 \cdot 10^5$
1.11	5283 Hz	5052 Hz	182.7 Hz	21.5 Hz	$5 \cdot 10^5$
1.54	5283 Hz	5052 Hz	182.7 Hz	21.5 Hz	$6 \cdot 10^5$

TABLE I: Parameters for the experiments reported in [4]. The Fermi temperature is defined in terms of the geometric mean $\bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$ of the trap frequencies, $T_F = (3N)^{1/3}\bar{\omega}$. After the optical trap is turned off, the gas expands in a magnetic bowl with frequencies ω_i^{mag}. The effect of $\omega_{x,y}^{mag}$ is negligible, and only ω_z^{mag} is given in the table.

fluid dynamics provides an accurate representation of kinetic theory at finite Knudsen number. Here we will verify this expectation by comparing numerical solutions of anisotropic fluid dynamics and the Boltzmann equation. The Boltzmann equation reads

$$\left(\partial_t + \vec{v}\cdot\vec{\nabla}_x - \vec{F}\cdot\vec{\nabla}_p\right) f_p(\vec{x},t) = C[f_p]\,, \qquad (5)$$

where $f_p(\vec{x},t)$ is the distribution function, $\vec{v} = \vec{\nabla}_p E_p$ is the quasi-particle velocity, E_p is the quasi-particle energy, $\vec{F} = -\vec{\nabla}_x E_p$ is a force, and $C[f_p]$ is the collision term. For simplicity, we have assumed the system to be spin-symmetric with $f_p^{\uparrow} = f_p^{\downarrow} = f_p$. In the high-$T$ limit $E_p = p^2/(2m)$ and $\vec{v} = \vec{p}/m$ [32]. In this limit the collision term is dominated by two-body collisions and

$$C[f_1] = -\prod_{i=2,3,4}\left(\int d\Gamma_i\right) w(1,2;3,4)\,(f_1f_2 - f_3f_4)\;, \qquad (6)$$

where $f_i = f_{p_i}$, $d\Gamma_i = \frac{d^3p_i}{(2\pi)^3}$ and the transition rate is given by

$$w(1,2;3,4) = (2\pi)^4\delta\Big(\sum_i E_i\Big)\delta\Big(\sum_i \vec{p}_i\Big)\,|\mathcal{A}|^2\,. \qquad (7)$$

The square of the scattering amplitude in the unitary limit is given by $|\mathcal{A}|^2 = 16\pi^2/(q^2m^2)$ where $2\vec{q} = \vec{p}_2 - \vec{p}_1$. Numerical solutions of the Boltzmann equation for the unitary Fermi gas using the test particle method were obtained in [33, 34]. In the test particle method the distribution function is represented by a sum of delta functions, which can be thought of as classical particles that follow trajectories governed by Newton's laws. Collisions occur when the particles approach to within the scaled geometrical cross section, where the scale factor is determined by the number of test particles.

A comparison between numerical results of anisotropic fluid dynamics and solutions of the Boltzmann equation is shown in Fig. 1 for three different values of T [40]. The parameters are given in Table I. The solutions of the Boltzmann equation were obtained for the cross section in the unitary limit. The shear viscosity in the anisotropic fluid dynamics code is determined by using this cross section together with the Chapman-Enskog method for solving the Boltzmann equation in approximate local equilibrium. The result of this calculation is $\eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2}$

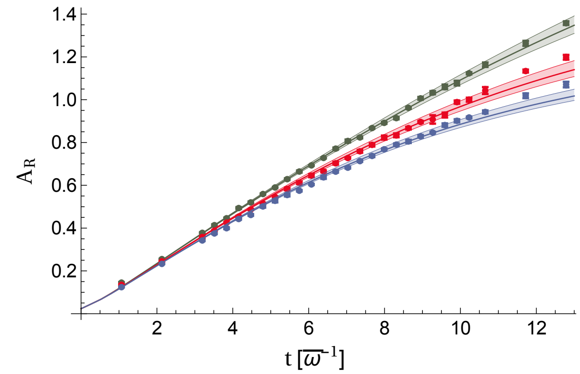

FIG. 2: This figure shows the aspect ratio A_R of an expanding unitary Fermi gas as a function of time t in units of $\bar{\omega}^{-1}$ for three different initial temperatures $T/T_F = 0.79, 1.11, 1.54$ (from top to bottom). The data are from Cao et al. [4]. The solid lines show fits obtained using anisotropic fluid dynamics, and the bands correspond to a $\pm 15\%$ uncertainty in the shear viscosity. The aspect ratio is defined using a Gaussian fit to two-dimensional densities.

[41]. The agreement between anisotropic fluid dynamics and the Boltzmann equation is essentially perfect. This is remarkable, because there are no free parameters, and, as explained in the introduction, there are no well-behaved solutions of the NS equation for $\eta \sim (mT)^{3/2}$.

One way to think about this is to note that it is possible to represent the Boltzmann equation as an infinite set of moment equations. Standard fluid dynamics corresponds to truncating this expansion after the first five moments, corresponding to the conserved quantities particle number, momentum, and energy. What we have demonstrated is that adding only two additional moments, corresponding to anisotropic components of the internal energy, dramatically improves the agreement between local moment equations and the underlying kinetic theory for a fluid in which the density varies significantly.

Fits to high temperature expansion data: We have shown that anisotropic fluid dynamics reproduces the NS equation in the short mean free path limit, and kinetic theory with two-body scattering in the dilute limit. The only input parameters are the equation of state and the shear viscosity. This approach is therefore ideally suited to determine the shear viscosity of the unitary Fermi gas. In this section we illustrate the method by reanalyzing the data of Cao et al. [4]. Cao et al. studied the expansion of the cloud for a range of energies $2.3NE_F \leq E \leq 4.6NE_F$, where E is the total energy of the cloud and $E_F = T_F$ is the Fermi energy. This is significantly above the critical energy $E_c = 0.7NE_F$ for the superfluid transition [35], and we can describe the initial density profile as a Gaussian [26]. We will also assume that the shear viscosity follows the high temperature law $\eta = \eta_0(mT)^{3/2}$. We will check this assumption below. Our goal here is to demonstrate that we can accurately extract the high temperature shear viscosity from data. This result provides a crucial and indispensable benchmark for any attempt to reliably extract the shear viscosity near T_c.

Fits to the data based on anisotropic fluid dynamics are shown in Fig. 2. We consider three different initial temperatures, spanning about a factor of two. As noted in [34] an important ingredient in obtaining a good fit to the data is to follow the experimental procedure and determine the aspect ratio from a Gaussian fit to the two-dimensional column density $n(x,z)=\int dy\, n(x,y,z)$. Note that the need to perform a Gaussian fit is related to viscous effects. In ideal fluid dynamics the evolution preserves the Gaussian shape of the initial density distribution, and there is no difference between rms and Gaussian fit radii. At $T/T_F = 0.79, 1.11, 1.54$ we find $\eta_0 = 0.266, 0.302, 0.288$. The fits to the data for these values of η_0, together with $\pm 15\%$ error bands, are shown in Fig. 2. There are some discrepancies at large t, but this is the regime in which systematic errors in the measurement of the aspect ratio are expected to be significant [42].

We observe that as the temperature of the cloud changes by a factor of 1.95, and the shear viscosity changes by a factor 2.72, the variance of the extracted values of η_0 is only 6%. This places strong constraints on deviations from the expected scaling behavior $\eta \sim T^{3/2}$. Combining all the data, and using a fit to the more general functional form $\eta = \eta_0(mT)^{3/2}(mT/n^{2/3})^a$, we find $a = 0.05 \pm 0.1$, consistent with $a = 0$ [43]. For $a = 0$ we obtain $\eta = 0.282(mT)^{3/2}$, which agrees to about 5% with the theoretical prediction $\eta = 0.269(mT)^{3/2}$ [21, 22, 36]. We note that the theoretical uncertainty inherent in the use of anisotropic fluid dynamics, which can be estimated from Fig. 1, is much smaller than that. Indeed, the difference between theory and experiment is consistent with the statistical uncertainty of the fit, which is about 10%.

Conclusions and outlook: In this work we have demonstrated that anisotropic fluid dynamics can be used to make high precision, model-independent, determinations of the shear viscosity of trapped atomic Fermi gases. The key feature of the method is that it interpolates between an exact realization of the Navier-Stokes equation in the short mean free path limit and ballistic expansion in the long mean free path limit. We have also shown that the method provides a very accurate representation of the Boltzmann equation in the limit of pure two-body scatterings. Together, these results imply that the method incorporates the most general description of a dense fluid in the normal phase, Navier-Stokes fluid dynamics, and the correct theory of a dilute gas, kinetic theory with two-body collisions.

In this work we have focused on high temperature data and verified the theoretical prediction for η in this regime. We have been able to extract, for the first time, the shear viscosity coefficient without uncontrolled assumptions about dissipative effects in the dilute corona. This is a crucial benchmark for the natural next step, which is to reanalyze data near the superfluid transition [10]. This will require initializing the density profile for a nontrivial equation of state, and extracting the full functional dependence of η on $n/(mT)^{3/2}$. In order to describe the data below T_c the method has to be extended to superfluid hydrodynamics. In principle this is straightforward, because in terms of fluid dynamics a superfluid can be viewed as a mixture of a normal, viscous, fluid with an inviscid fluid.

Finally, we emphasize that the method presented in this work is quite general, and can be applied to a variety of physical problems. This includes problems in fluid dynamics which involve the expansion into a vacuum, or large changes in the density, so that the Knudsen number of the flow varies by orders of magnitude. The basic idea of the method can also be applied to determine other transport coefficients, for example the spin diffusion constant in trapped atomic gases [13].

Acknowledgments: This work was supported in parts by the US Department of Energy grant DE-FG02-03ER41260. We would like to thank Michael Urban for providing us with the data points shown in Fig. 1. We also thank James Joseph and John Thomas for many useful discussions, and John Blondin for help with the VH1 code.

[1] J. Kinast, A. Turlapov, J. E. Thomas, "Two Transitions in the Damping of a Unitary Fermi Gas," Phys. Rev. Lett. **94**, 170404 (2005) [cond-mat/0502507].

[2] T. Schäfer, "The Shear Viscosity to Entropy Density Ratio of Trapped Fermions in the Unitarity Limit," Phys. Rev. A **76**, 063618 (2007) [arXiv:cond-mat/0701251].

[3] A. Turlapov, J. Kinast, B. Clancy, L. Luo, J. Joseph, J. E. Thomas, "Is a Gas of Strongly Interacting Atomic Fermions a Nearly Perfect Fluid" J. Low Temp. Phys. **150**, 567 (2008) [arXiv:0707.2574].

[4] C. Cao, E. Elliott, J. Joseph, H. Wu, J. Petricka, T. Schäfer and J. E. Thomas, "Universal Quantum Viscosity in a Unitary Fermi Gas," Science 331, 58 (2011) [arXiv:1007.2625 [cond-mat.quant-gas]].

[5] E. Vogt, M. Feld, B. Frohlich, D. Pertot, M. Koschorreck and M. Kohl, "Scale invariance and viscosity of a two-dimensional Fermi gas," Phys. Rev. Lett. **108**, 070404 (2012) [arXiv:1111.1173 [cond-mat.quant-gas]].

[6] T. Schäfer, "Shear viscosity and damping of collective modes in a two-dimensional Fermi gas," Phys. Rev. A **85**, 033623 (2012) [arXiv:1111.7242 [cond-mat.quant-gas]].

[7] T. Enss, C. Kuppersbusch and L. Fritz, "Shear viscosity and spin diffusion in a two-dimensional Fermi gas," Phys. Rev. A **86**, 013617 (2012) [arXiv:1205.2376 [cond-mat.quant-gas]].

[8] E. Elliott, J. A. Joseph, J. E. Thomas, "Anomalous minimum in the shear viscosity of a Fermi gas," Phys. Rev. Lett. **113**, 020406 (2014) [arXiv:1311.2049 [cond-

mat.quant-gas]].

[9] M. Bluhm and T. Schäfer, “Medium effects and the shear viscosity of the dilute Fermi gas away from the conformal limit,” Phys. Rev. A **90**, no. 6, 063615 (2014) [arXiv:1410.2827 [cond-mat.quant-gas]].

[10] J. A. Joseph, E. Elliott, J. E. Thomas, “Shear viscosity of a universal Fermi gas near the superfluid phase transition,” Phys. Rev. Lett. **115**, 020401 (2015) [arXiv:1410.4835 [cond-mat.quant-gas]].

[11] J. Brewer, M. Mendoza, R. E. Young and P. Romatschke, “Lattice Boltzmann simulations of a two-dimensional Fermi gas at unitarity,” arXiv:1507.05975 [cond-mat.quant-gas].

[12] E. Elliott, J. A. Joseph, J. E. Thomas, “Observation of conformal symmetry breaking and scale invariance in expanding Fermi gases,” Phys. Rev. Lett. **112**, 040405 (2014) [arXiv:1308.3162 [cond-mat.quant-gas]].

[13] A. Sommer, M. Ku, G. Roati, and M. W. Zwierlein. “Universal spin transport in a strongly interacting Fermi gas,” Nature **472**, 201 (2011) [arXiv:1103.2337v1 [cond-mat.quant-gas]].

[14] G. M. Bruun, C. J. Pethick, “Spin diffusion in trapped clouds of strongly interacting cold atoms,” Phys. Rev. Lett. **107**, 255302 (2011) [arXiv:1109.5709 [cond-mat.quant-gas]].

[15] M. Koschorreck, D. Pertot, E. Vogt, M. Köhl, “Universal spin dynamics in two-dimensional Fermi gases,” Nature Physics 9, 405 (2013) [arXiv:1304.4980 [cond-mat.quant-gas]].

[16] H. Guo, D. Wulin, C.-C. Chien, K. Levin, “Perfect Fluids and Bad Metals: Transport Analogies Between Ultracold Fermi Gases and High T_c Superconductors,” New J. Phys. **13**, 075011 (2011) [arXiv:1009.4678 [cond-mat.supr-con]].

[17] T. Schäfer and D. Teaney, “Nearly Perfect Fluidity: From Cold Atomic Gases to Hot Quark Gluon Plasmas,” Rept. Prog. Phys. **72**, 126001 (2009) [arXiv:0904.3107 [hep-ph]].

[18] A. Adams, L. D. Carr, T. Schäfer, P. Steinberg and J. E. Thomas, “Strongly Correlated Quantum Fluids: Ultracold Quantum Gases, Quantum Chromodynamic Plasmas, and Holographic Duality,” New J. Phys. **14**, 115009 (2012) [arXiv:1205.5180 [hep-th]].

[19] T. Schäfer and C. Chafin, “Scaling Flows and Dissipation in the Dilute Fermi Gas at Unitarity,” Lect. Notes Phys. **836**, 375 (2012) [arXiv:0912.4236 [cond-mat.quant-gas]].

[20] K. M. O’Hara, S. L. Hemmer, M. E. Gehm, S. R. Granade, J. E. Thomas, “Observation of a Strongly-Interacting Degenerate Fermi Gas of Atoms,” Science Vol. 298, No. **5601**, 2179 (2002). [cond-mat/0212463].

[21] G. M. Bruun, H. Smith, “Viscosity and thermal relaxation for a resonantly interacting Fermi gas,” Phys. Rev. A **72**, 043605 (2005) [cond-mat/0504734].

[22] G. M. Bruun, H. Smith, “Shear viscosity and damping for a Fermi gas in the unitarity limit,” Phys. Rev. A **75**, 043612 (2007) [cond-mat/0612460].

[23] M. Bluhm and T. Schäfer, “Dissipative fluid dynamics for the dilute Fermi gas at unitarity: Anisotropic fluid dynamics,” Phys. Rev. A **92**, no. 4, 043602 (2015) [arXiv:1505.00846 [cond-mat.quant-gas]].

[24] W. Florkowski and R. Ryblewski, “Highly-anisotropic and strongly-dissipative hydrodynamics for early stages of relativistic heavy-ion collisions,” Phys. Rev. C **83**, 034907 (2011) [arXiv:1007.0130 [nucl-th]].

[25] M. Martinez and M. Strickland, “Dissipative Dynamics of Highly Anisotropic Systems,” Nucl. Phys. A **848**, 183 (2010) [arXiv:1007.0889 [nucl-th]].

[26] T. Schäfer, “Dissipative fluid dynamics for the dilute Fermi gas at unitarity: Free expansion and rotation,” Phys. Rev. A **82**, 063629 (2010) [arXiv:1008.3876 [cond-mat.quant-gas]].

[27] P. Colella, P. R. Woodward, “The Piecewise Parabolic Method (PPM) for Gas-Dynamical Simulations,” J. Comp. Phys. **54**, 174 (1984).

[28] J. M. Blondin, E. A. Lufkin, “The piecewise-parabolic method in curvilinear coordinates,” Astrophys. J. Supp. Ser. **88**, 589 (1993).

[29] C. Menotti, P. Pedri, S. Stringari, “Expansion of an interacting Fermi gas,” Phys. Rev. Lett. **89**, 250402 (2002) [cond-mat/0208150].

[30] P. Pedri, D. Guéry-Odelin and S. Stringari, “Dynamics of a classical gas including dissipative and mean-field effects,” Phys. Rev. A **68**, 043608 (2003) [cond-mat/0305624].

[31] K. Dusling and T. Schäfer, “Elliptic flow of the dilute Fermi gas: From kinetics to hydrodynamics,” Phys. Rev. A **84**, 013622 (2011) [arXiv:1103.4869 [cond-mat.stat-mech]].

[32] K. Dusling and T. Schäfer, “Bulk viscosity and conformal symmetry breaking in the dilute Fermi gas near unitarity,” Phys. Rev. Lett. **111**, 120603 (2013) [arXiv:1305.4688 [cond-mat.quant-gas]].

[33] T. Lepers, D. Davesne, S. Chiacchiera, M. Urban, “Numerical solution of the Boltzmann equation for the collective modes of trapped Fermi gases,” Phys. Rev. A **82**, 023609 (2010) [arXiv:1004.5241 [cond-mat.quant-gas]].

[34] P. A. Pantel, D. Davesne and M. Urban, “Numerical solution of the Boltzmann equation for trapped Fermi gases with in-medium effects,” Phys. Rev. A **91**, 013627 (2015) [arXiv:1412.3641 [cond-mat.quant-gas]].

[35] M. J. H. Ku, A. T. Sommer, L. W. Cheuk, and M. W. Zwierlein, “Revealing the Superfluid Lambda Transition in the Universal Thermodynamics of a Unitary Fermi Gas,” Science 335, 563 (2012) [arXiv:1110.3309 [cond-mat.quant-gas]].

[36] T. Schäfer, “Second order fluid dynamics for the unitary Fermi gas from kinetic theory,” Phys. Rev. A **90**, 043633 (2014) [arXiv:1404.6843 [cond-mat.quant-gas]].

[37] This estimate is based on properties of the scaling solution of the Euler equation for an expanding gas. The velocity field is linear $u_i = \alpha_i(t)x_i$, where $\alpha_i = \dot{b}_i(t)/b_i(t)$ and b_i is the scale factor of the expansion in the i direction. The density of a co-moving fluid element scales as $n \sim 1/b_\perp^2$, and the temperature scales as $T \sim n^{2/3}$. Finally, after the initial acceleration period we have $b_i \sim \omega_i t$ and $\alpha_i \sim \omega_i/b_i$.

[38] The stresses are diagonal because of the symmetries of the trapping potential. The Euler equation implies $\partial_0 u_i \sim \nabla_i P \sim x_i$ so that $\nabla_i u_j = 0$ for $i \neq j$. This feature is preserved by viscous corrections.

[39] In kinetic theory $\eta = \tau_0 P$, where τ_0 is the collision time, and this parameter is the Knudsen number Kn of the flow. In fluid dynamics $Kn = Re^{-1}Ma^2$, where Re is the Reynolds number, and Ma is the Mach number.

[40] The solutions of the Boltzmann equation shown in Fig. 1 correspond to results of [34] with all quantum corrections and in-medium effects removed. In the temperature-regime considered here, these effects are small.

[41] The Chapman-Enskog result is formally exact in the limit $n/(mT)^{3/2} \to 0$. The coefficient $\frac{15}{32\sqrt{\pi}}$ is an approximation that arises at leading order in an expansion of the solution of the Boltzmann equation in Laguerre polynomials. The next-to-leading order correction gives a result which is larger by a factor 193/190 [22, 36]. This is a 2% correction, suggesting that the full result is well approximated, and that the correction to Fig. 1 is very small.

[42] The goodness of fit χ^2/ν for the fits shown in Fig. 2 is between 5 and 10. This is comparable to the quality of the model-dependent fits in the original work [4]. Note that the error bars in the data only include statistical fluctuations in the measurement of the aspect ratio.

[43] The best fit is $\eta = \eta_0 (mT)^{3/2} (T/T^*)^a$ with $\eta_0 = 0.282$, $a = 0.05$ and $T^* = 10.1\, T_F$. Here, $T_F = k_F^2/(2m)$ is the local Fermi temperature. We have fixed the dimensionless coefficient η_0 from the fit for $a = 0$.