Model-Independent Determination of the Shear Viscosity of a Trapped Unitary Fermi gas: Application to High-Temperature Data

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Determinations of the shear viscosity of trapped ultracold gases suffer from systematic, uncontrolled uncertainties related to the treatment of the dilute part of the gas cloud. In this work we present an analysis of expansion experiments based on a new method, anisotropic fluid dynamics, that interpolates between Navier-Stokes fluid dynamics at the center of the cloud and ballistic behavior in the dilute corona. We validate the method using a comparison between anisotropic fluid dynamics and numerical solutions of the Boltzmann equation. We then apply anisotropic fluid dynamics to the expansion data reported by Cao et al. In the high temperature limit we find \( \eta = 0.282(mT)^{3/2} \), which agrees within about 5\% with the theoretical prediction \( \eta = 0.269(mT)^{3/2} \).

**Introduction:** A number of studies have been devoted to extracting the transport properties of dilute atomic Fermi gases. Quantities of interest include the shear viscosity [1–11], the bulk viscosity [12], and the spin diffusion constant [13–15]. These transport coefficients provide valuable information about the nature of the low energy degrees of freedom. Strongly correlated Fermi gases also contribute important insights into the transport properties of other quantum many-body systems, such as high-\( T_c \) superconductors or the quark-gluon plasma [16–18]. Truly model-independent determinations of the transport coefficients of trapped atomic gases have so far been precluded, however, by the fact that there is a transition from fluid dynamical behavior in the dense part of the cloud to weakly collisional kinetic behavior in the dilute corona.

Consider, for example, a unitary Fermi gas expanding after release from a deformed harmonic trap [19]. Fluid dynamics predicts that the difference in pressure gradients along the short and the long axis of the cloud translates into a larger acceleration along the short direction. This implies that the aspect ratio \( \alpha \) of the cloud, which is initially much smaller than one, quickly grows and eventually exceeds unity, as was first observed by O’Hara et al. [20]. Shear viscosity \( \eta \) slows down the acceleration in the transverse direction, and measurements of \( A_B(t) \) for different initial values of \( T/T_F \), where \( T_F \) is the Fermi temperature, can be used to constrain the dependence of \( \eta(n,T) \) on density \( n \) and temperature \( T \). This task is simplified by the scale invariance of the unitary Fermi gas, which implies that the bulk viscosity vanishes, and that \( \eta = \frac{(mT)^{3/2}}{f(n/(mT)^{3/2})} \), where \( f(x) \) is a universal function. Note that we use units \( \hbar = k_B = 1 \).

The natural tool for extracting \( \eta(n,T) \) is the Navier-Stokes (NS) equation. The problem in determining \( \eta(n,T) \) is that \( A_B(t) \) is a global property of the cloud, and that the NS equation breaks down in the dilute corona, where the mean free path is large compared to the density and the pressure scale heights. Because the total number of particles in the corona is small, one might hope that this does not lead to serious difficulties. Unfortunately, this is not the case: The rate of dissipative heating is \( \dot{q} = \frac{\eta}{2}(\sigma_{ij})^2 \), where \( \sigma_{ij} = \nabla_i u_j + \nabla_j u_i - \frac{2}{3} \delta_{ij} \nabla \cdot \vec{u} \) is the strain tensor, and \( \vec{u} \) is the fluid velocity. In the dilute limit kinetic theory predicts that the shear viscosity is only a function of temperature, and not of density, \( \eta \sim (mT)^{3/2} \) [21, 22]. The square of the strain tensor scales as \( (\sigma_{ij})^2 \sim \tau_{\exp}^{-2} \), where \( \tau_{\exp}^{-1} = \nabla \cdot \vec{u} \) is the expansion rate of the fluid. This means that the local heating rate is \( \dot{q} \sim T^{3/2} \tau_{\exp}^{-2} \sim T^3 \), independent of density [37]. Thus, integrating the NS equation over volume leads to the prediction that dissipation produces an infinite amount of heat. This result is, of course, an artifact of applying the NS equation in a regime where the mean free path is large. It implies, however, that any attempt to address this problem by imposing a cutoff radius will give results that are very sensitive to the precise nature of the cutoff.

**Prior work:** Previous analyses have dealt with this issue in a variety of ways. In [2] it was argued that collective mode and expansion experiments primarily constrain the trap integral of the shear viscosity, \( \alpha_n \equiv \frac{1}{N} \int d^3x \eta(n_0(\vec{x}),T_0) \), where \( N \) is the total number of particles, \( n_0(\vec{x}) \) is the initial density, and \( T_0 \) is the initial temperature. The integration volume was restricted to lie within the surface of last scattering, defined using the mean free path computed in kinetic theory. Later, Cao et al. [4] assumed that the local shear viscosity scales as \( \eta(\vec{x}) = n(\vec{x})\eta(0)/n(0) \), so that \( \alpha_n = \eta(0)/n(0) \) is determined by \( \eta \) and \( n \) at the trap center. This assumption has a number of nice properties, because for a scaling expansion \( \eta(0)/n(0) \) is approximately independent of time.

In the more recent work by Joseph et al. [10] the integration volume was restricted to the interior of an ellipsoid. The length of the principle axes was taken to be \( R_i = \gamma(x_i^2)^{1/2} \), where \( x_i^2 \) is the rms radius, and \( \gamma \) is a temperature-independent coefficient that was fitted in order to reproduce the theoretically computed high-\( T \) limit of the shear viscosity, \( \eta = \frac{15}{8\sqrt{2\pi}}(mT)^{3/2} \) [21, 22].

**Anisotropic fluid dynamics:** These methods are clearly not fully satisfactory, because they involve model assumptions for which the error cannot be quantified. For example, the analysis of Cao et al. gives \( \eta = 0.33(mT)^{3/2} \) in the high temperature limit \( T \gg T_F \) [4]. This agrees to within 25% with the theoretical prediction, but there is no a priori estimate of the theoretical error related to
the assumption \( n(\vec{x}) = n(\vec{x})[n(0)/n(0)] \). Also, there is no reliable method for estimating the systematic uncertainty in the low temperature data obtained by Joseph et al. [10].

A possible approach that resolves the difficulty is to couple a fluid dynamical calculation for the center of the cloud with a kinetic treatment based on the Boltzmann equation for the dilute corona. However, this method is computationally very demanding, and extensive studies would be required to establish that the results are independent of the prescription for switching between thermodynamic variables in fluid dynamics and distribution functions in kinetic theory. A much simpler approach, termed anisotropic fluid dynamics, was recently proposed in [23]. This method has also been studied in connection with relativistic heavy-ion collisions [24, 25]. The idea is to include certain non-hydrodynamic variables in the fluid dynamical description. In the limit of short mean free paths, these variables relax to their equilibrium values on a microscopic time scale, and NS theory is recovered. In the limit of long mean free paths, in contrast, the non-hydrodynamic modes are approximately conserved, and the additional conservation laws ensure a smooth transition to free streaming.

The fluid dynamical variables describing a non-relativistic fluid in the normal phase are the mass density \( \rho \), the momentum density \( \vec{p} = \rho \vec{u} \), and the energy density \( \mathcal{E} \). The conservation laws can be written as

\[
D_{0} \rho = -\rho \nabla \cdot \vec{u},
\]

\[
D_{0} \rho u_{i} = -\frac{1}{\rho} (\nabla_{i} P + \nabla_{j} \delta \Pi_{ij} ),
\]

\[
D_{0} \mathcal{E} = -\frac{1}{\rho} \nabla_{i} (u_{i} P + \delta \mathcal{E}_{i} ) ,
\]

where we have introduced the comoving time derivative \( D_{0} = \partial_{0} + \vec{u} \cdot \nabla \), the energy per mass \( \epsilon = \mathcal{E}/\rho \), and the pressure \( P \). In NS theory the dissipative stress tensor is given by \( \delta \Pi_{ij} = -\eta \sigma_{ij} \) and the dissipative energy current is \( \delta \mathcal{E}_{i} = u_{i} \delta \mathcal{E} / u \). For simplicity, we neglect the effects of heat conduction, which are not important for the physical systems studied in this work [26]. The fluid dynamical equations close once we provide an equation of state \( P = P(\mathcal{E}) \), where \( \mathcal{E} = \mathcal{E} - \frac{1}{2} \rho \vec{u}^{2} \) is the energy density in the fluid rest frame. The unitary Fermi gas is scale invariant and \( P = \frac{5}{3} \mathcal{E} \).

In anisotropic fluid dynamics we treat the components of the dissipative stress tensor as additional, independent, fluid dynamical variables. In the present case the stresses remain diagonal and we only have to keep the diagonal components of \( \delta \Pi_{ij} [38] \). We define anisotropic components of the pressure, \( P_{a} \), for \( a = 1, 2, 3 \), and write \( \delta \Pi_{ij} = \sum_{a} \delta \mathcal{E}_{a} \delta \rho_{a} \Delta P_{a} \), where \( \Delta P_{a} = P_{a} - P \). We also define anisotropic components of the energy density \( \mathcal{E}_{a} \) such that \( \mathcal{E} = \sum_{a} \mathcal{E}_{a} \). The anisotropic components of the energy per mass satisfy the fluid dynamical equations [23]

\[
D_{0} \mathcal{E}_{a} = -\frac{1}{\rho} \nabla_{i} [\delta \mathcal{E}_{a} u_{i} P + (\delta \mathcal{E}_{a} )_{i} ] - \frac{P}{2\eta} \Delta P_{a} ,
\]

where \( \mathcal{E}_{a} = \mathcal{E}_{a} / \rho \) and \( (\delta \mathcal{E}_{a} )_{i} = \delta \mathcal{E}_{a} u_{i} \delta \Pi_{ij} \). To close the fluid dynamical equations we have to provide an equation of state. For a scale invariant fluid we have \( P_{a} (\mathcal{E}_{a}^{0}) = 2 \mathcal{E}_{a}^{0} \) with \( \mathcal{E}_{a}^{0} = \mathcal{E}_{a} - \frac{1}{2} \rho u_{a}^{2} \). Then \( P = \frac{1}{3} \sum_{a} P_{a} \) satisfies the isotropic equation of state and eqns. (4) reproduces the equation of energy conservation eqn. (3) when summed over \( a \). Equations (1)-(4) can be solved using standard techniques in computational fluid dynamics. We have developed a code based on the PPM scheme of Colella and Woodward [23, 26–28].

The precise form of the fluid dynamical equations (1-4) can be derived using moments of the Boltzmann equation [23]. In particular, the new equation (4) arises from taking moments with \( p_{a}^{2} / (2m) \), where \( p_{a} \) is a Cartesian component of the quasi-particle momentum. Note that physically eqn. (4) is a relaxation time equation for the viscous stresses. To demonstrate the relation to the NS equation we solve eqn. (4) for \( \Delta P_{a} \) order by order in the small parameter \( Kn = (\eta/P) \nabla \cdot \vec{u} [39] \). At leading order we find \( \delta \Pi_{ij} = -\eta \sigma_{ij} \) and, thus, recover NS theory [23]. This is true for any functional form of the shear viscosity \( \eta(n, T) \). In the opposite limit, \( Kn \gg 1 \), the components of \( \mathcal{E}_{a} \) are independently conserved. This corresponds to the ballistic limit, because without collisions the components of the internal energy corresponding to motion in different directions are individually conserved.

**Comparison to solutions of the Boltzmann equation:**

Anisotropic fluid dynamics can be viewed as a low density regulator for the NS equation. The theory exactly reduces to NS theory in a dense fluid, and the relaxation time equation ensures that in the dilute limit free streaming is recovered. Given that the crossover between these limits is smooth [29–31] we expect that anisotropic
fluid dynamics provides an accurate representation of kinetic theory at finite Knudsen number. Here we will verify this expectation by comparing numerical solutions of anisotropic fluid dynamics and the Boltzmann equation. The Boltzmann equation reads

\[ \left( \partial_t + \vec{v} \cdot \nabla_x - \vec{F} \cdot \nabla_p \right) f_p(\vec{x}, t) = C[f_p], \tag{5} \]

where \( f_p(\vec{x}, t) \) is the distribution function, \( \vec{v} = \nabla_x E_p \) is the quasi-particle velocity, \( \vec{F} = -\nabla_x E_p \) is a force, and \( C[f_p] \) is the collision term. For simplicity, we have assumed the system to be spin-symmetric with \( f_p^* = f_p^0 = f_p \). In the high-\( T \) limit \( E_p = p^2/(2m) \) and \( \vec{v} = \vec{p}/m \) \[32\]. In this limit the collision term is dominated by two-body collisions and

\[ C[f_i] = -\prod_{i=2,3,4} \left( \int d\Gamma_i \right) w(1, 2; 3, 4) (f_i f_2 - f_3 f_4), \tag{6} \]

where \( f_i = f_{p_i}, \right. \left. d\Gamma_i = \frac{dp_i}{(2\pi)^3} \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \r...
Fits to the data based on anisotropic fluid dynamics are shown in Fig. 2. We consider three different initial temperatures, spanning a factor of two. As noted in [34] an important ingredient in obtaining a good fit to the data is to follow the experimental procedure and determine the aspect ratio from a Gaussian fit to the two-dimensional column density \( n(x, z) = \int dy n(x, y, z) \). Note that the need to perform a Gaussian fit is related to viscous effects. In ideal fluid dynamics the evolution preserves the Gaussian shape of the initial density distribution, and there is no difference between rms and Gaussian fit radii. At \( T/T_F = 0.79, 1.11, 1.54 \) we find \( \eta_0 = 0.266, 0.302, 0.288 \). The fits to the data for these values of \( \eta_0 \), together with \( \pm 15\% \) error bands, are shown in Fig. 2. There are some discrepancies at large \( t \), but this is the regime in which systematic errors in the measurement of the aspect ratio are expected to be significant [42].

We observe that as the temperature of the cloud changes by a factor of 1.95, and the shear viscosity changes by a factor 2.72, the variance of the extracted values of \( \eta_0 \) is only 6\%. This places strong constraints on deviations from the expected scaling behavior \( \eta \sim T^{3/2} \). Combining all the data, and using a fit to the more general functional form \( \eta = \eta_0 (mT)^{3/2} (mT/n^{3/2})^a \), we find \( a = 0.05 \pm 0.1 \), consistent with \( a = 0 \) [43]. For \( a = 0 \) we obtain \( \eta = 0.282 (mT)^{3/2} \), which agrees to about 5\% with the theoretical prediction \( \eta = 0.269 (mT)^{3/2} \) [21, 22, 36]. We note that the theoretical uncertainty inherent in the use of anisotropic fluid dynamics, which can be estimated from Fig. 1, is much smaller than that. Indeed, the difference between theory and experiment is consistent with the statistical uncertainty of the fit, which is about 10\%.

Conclusions and outlook: In this work we have demonstrated that anisotropic fluid dynamics can be used to make high precision, model-independent, determinations of the shear viscosity of trapped atomic Fermi gases. The key feature of the method is that it interpolates between an exact realization of the Navier-Stokes equation in the short mean free path limit and ballistic expansion in the long mean free path limit. We have also shown that the method provides a very accurate representation of the Boltzmann equation in the limit of pure two-body scatterings. Together, these results imply that the method incorporates the most general description of a dense fluid in the normal phase, Navier-Stokes fluid dynamics, and the correct theory of a dilute gas, kinetic theory with two-body collisions.

In this work we have focused on high temperature data and verified the theoretical prediction for \( \eta \) in this regime. We have been able to extract, for the first time, the shear viscosity coefficient without uncontrolled assumptions about dissipative effects in the dilute corona. This is a crucial benchmark for the natural next step, which is to reanalyze data near the superfluid transition [10]. This will require initializing the density profile for a non-trivial equation of state, and extracting the full functional dependence of \( \eta \) on \( n/(mT)^{3/2} \). In order to describe the data below \( T_c \) the method has to be extended to superfluid hydrodynamics. In principle this is straightforward, because in terms of fluid dynamics a superfluid can be viewed as a mixture of a normal, viscous, fluid with an inviscid fluid.

Finally, we emphasize that the method presented in this work is quite general, and can be applied to a variety of physical problems. This includes problems in fluid dynamics which involve the expansion into a vacuum, or large changes in the density, so that the Knudsen number of the flow varies by orders of magnitude. The basic idea of the method can also be applied to determine other transport coefficients, for example the spin diffusion constant in trapped atomic gases [13].

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[37] This estimate is based on properties of the scaling solution of the Euler equation for an expanding gas. The velocity field is linear $u_i = \alpha_i(t) x_i$, where $\alpha_i = b_i(t)/b_0(t)$ and $b_i$ is the scale factor of the expansion in the $i$ direction. The density of a co-moving fluid element scales as $n \sim 1/b_i^3$, and the temperature scales as $T \sim n^{2/3}$. Finally, after the initial acceleration period we have $b_i \sim \omega_t$ and $\alpha_i \sim \omega_i/b_i$.

[38] The stresses are diagonal because of the symmetries of the trapping potential. The Euler equation implies $\partial_i u_i \sim \nabla_i P \sim \omega_t x_i$ so that $\nabla_i u_j = 0$ for $i \neq j$. This feature is preserved by viscous corrections.

[39] In kinetic theory $\eta = n_0 P$, where $n_0$ is the collision time, and this parameter is the Knudsen number $Kn$ of the flow. In fluid dynamics $Kn = Re^{-1} Ma^2$, where $Re$ is the Reynolds number, and $Ma$ is the Mach number.

[40] The solutions of the Boltzmann equation shown in Fig. 1 correspond to results of [34] with all quantum corrections and in-medium effects removed. In the temperature regime considered here, these effects are small.
The Chapman-Enskog result is formally exact in the limit $n/(mT)^{3/2} \to 0$. The coefficient $\frac{15}{32\sqrt{\pi}}$ is an approximation that arises at leading order in an expansion of the solution of the Boltzmann equation in Laguerre polynomials. The next-to-leading order correction gives a result which is larger by a factor $193/190$ [22, 36]. This is a 2% correction, suggesting that the full result is well approximated, and that the correction to Fig. 1 is very small.

The goodness of fit $\chi^2/\nu$ for the fits shown in Fig. 2 is between 5 and 10. This is comparable to the quality of the model-dependent fits in the original work [4]. Note that the error bars in the data only include statistical fluctuations in the measurement of the aspect ratio.

The best fit is $\eta = \eta_0 (mT)^{3/2}(T/T^*)^a$ with $\eta_0 = 0.282$, $a = 0.05$ and $T^* = 10.1 T_F$. Here, $T_F = k_B^2/(2m)$ is the local Fermi temperature. We have fixed the dimensionless coefficient $\eta_0$ from the fit for $a = 0$. 

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[42] The goodness of fit $\chi^2/\nu$ for the fits shown in Fig. 2 is between 5 and 10. This is comparable to the quality of the model-dependent fits in the original work [4]. Note that the error bars in the data only include statistical fluctuations in the measurement of the aspect ratio.

[43] The best fit is $\eta = \eta_0 (mT)^{3/2}(T/T^*)^a$ with $\eta_0 = 0.282$, $a = 0.05$ and $T^* = 10.1 T_F$. Here, $T_F = k_B^2/(2m)$ is the local Fermi temperature. We have fixed the dimensionless coefficient $\eta_0$ from the fit for $a = 0$. 

[41] The Chapman-Enskog result is formally exact in the limit $n/(mT)^{3/2} \to 0$. The coefficient $\frac{15}{32\sqrt{\pi}}$ is an approximation that arises at leading order in an expansion of the solution of the Boltzmann equation in Laguerre polynomials. The next-to-leading order correction gives a result which is larger by a factor $193/190$ [22, 36]. This is a 2% correction, suggesting that the full result is well approxi-