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Unitary Limit of Two-Nucleon Interactions in Strong Magnetic Fields

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Two-nucleon systems are shown to exhibit large scattering lengths in strong magnetic fields at unphysical quark masses, and the trends toward the physical values indicate that such features may exist in nature. Lattice QCD calculations of the energies of one and two nucleons systems are performed at pion masses of $m_\pi \sim 450$ and 806 MeV in uniform, time-independent magnetic fields of strength $|\mathbf{B}| \sim 10^{19}$—$10^{20}$ Gauss to determine the response of these hadronic systems to large magnetic fields. Fields of this strength may exist inside magnetars and in peripheral relativistic heavy ion collisions, and the unitary behavior at large scattering lengths may have important consequences for these systems.

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In most physical situations, external electromagnetic (EM) fields have only small effects on hadronic and nuclear systems, whose structure and dynamics are dominated by the internal strong interactions arising from Quantum Chromodynamics (QCD) and internal EM interactions. However, there are specific situations involving extremely large EM fields, created either naturally in astrophysical environments or in particle colliders, for which the effects of external fields are important. In *magnetars*, high magnetic field rotating neutron stars [1], surface magnetic fields are observed up to $\mathcal{O}(10^{14})$ Gauss (for reviews, see e.g. Ref. [2, 3]), and it is conjectured that interior magnetic fields reach up to $\mathcal{O}(10^{19})$ Gauss [4]. In heavy ion collisions, the currents produced by relativistic nuclei lead to large magnetic fields within the projectiles, particularly during (ultra-)peripheral collisions [5]. It is estimated that fields of $\mathcal{O}(10^{19})$ Gauss are experienced by the nuclei during the femtoseconds of the nuclear crossings [5]. Neither of these environments are easy to probe in a controlled way, and the detailed behavior of nuclei in such fields is an open question. As a step toward exploring nuclei in these extreme magnetic environments, we present the results of calculations of the effects of uniform, time-independent magnetic fields on two-nucleon (as well as two-hyperon) systems performed with the underlying quark and gluon degrees of freedom. We find that such fields can potentially unbind the deuteron and significantly modify the nucleon-nucleon (NN) interactions in the $^1S_0$ channel. At the unphysical quark masses where the calculations are performed, the scattering lengths in both the $^3S_1$–$^3D_1$ and $^1S_0$ channels diverge at particular values of the field strength. Near these values, the low energy dynamics of these systems will become unitary. The trends seen towards the physical values of the quark masses suggest that this feature may exist in nature in some of these systems. The prospect of such resonant behavior in nuclear systems is exciting and it will be important to incorporate this effect into models of magnetars and heavy ion collisions in which the relevant field strengths are probed.

Before presenting the results of our calculations, it is interesting to consider phenomenological expectations for the behavior of such systems.[1] For small, constant magnetic fields, the responses of the two-nucleon systems be-

[1] Significant effort has been devoted to understanding the nature of the QCD vacuum in strong magnetic fields (see Ref. [9] for a review), but effects specific to hadronic systems are not well studied.

yond their charges are governed by their magnetic moments if the system has spin, and otherwise by their magnetic polarizabilities. The deuteron has a magnetic moment such that in a magnetic field in the z direction the $j_z = +1$ component is positively shifted in energy with respect to the breakup threshold[2] and so an approach toward unbinding in a magnetic field is plausible. However, higher order responses to the magnetic field may be important, and at intermediate field strengths, $|e\mathbf{B}| \sim m_\pi^2$, significant deviations from linearity should be anticipated. In the opposite limit of extremely large magnetic fields, where $|e\mathbf{B}| \gg \Lambda_{\rm QCD}^2$, the asymptotic freedom of QCD implies [10] that the eigenstates evolve towards weakly-interacting up and down quarks in Landau levels. Hence, as the magnetic field tends to infinity, the ground states of dilute systems tend to threshold. When the density of the system is also large and comparable to the scale of Landau orbits, more exotic phases may occur (see Ref. [11] for a review).

In this work, the numerical technique of Lattice QCD (LQCD) is applied to study two-nucleon systems in uniform, time-independent background magnetic fields, following methods used in previous studies of the magnetic moments [12] and polarizabilities [13] of nucleons and light nuclei up to atomic number $A = 4$. To understand the phenomenological effects of the strong fields in nuclear environments, a first task is to ascertain the effects on the two-nucleon interactions. Two particle scattering phase shifts can be accessed in LQCD from the volume dependence of two-nucleon energies (the Lüscher method [14, 15]), but here a simpler approach is undertaken in which only the bound states of the two-nucleon sector are addressed.[3] The primary goal of these calculations is to investigate how the binding energies of the two-nucleon states respond to applied magnetic fields.

LQCD calculations were performed using two ensembles of gauge-field configurations generated with a clover-improved fermion action [16] and the Lüscher-Weisz gauge action [17]. The first ensemble had $N_f = 3$ degenerate light-quark flavors with masses tuned to the physical strange quark mass, producing a pion of mass $m_\pi \sim 806$ MeV, and used a volume of $L^3 \times T = 32^3 \times 48$. The second ensemble used $N_f = 2 + 1$ quark flavors with the same strange quark mass and degenerate up and down quarks with masses corresponding to a pion mass of $m_\pi \sim 450$ MeV and a volume of $L^3 \times T = 32^3 \times 96$. Both ensembles had a gauge coupling of $\beta = 6.1$, corresponding to a lattice spacing of $a \sim 0.11$ fm. The ensembles consisted of $\sim 1,000$ gauge-field configurations at the SU(3) point and ~ 650 configurations at the lighter pion mass, each taken at intervals of 10 hybrid Monte-Carlo trajectories. We have extensively studied these ensembles in previous works, and have found that the finite-volume effects to both the single nucleon and two-nucleon bound state energies are small [18, 19].

As in Refs. [12, 13, 20], background EM ($U_Q(1)$) gauge fields were implemented through the gauge-links,

$$U_\mu^{(Q)}(x) \;=\; e^{i\frac{6\pi Q_q \tilde{n}}{L^2}x_1\delta_{\mu,2}} \times e^{-i\frac{6\pi Q_q \tilde{n}}{L}x_2\delta_{\mu,1}\delta_{x_1,L-1}}\,, \quad (1)$$

that give rise to uniform magnetic fields along the x_3-direction. These were multiplied onto each QCD gauge field in each ensemble (separately for each quark flavor of charge Q_q). The combined QCD+EM gauge fields were used to calculate up-, down-, and strange-quark propagators, which were then contracted to form the requisite nuclear correlation functions using the techniques of Ref. [21]. To ensure periodicity, $\tilde{n} \in \mathbb{Z}$, and the values $\tilde{n} = 0, 1, -2, 3, 4, -6, 12$ were used on the SU(3) symmetric ensemble, while $\tilde{n} = 0, 1, -2, 4$ were used on the $m_\pi \sim 450$ MeV ensemble. The corresponding field strengths are quantized as $|e\mathbf{B}| = 6\pi|\tilde{n}|/(aL)^2$, giving a field of $\mathcal{O}(10^{19})$ Gauss for $\tilde{n} = 1$. On each configuration, quark propagators were generated from 48 uniformly distributed Gaussian-smeared sources for each magnetic field. For further details of the production at the SU(3)-symmetric point, see Refs. [12, 18, 19] and in particular, Ref. [13]. Analogous methods were used for the light mass ensemble.

This work focuses on the dineutron, the diproton, and the maximal $|j_z| = j = 1$ spin state of the deuteron, all of which remain isolated, sub-threshold states in the presence of a magnetic field. The $I_z = j_z = 0$ neutron-proton systems with $(j = 1; I = 0)$ and $(j = 0; I = 1)$ mix in a magnetic field and have been considered previously in Ref. [20] to determine the cross section for the radiative capture process $np \to d\gamma$. States with the quantum numbers of $h = n, p, nn,\ pp,\ d_{|j_z|=1}$ are accessed from correlation functions $C_h(t;\mathbf{B}) = \langle 0|\chi_h(t)\overline{\chi}_h(0)|0\rangle_{\mathbf{B}}$ computed in the presence of the background magnetic field $\mathbf{B}$ from source and sink interpolating operators with the requisite quantum numbers, as discussed in detail in Ref. [13]. Representative correlation functions for the heavier mass ensemble can be found in Ref. [13] for each hadron/nucleus and background magnetic field. Ratios of these correlation functions to those without the magnetic field, $R_h(t;\mathbf{B}) \equiv C_h(t;\mathbf{B})/C_h(t;\mathbf{0})$, are also shown in Ref. [13], and are used to extract the magnetic moments and polarizabilities of the respective systems. For the $m_\pi \sim 450$ MeV ensemble, the ratios behave in a qualitatively similar manner and the signals are of comparable quality. As the central focus of this study is on

[2] While the deuteron magnetic moment is positive, it is less than the sum of the neutron and proton magnetic moments. In a potential model the difference is due to the d-state admixture into the predominantly s-wave deuteron wave function, while in NN effective field theories (EFTs) this is encapsulated in short-distance two-nucleon interactions with the magnetic field.

[3] At unphysically large values of the light quark masses, both the deuteron and dineutron are bound, as are various two baryon hypernuclei [18].

the difference between the effect of the field on the two-nucleon systems and on the nucleons in isolation, the further ratios

$$\delta R_{\mathcal{A}}(t;\mathbf{B}) \;=\; R_{\mathcal{A}}(t;\mathbf{B}) \Big/ \prod_{h\in\mathcal{A}} R_h(t;\mathbf{B}) \quad , \qquad (2)$$

are of primary importance. In this expression, $\mathcal{A}$ refers to the composite system and the product is over its constituent nucleon correlator ratios (e.g., for $\mathcal{A} = d_{j_z=+1}$ the contributions are from $p^{\uparrow}$ and $n^{\uparrow}$). The late time exponential decay of this ratio is dictated by the binding energy of the system in the presence of the field [13],

$$\delta R_{\mathcal{A}}(t;\mathbf{B}) \;\overset{t\to\infty}{\longrightarrow}\; Z_{\mathcal{A}}(\mathbf{B}) e^{-\left(\delta E_{\mathcal{A}}(\mathbf{B}) - \sum_{h\in\mathcal{A}} \delta E_h(\mathbf{B})\right)t} \, . \quad (3)$$

Fig. 1, shows these ratios for the $m_\pi \sim 450$ MeV ensemble along with the results of single exponential fits to time ranges in which the individual correlation functions entering the ratios are consistent with single exponential behavior. As discussed in Ref [13], multiple different interpolating operators are investigated for each state in this study and the resulting differences are used to gauge, in part, the systematic uncertainty. In the figures below, we focus on a particular set of interpolating operators for clarity but have verified that other choices of interpolators provide consistent results. The analogous results for the heavier mass ensemble are presented in Ref. [13].

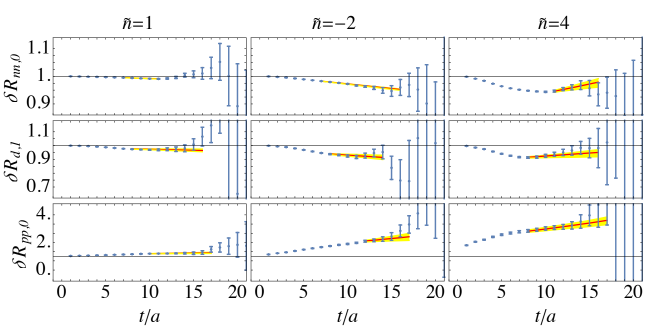

FIG. 1: Correlator ratios defined in Eq. (2) for the nn, the $j_z = +1$ deuteron and pp systems for field strengths $\tilde{n} = 1, -2, 4$, for the $m_\pi \sim 450$ MeV ensemble. The bands correspond to the exponential fit and its statistical uncertainties associated with the shown fit interval. Systematic uncertainties from the choice of fit range are separately assessed.

The energy shifts

$$\Delta_{\mathcal{A}}(\tilde{n}) \equiv \delta E_{\mathcal{A}}(\mathbf{B}) - \sum_{h\in\mathcal{A}} \delta E_h(\mathbf{B}) \qquad (4)$$

in the dineutron and deuteron ($j_z = +1$) channels are shown in Figs. 2 and 3, respectively. As the strength of the applied magnetic field is increased, the ground state energies of the systems are shifted closer to threshold, and at a given field strength it appears that the states

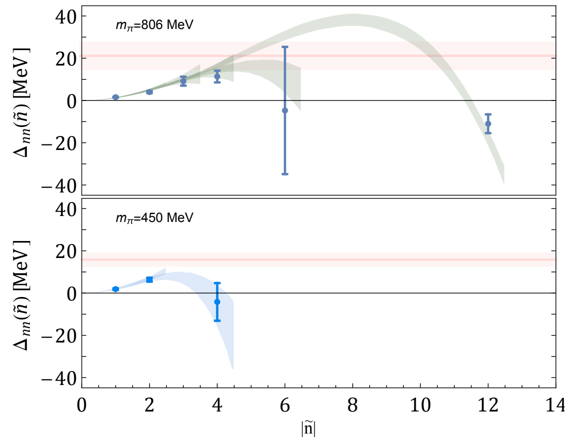

FIG. 2: Response of the binding of the dineutron system to applied magnetic fields. The upper panel shows the result at $m_\pi = 806$ MeV, while the lower panel is for $m_\pi = 450$ MeV. The shaded regions correspond to the envelopes of successful fits to the energy shifts using linear and quadratic polynomials in $\tilde{n}^2$ to data points in the corresponding range indicated by the shaded region. The horizontal bands indicate the binding threshold.

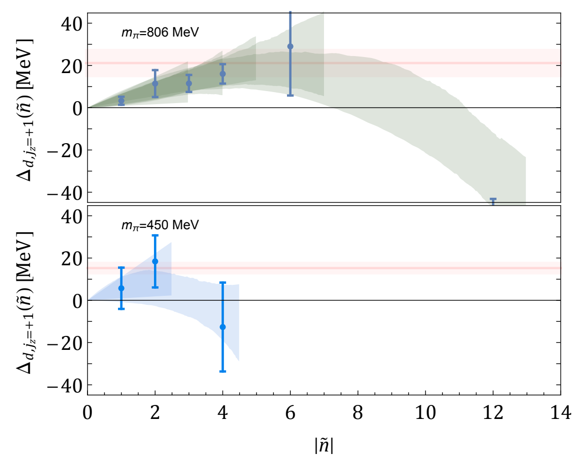

FIG. 3: Response of the binding of the $j_z = +1$ state of the deuteron to applied magnetic fields. The shaded regions correspond to the envelopes of successful fits to the energy shifts using polynomials in $\tilde{n}$ of up to 4^{th} (2^{nd}) order for the $m_\pi = 806$ (450) MeV ensemble. The horizontal bands indicate the binding threshold.

unbind. For the deuteron, this behavior is not clearly resolved at the lighter mass because of the uncertainties. The approach to threshold and subsequent turnover is seen at both quark masses in the dineutron system, and the point of minimum binding decreases as the quark mass is lowered, $\tilde{n}_{nn}^{(\max)} \sim 6$ at $m_\pi \sim 806$ MeV and $\tilde{n}_{nn}^{(\max)} \sim 3$ at $m_\pi \sim 450$ MeV. The dineutron is unbound in nature and the present results suggest that magnetic effects would push the system further into the continuum. On the other hand, it is possible that the deuteron

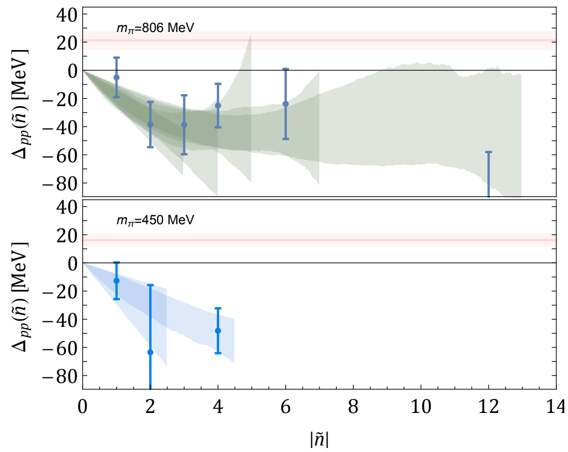

FIG. 4: Response of the binding of the diproton to applied magnetic fields. The shaded regions correspond to the envelopes of successful fits to the energy shifts using polynomials in $\tilde{n}$ of up to 4^{th} (2^{nd}) order for the $m_\pi = 806$ (450) MeV ensemble. The horizontal bands indicate the binding threshold.

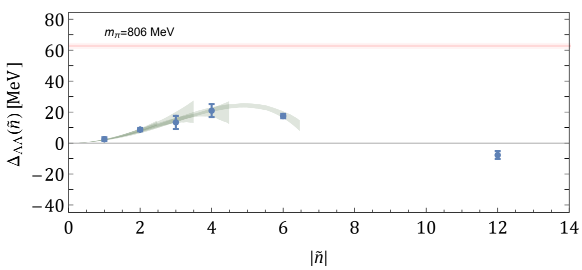

FIG. 5: The energy splitting in the H dibaryon channel at $m_\pi = 806$ MeV. The horizontal band indicates the binding threshold.

could be unbound by the presence of magnetic fields of strength comparable to those expected in magnetars and heavy ion collisions, potentially modifying the dynamics of those systems. A particularly interesting aspect of the behavior in both of these channels is the approach to the unitary regime in which the binding energies decrease to zero and consequently the scattering lengths diverge.[4] In atomic physics, such behavior is routinely used to investigate the universal physics that emerges in systems interacting near unitarity [22], but they have not been observed in nuclear physics.

The energy shifts of the diproton are shown in Fig. 4. For this system, the extracted energies are not as cleanly determined as for the dineutron, but a trend toward strengthening attraction is seen at both quark masses as the field strength increases. This is interesting in light of a recent suggestion [23] that the diproton can overcome the Coulomb repulsion and form a bound state in sufficiently large magnetic fields. A naive extrapolation of the slope of the shift linearly in m_π^2 indicates that for a field of $|e\mathbf{B}| \sim 10^{17}$ Gauss, corresponding to $\tilde{n} \sim 0.01$, the additional attraction is enough to bind the diproton system. While such a result would be interesting, further calculations at lighter quark masses are necessary to refine the extrapolation.

Two baryon systems containing strange quarks have also been investigated. Figure 5 shows the energy splittings of the ground state in the channel with the quantum numbers of two Λ-baryons, which contains a deeply bound H-dibaryon at heavier quark masses [24, 25]. This channel exhibits a slight reduction of the binding energy for intermediate field strengths, comparable in size to that of the dineutron system, but does not exhibit resonant behavior in the range of field strengths that are probed as the binding energy is significantly larger.

Discussion: Having found significant changes in the binding of two-nucleon systems immersed in strong magnetic fields at two values of unphysical quark masses, it is conceivable that similar modifications occur in nature. To solidify this discussion, calculations would need to be performed at or near the physical quark masses and the continuum and infinite volume limits would require careful investigation.[5] While the responses of these systems can as yet only be estimated at the physical quark masses, the calculated trends provide an interesting starting point to consider possible consequences. To this end, it is conjectured that two-nucleon systems will exhibit unitary behavior, with the deuteron unbinding in a large magnetic field and the diproton system becoming bound. On the other hand, the dineutron will be pushed further into the continuum as the field strength increases. Interestingly, it may be possible to find values of the field strength and quark masses where all NN states are at threshold simultaneously, realizing the low energy conformal symmetry postulated by Braaten and Hammer [26]. Given the observed behavior of bound states, it is natural to expect that the NN scattering phases shifts and mixing angles will also be modified at a similar level in such fields. These modifications would be interesting to probe in future LQCD calculations utilizing the Lüscher method [14, 15] to analyze the spectra of NN systems.

In ultra-peripheral heavy ion collisions, one can speculate that the reduced binding between pairs of nucleons, along with the reduction in the nucleon mass, will increase the size of each nucleus as they interact with

[4] It is expected that the range of the interaction (set by hadronic scales) is only weakly affected by the magnetic field, so the volume effects in the two-nucleon systems are not expected to be unmanageable even as the scattering length diverges.

[5] Based on studies of binding energies on these and other related ensembles, we are confident that the current calculations do not suffer from large volume or scaling artifacts.

the field of the other nucleus. Ignoring other potential effects, purely geometrical considerations will result in larger than expected interaction cross-sections that will increase with the collision energy for a given impact parameter and potentially larger fluctuations in collision cross sections. However, considering the transient nature of such a collision, and the difference between the internal time-scales associated with a rearrangement of the nucleons comprising each nucleus and that of the collision, more detailed analyses must be performed before even a qualitative understanding can be established. The effects of large magnetic fields in magnetars through the magnetic moments of nucleons and electrons have been considered through a number of model approaches [4, 27–31], and in some cases lead to significant modifications. The more complicated effects from magnetic shifts in binding and hadronic interactions likely also induce significant modifications that deserve further investigation.

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