Overwhelming Thermomechanical Motion with Microwave Radiation Pressure Shot Noise

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Overwhelming thermomechanical motion with microwave radiation pressure shot noise

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We measure the fundamental noise processes associated with a continuous linear position measurement of a micromechanical membrane incorporated in a microwave cavity optomechanical circuit. We observe the trade-off between the two fundamental sources of noises that enforce the standard quantum limit: the measurement imprecision and radiation-pressure backaction from photon shot noise. We demonstrate that the quantum backaction of the measurement can overwhelm the intrinsic thermal motion by 24 dB, entering a new regime for cavity optomechanical systems.

Quantum mechanics places limits on the precision with which one can simultaneously measure canonically conjugate variables, such as the position and momentum of an object. For example, if one scatters light off an object to determine its position, the momentum kicks from individual photons will necessarily apply forces back on that object. In the limit of a continuous measurement in which a coherent state of light is used to infer the state of the mechanical degree of freedom, it is the quantum statistics of the photons that will place simultaneous limits on the information gained and the backaction imparted. So while a stronger measurement reduces the imprecision, this comes at the cost of increasing radiation pressure shot noise forces. The minimum noise added by the measurement process is bounded by the standard quantum limit (SQL) and occurs when the imprecision noise and backaction noise are perfectly balanced [1–4]. Experimentally observing this fundamental trade-off requires a system that interacts strongly with the measurement yet weakly with its thermal environment.

One successful strategy for the measurement of mechanical systems has been that of cavity optomechanics [5], in which the displacement of a mechanical oscillator of frequency Ω_m tunes the resonance frequency ω_c of a high-frequency electromagnetic cavity. When the cavity is excited with a coherent drive of power P at ω_c, the mechanical motion becomes encoded as phase fluctuations of the reflected light. As the measurement power is increased, this mechanical signal rises above the measurement noise floor, ideally limited by the vacuum fluctuations in the phase quadrature of the drive. This improvement in the measurement imprecision comes at the expense of an increasing backaction force, ideally solely from the vacuum fluctuations in the amplitude quadrature of the drive. This behavior is most evident at the mechanical resonance frequency, where the mechanical susceptibility is resonantly enhanced and the influence of radiation pressure is maximized.

The measured total displacement spectral density S_x at the mechanical resonance frequency is a combination of the zero-point motion S_x^zp, the thermal motion S_x^th, the measurement imprecision S_x^imp and measurement backaction S_x^ba. The SQL can be stated by the inequality $S_x^\mathrm{imp} + S_x^\mathrm{ba} \geq S_x^\mathrm{zp}$. For an ideal homodyne detection of

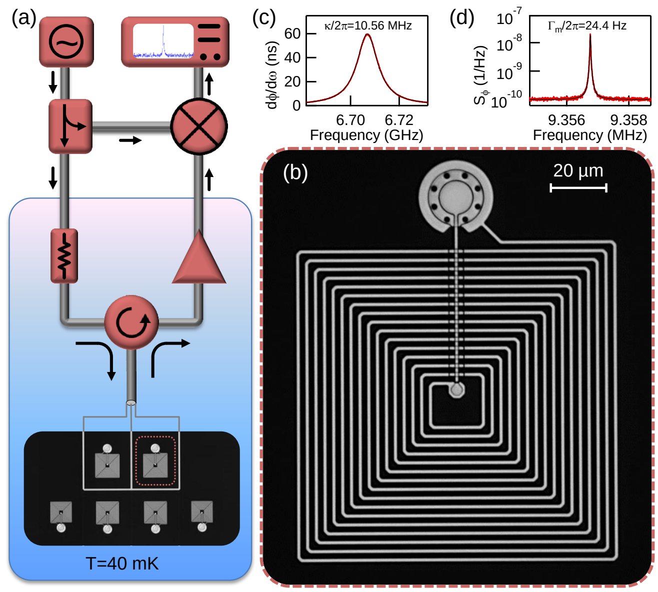

FIG. 1. **Experimental schematic and device characterization.** **(a)** We interrogate an array of microwave cavity optomechanical circuits with a cryogenic interferometer. A coherent microwave drive at the cavity resonance frequency is inductively coupled to the resonators. The reflected microwave field acquires phase modulation sidebands from the mechanical mode that are amplified cryogenically and demodulated at room temperature. **(b)** Optical micrograph of a single microwave cavity optomechanical circuit, microfabricated out of aluminum (grey) on a sapphire substrate (black). The mechanically compliant capacitor resonates with a spiral inductor to form a superconducting LC resonant circuit. **(c)** A Lorentzian fit (black) to the microwave group delay (red) shows a highly overcoupled electromagnetic resonance at $\omega_\mathrm{c}/2\pi = 6.707$ GHz with a total linewidth $\kappa/2\pi = 10.56$ MHz (FWHM). **(d)** When this microwave resonance is driven at ω_c, the noise spectrum of the microwave phase fluctuations shows a mechanical resonance (red) whose fit (black) yields $\Omega_\mathrm{m}/2\pi = 9.357$ MHz with intrinsic linewidth of $\Gamma_\mathrm{m}/2\pi = 24.4$ Hz (FWHM).

the phase quadrature of the light, the added noise of the measurement at Ω_m is

$$S_x^\mathrm{imp} + S_x^\mathrm{ba} = \frac{S_x^\mathrm{zp}}{2}\left(\frac{P_\mathrm{SQL}}{P} + \frac{P}{P_\mathrm{SQL}}\right). \tag{1}$$

The power required to reach the SQL for an ideal, lossless cavity optomechanical system is $P_{\rm SQL} = \hbar\omega_{\rm c}\Gamma_{\rm m}\left(\kappa^2+4\Omega_{\rm m}^2\right)/64g_0^2$, where $\Gamma_{\rm m}$ and κ are the mechanical and cavity energy dissipation rates, respectively, and g_0 is the vacuum optomechanical coupling rate[3, 5]. It is only at the optimum power $P_{\rm SQL}$ that the measurement is impedance matched to the mechanical system [3], reaching the minimum added noise of $S_x^{\rm zp} = 2\hbar/\left(m\Omega_{\rm m}\Gamma_{\rm m}\right)$, where m is the mass of the oscillator.

In practice, any technical deficiencies will preclude a measurement from reaching this minimum of noise added. For example, internal loss of the cavity and inefficient homodyne detection degrade the measurement imprecision. Excess amplitude noise on the coherent drive and thermal population of the cavity increase the measurement backaction. Lastly, the power required to reach the minimum added noise must be experimentally feasible. In typical experiments, it is not the available drive power, but rather the number of drive photons in the cavity that limits the strength of the measurement. For example, while nonidealities of optical or microwave cavities can arise from dielectric nonlinearity, conductor nonlinearity, thermal effects or even the optomechanical interaction itself, all of these effects scale with the intra-cavity power [6]. As such, one important figure of merit is the number of intra-cavity photons required to reach the SQL $n_{\rm SQL} = 4P_{\rm SQL}/\hbar\omega_{\rm c}\kappa = \Gamma_{\rm m}\left(\kappa^2+4\Omega_{\rm m}^2\right)/16g_0^2\kappa$. Thus, while engineering κ to be much less than $\Omega_{\rm m}$ is beneficial for sideband cooling [3], this regime increases $n_{\rm SQL}$ and greatly diminishes the measurement strength per resonant cavity photon.

While many experiments are now able to engineer low enough $n_{\rm SQL}$ to be achieved in the laboratory, the quantum backaction from the radiation pressure shot noise is often obscured by residual thermal motion of the oscillator [7–9]. It is only recently that cavity optomechanical experiments observe backaction noise beyond the thermal occupancy[10–13]. However, one would ideally like to operate at much larger powers where the quantum backaction dominates all other sources of noise including the thermal motion, $S_x^{\rm ba} \gg S_x^{\rm th}$. It is in this high power limit where many longstanding theoretical proposals reach their full potential; these include, for example, ponderomotive squeezing [14] or amplification [15] of light and coherent feedback cooling of motion [16].

We achieve this limit in a microwave cavity optomechanical circuit incorporated in a cryogenic interferometer, as shown in Fig.1. The circuit is made out of aluminum and consists of a superconducting 15 nH spiral inductor resonating with a vacuum-gap, parallel-plate capacitor [17]. This LC resonant circuit creates a microwave "cavity" whose resonance frequency depends sensitively on the separation between the capacitor electrodes. The mean plate separation of 40 nm yields a cavity frequency of $\omega_{\rm c}/2\pi = 6.707$ GHz. The top plate of the capacitor is mechanically compliant and has a fundamental flexural mode at $\Omega_{\rm m}/2\pi = 9.357$ MHz, which strongly couples to the microwave resonance [18]. With a total mass of 85 pg, it has a zero-point motion of $x_{\rm zp} = \sqrt{\hbar/(2m\Omega_{\rm m})} = 3.3$ fm. At a cryostat temperature of 40 mK, the mechanical mode has an equilibrium occupancy of $n_{\rm th} = [\exp(\hbar\Omega_{\rm m}/k_{\rm B}T)-1]^{-1} = 90$ phonons.

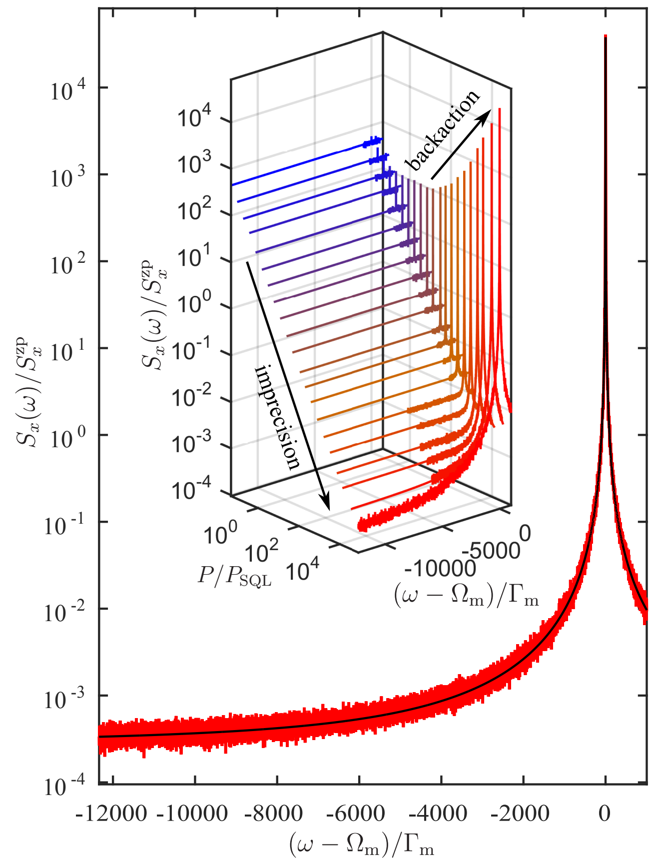

FIG. 2. **Calibrated mechanical noise spectra.** Phase noise spectra in units of normalized displacement spectral density allow for direct noise thermometry of the occupancy of the mechanical mode for various measurement strengths. Even at weak microwave drive powers (≈ 10 fW), we readily resolve the thermomechanical motion of the oscillator above measurement imprecision with negligible backaction. As the drive power is increased, the imprecision noise floor is lowered, and radiation pressure shot noise of the microwave photons imparts quantum backaction on the mechanical mode as visible from the increased height of the Lorentzian peak. At the largest measurement power, the quantum backaction completely overwhelms the intrinsic thermal motion, and the backaction exceeds the measurement imprecision over a frequency span of more than 10^4 mechanical linewidths.

Critically, and contrary to previous work[19], the cavity is highly overcoupled to its feedline, with a total linewidth $\kappa/2\pi = 10.56$ MHz (Fig. 1c). This modification of κ solves three significant technical issues. Most importantly, the measurement strength per cavity photon is maximized by engineering $\kappa \approx \Omega_{\rm m}$. Next, with internal losses contributing to less than 1% to the total cavity linewidth, very few photons are dissipated at the device, optimizing the collection efficiency by the measurement apparatus. Lastly, as internal losses are responsible for excess cavity thermal population at high drive power [12, 19], strongly overcoupling to a shot-noise-limited feedline thermalizes the cavity into its ground state.

When the overcoupled optomechanical cavity is driven exactly on resonance, the thermomechanical signal appears only in the phase quadrature of the microwave light. The measured power spectral density of the phase quadrature of the microwave field S_ϕ is directly related to the displacement spectral density S_x via the relation

$$\frac{S_x}{x_{\mathrm{zp}}^2} = \left(\frac{\kappa^2 + 4\Omega_{\mathrm{m}}^2}{64 g_0^2}\right) S_\phi. \quad (2)$$

We probe the cavity with a coherent microwave drive applied at ω_{c}, heavily filtered and attenuated at cryogenic temperatures to ensure that the microwave field is in a pure coherent state. The reflected signal is separated from the incident wave through a microwave circulator, amplified with a cryogenic low-noise amplifier, and the phase quadrature is measured with a homodyne detection scheme. In Fig.1d the mechanical line shape is evident from the Lorentzian peak in the power spectral density of the phase fluctuations S_ϕ centered at Ω_{m}, yielding a intrinsic mechanical damping rate of $\Gamma_{\mathrm{m}}/2\pi = 24.4$ Hz. As with previous experiments [7, 12, 19, 20], we use the thermal motion at several cryostat temperatures in the weak measurement regime to calibrate $g_0/2\pi = 230$ Hz and $P_{\mathrm{SQL}} = 93$ fW. The absolute power at the device and the measurement efficiency $\eta = 0.02$ are calibrated from the measured optical damping from a red-detuned drive [19]. With these parameters determined, we may explore the power dependence of the calibrated noise spectra.

As shown in Fig. 2, increasing the measurement power has two effects on the measured displacement spectral density. First, the measurement imprecision decreases with increasing power as expected. For moderate measurement strengths ($P \lesssim 100$ fW), this reduced imprecision is the only observable effect as the expected backaction remains well below the thermal motion. Second, beyond this power, the observed height of the Lorentzian peak begins to grow linearly with increasing drive power. It is in this regime where we can begin to observe the fundamental trade-off expected from Eq. 1.

Crucially, the ideal power scaling of imprecision and backaction in Eq. 1 requires the mechanical susceptibility to remain unchanged by the measurement despite the presence of strong radiation pressure forces. If the cavity is not driven exactly on resonance, the imbalance between Stokes and anti-Stokes scattering processes would lead to dynamical backaction, greatly damping or anti-damping the mechanical oscillator [3]. For all of the data shown here, the measured mechanical linewidth remains within 5% of the intrinsic value Γ_{m}, corresponding to residual detuning of less than 1 part per million of κ.

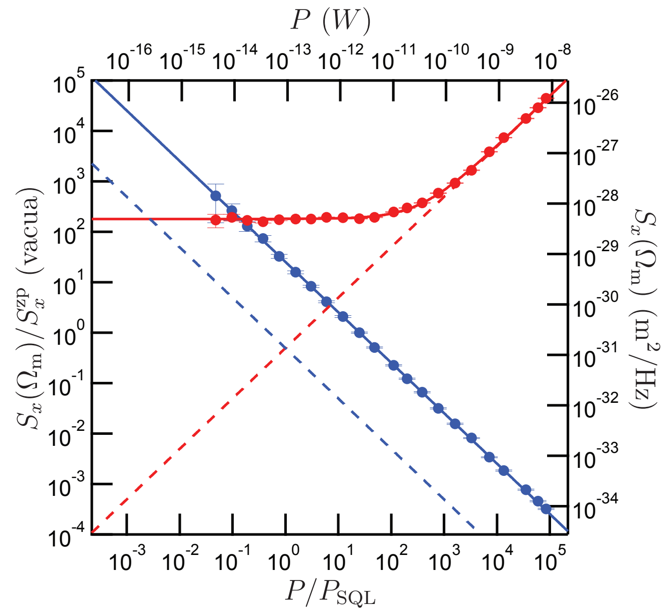

FIG. 3. **Fundamental trade-off between imprecision and measurement backaction.** The total noise power at the mechanical resonance frequency is decomposed into its two contributions. The data show the imprecision from the noise floor of the measurement (blue circles) and the total motion from the peak height of the Lorentzian (red circles) as a function of measurement power. The dashed lines show the expected imprecision and backaction of the perfect quantum measurement. The solid lines are theoretical predictions for the imprecision (blue) and total motion (red) based on the independently determined measurement efficiency and thermal occupancy. At the highest power, the imprecision is lowered to 8.8×10^{-35} m^2/Hz, a factor of 1700 below that at the standard quantum limit. Likewise, the quantum backaction from momentum kicks of the measurement photons overwhelms the intrinsic thermal motion of $n_{\mathrm{th}} = 90$, demonstrating a quantum measurement rate 250 times larger than the thermal decoherence rate.

From Lorentzian fits to these noise spectra, we independently determine the imprecision noise and actual motion of the oscillator in the presence of backaction. Figure 3 shows these two components of the noise as a function of the measurement strength. The imprecision follows the expected $1/P$ dependence with no visible deviation even at the highest power. The solid blue line is the predicted imprecision, degraded from the ideal imprecision (blue dashed line) by the homodyne efficiency η. The red dashed line is the expected quantum backaction of the measurement. The measured data agrees with the theoretically predicted backaction from a true coherent state of the microwave field. We do not observe any measurable deviation from linear scaling that would indicate cavity heating or excess amplitude noise of the drive. From the error bars of the measurement, we can place an upperbound on the excess noise at less than 5% of the shot noise at the highest drive power.

At the optimum measurement power of $P = P_{\mathrm{SQL}}/\sqrt{\eta} \approx 600$ fW, the total added noise is optimized, yielding $S_x^{\mathrm{imp}} \approx S_x^{\mathrm{ba}} \approx 3.6 \times S_x^{\mathrm{zp}}$. It is at this power that the measurement is most sensitive to both the thermal mechanical environment and external forces, demonstrating a force sensitivity of 5.5 aN/$\sqrt{\mathrm{Hz}}$ at the mechanical resonance frequency. At the highest measurement power of $P = 7.8$ nW, a power approaching $\approx 10^5 \times P_{\mathrm{SQL}}$, we interrogate the system with approximately 10^{15} photons per second. The primary limit to the measurement power was the ability to stably maintain precisely zero detuning, ensuring negligible change in mechanical susceptibility from the measurement. This limitation could be improved in future experiments by actively locking the drive to the cavity. Despite a modest detection efficiency, we achieve an imprecision noise of 8.8×10^{-35} m^2/Hz, 35 dB below the zero-point level S_x^{zp}. Concurrently, the quantum backaction increases the total mechanical motion to 2.2×10^4 quanta. Comparing this to the intrinsic thermal occupancy of the mechanical mode, we observe microwave radiation pressure shot noise 24 dB above the thermal motion. In other words, the mechanical oscillator is so strongly coupled to the measurement that its dissipative thermal environment should only contribute less than 1% of its total motion.

Future experiments can further improve the total displacement noise by addressing each of the fundamental noise sources. First, the excess thermal noise can be greatly reduced by using sideband cooling to prepare the mechanical mode near its ground state. Next, the imprecision noise can be lowered by incorporating Josephson parametric circuits as nearly quantum limited, phase-sensitive, microwave amplifiers [7, 19]. Lastly, now that we can achieve $P \gg P_{\mathrm{SQL}}$, the backaction noise that dominates the measurement can be suppressed by implementing techniques that do not simultaneously monitor both quadratures of mechanical motion [2, 3, 21]. For example, amplitude modulating the measurement at precisely twice the mechanical frequency decouples the radiation pressure backaction from the measurement. As such measurements are not bound by the traditional quantum limits, they are actively being pursued for improved force sensing and mechanical state tomography [12, 22].

These results demonstrate the fundamental sensitivity limits for mechanical measurements, with numerous applications [23] including areas such as magnetic resonance force microscopy [24] and the detection of gravitational waves [25, 26]. The regime of strong measurement backaction demonstrated here opens the door toward an array of experiments utilizing quantum states of both light and motion. In order to reduce the excess thermal motion of the oscillator, one could implement active feedback to cool the motion close to the ground state [13, 27]. Beyond simply using the light to interrogate the state of the mechanical oscillator, it is in the backaction-dominated regime that an optomechanical system can prepare and measure nonclassical states of the light field, even in the presence of thermal motion. One could achieve strong ponderomotive squeezing of the outgoing light [28, 29], and the parameters demonstrated here would predict ponderomotive squeezing of microwave light more than 20 dB below vacuum without the need to cool the mechanical mode. Likewise, the same correlations that can squeeze the microwave field can also be used to amplify the light in a nearly ideal way [15, 30]. Finally, it is in this regime that an optomechanical system realizes a quantum nondemolition measurement of light [31].

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