Precision Test of Gauge-Gravity Duality with Flavor

Andreas Karch, Brandon Robinson, and Christoph F. Uhlemann

Phys. Rev. Lett. **115**, 261601 — Published 22 December 2015

DOI: 10.1103/PhysRevLett.115.261601

A Precision Test of AdS/CFT with Flavor

Andreas Karch,[*] Brandon Robinson,[†] and Christoph F. Uhlemann[‡]
Department of Physics, University of Washington, Seattle, WA 98195-1560, USA

We put AdS/CFT dualities involving probe branes to a precision test. On the holographic side we use a new class of supersymmetric D7-brane embeddings into $\mathrm{AdS}_5\times\mathrm{S}^5$, which allow us to describe $\mathcal{N}=4$ SYM coupled to massive $\mathcal{N}=2$ supersymmetric flavors on S^4. With these embeddings we can compare holographic results to a field theory analysis of the free energy using supersymmetric localization. Localization allows us to get results at strong coupling, and hence to compare in detail to AdS/CFT. We find analytically matching results: a phase transition at the same critical mass in both calculations and matching free energies up to a scheme-dependent constant in both phases.

I. INTRODUCTION

Probe branes have found a wide range of applications in holographic studies, as the simplifications provided by the probe approximation make them a very versatile tool. They are used, e.g., to add quarks to holographic duals of QCD-like theories [1, 2] and give one of the simplest holographic realizations of compressible and conducting matter [3]. Strictly speaking, the addition of probe branes is an extra ingredient in holography. It does not directly follow from the basic postulates, and one may be worried about the probe limit being well defined. Conducting a decisive test of these dualities is tough, however. The virtue of the dualities, i.e. that involved questions on one side are mapped to simple ones on the other, becomes an obstacle when it comes to testing: it is difficult to calculate the same quantity in the same regime on both sides of the dualities. Building on recent progress in the study of supersymmetric gauge theories on curved, compact manifolds, and in particular supersymmetric localization [4], we give a detailed test in this work.

In [4], a massive deformation of $\mathcal{N}=4$ Super Yang-Mills (SYM) theory, called $\mathcal{N}=2^*$, was constructed on S^4. Preserving a subset of the supersymmetries allowed for the use of supersymmetric localization. This procedure reduces the partition function from an infinite-dimensional path integral to an ordinary integral over a modified Gaussian matrix model. This dramatic simplification makes exact calculations possible and has led to a large volume of work studying its application in the context of AdS/CFT [5–7]. In particular, the authors of [5] constructed, albeit numerically, the gravitational dual to $\mathcal{N}=2^*$ on an S^4, and were able to perform a rigorous test of AdS/CFT by matching derivatives of free energies.

The methods of [4] can be applied to more general $\mathcal{N}=2$ supersymmetric gauge theories on S^4. Of particular interest are QCD-like theories with N_c colors and N_f matter multiplets in the fundamental representation of the gauge group. In the limit of large N_c, fundamental matter offers a new small parameter, $\zeta\equiv N_f/N_c$. The matter fields experience non-trivial dynamics in the background of the gauge field even in the $\zeta\rightarrow 0$ limit. However, there are not enough matter degrees of freedom to alter the dynamics of the color fields. That simplification is captured holographically by the probe limit [8]. The fundamental flavor multiplets get incorporated via a brane that minimizes its action in a fixed background geometry. Its backreaction can be neglected. Building on our recent construction of supersymmetric probe brane embeddings dual to $\mathcal{N}=4$ SYM coupled to massive fundamental matter on curved spaces [9], we are now in a position to perform a precise check of this duality using localization, with both sides of the correspondence under complete analytic control.

Field theory calculations based on localization with fundamental matter revealed a complicated and sometimes poorly understood phase structure [10] at large N_c, where the large N_c limit is what allows non-trivial phase transitions even on a compact manifold [11]. In the theory we are studying we have complete control over the localization calculation and can identify a single well-characterized phase transition as a function of mass.

To begin, we start with the holographic side. We discuss the brane embedding and evaluate the chiral and scalar condensates as well as dF/dM, where M is the mass of the flavors and F the free energy. Second, we turn to the localization computation. We discuss the quenched approximation of the matrix model and calculate dF/dM, to compare to the holographic result. Lastly, we end with a discussion.

II. HOLOGRAPHIC PROBE BRANE ANALYSIS

The essential ingredient to finding supersymmetric brane embeddings is κ-symmetry, an extra fermionic gauge symmetry used to project out part of the fermionic modes and obtain matching numbers of bosonic and fermionic degrees of freedom [12–15].

* akarch@uw.edu
† robinb22@uw.edu
‡ uhlemann@uw.edu

A. Supersymmetric Embeddings

To describe the dual theory on S^4, we start with an S^4–sliced $AdS_5\times S^5$ background in Euclidean signature. In Fefferman-Graham gauge the line element reads

$$ds^2 = \frac{dz^2}{z^2} + \frac{(1-\frac{z^2}{4})^2}{z^2} d\Omega_4^2 + d\Omega_5^2 \ , \tag{1a}$$

$$d\Omega_5^2 = d\theta^2 + \sin^2\theta d\Omega_3^2 + \cos^2\theta d\psi^2 \ . \tag{1b}$$

The D7-branes, described by the action

$$S_{\rm D7} = -T_7 \int d^8x \sqrt{\det\left[g + 2\pi\alpha' F\right]} + 2(2\pi\alpha')^2 \int_{\Sigma_8} C_4 \wedge F \wedge F \ , \tag{2}$$

are embedded into this background. In static gauge the embedding is characterized by the slipping mode $\theta(z)$ alone. The induced metric reads

$$ds^2_{D7} = \frac{1+z^2\theta'^2}{z^2} dz^2 + \frac{(1-\frac{z^2}{4})^2}{z^2} d\Omega_4^2 + \sin^2\theta d\Omega_3^2 \ . \tag{3}$$

The asymptotic D7-brane geometry is $AdS_5\times S^3$, and the profile of $\theta(z)$ determines whether and where the branes cap off via the internal cycle collapsing. $\mathcal{N}=2$ supersymmetric field theories on S^4 with massive fields require the addition of a dimension-2 scalar-bilinear compensating term in the Lagrangian, in order to restore the supersymmetry that is otherwise broken by the curvature [4, 16]. To source these compensating terms holographically, we turn on a worldvolume gauge field on the D7, $A = f(z)\omega$. To reflect the properties of the field-theory mass term, A has to transform in a specific way under the SO(4) isometries of the S^3 that the D7-branes wrap in the internal space. In the language of [17], this translates to ω transforming as (0, 1) under $SU(2)\times SU(2)$.

We now turn to the κ-symmetry analysis. The brane embedding preserves those supersymmetries of the background which are generated by Killing spinors that satisfy a projection condition, $\Gamma_\kappa\epsilon = \epsilon$. The matrix Γ_κ encodes the brane embedding. To find supersymmetric embeddings, we feed in the explicit $AdS_5\times S^5$ Killing spinors and demand that there be non-trivial solutions to the projection condition. This yields a set of necessary conditions for the embedding and gauge field. We give the details in [9] and content ourselves with an outline of the main points here. Demanding the projection condition spelled out explicitly in [9] to have non-trivial solutions, such that $\mathcal{N}=2$ supersymmetry is preserved, fixes ω to be precisely what we argued for and gives us a non-linear relation between the gauge field and slipping mode. In addition, we find a 2nd-order differential equation for the slipping mode alone. That equation can be solved analytically:

$$\cos\theta(z) = 2\cos\left(\frac{4\pi + \cos^{-1}\tau(z)}{3}\right) \ , \tag{4a}$$

$$\tau(z) = \frac{96z^3(c - m\log\frac{z}{2}) + 6mz(z^4-16)}{(z^2-4)^3} \ , \tag{4b}$$

$$f(z) = -i\sin^3\theta \, \frac{z(z^2-4)\theta' - (z^2+4)\cot\theta}{8z} \ . \tag{4c}$$

The parameter m is identified, up to a factor of the tension of a fundamental string, with the mass of the flavor fields, $M = m\sqrt{\lambda}/2\pi$ [17]. The factor $\mu \equiv \sqrt{\lambda}/(2\pi)$ will be crucial in the field theory analysis: for any m which is not infinitesimally small, the flavors in the field theory are heavy, with mass of order $\sqrt{\lambda}$ in units of the S^4 radius. The relation (4c) in particular links the near-boundary expansions of f and θ, which should be expected given that, on the field theory side, the coefficient of the compensating term is fixed by the superpotential mass [4].

The D7 brane embeddings come in two distinct classes: the branes can either smoothly cap off at a $z_* \in (0,2)$, or they can extend all the way to the center of AdS at $z=2$. The D7-brane geometry is a cone with $S^3 \times S^4$ base, where the S^3 lives in the internal space and the S^4 is the radial slice in AdS. For the first type of embeddings, the S^3 shrinks at the tip of the cone, whereas for the second it is the S^4 that shrinks. These two types of embeddings are connected by a critical embedding, where the brane caps off at $z_*=2$ and the spheres collapse simultaneously at the tip. The condition for the branes to cap off at a $z_\star \in (0,2)$ is $\theta(z_*) \in \{0, \pi\}$, which determines c as

$$c = \frac{96mz_*^3\log\frac{z_*}{2} - 6mz_*(z_*^4-16) \pm (z_*^2-4)^3}{96z_*^3} \ . \tag{5a}$$

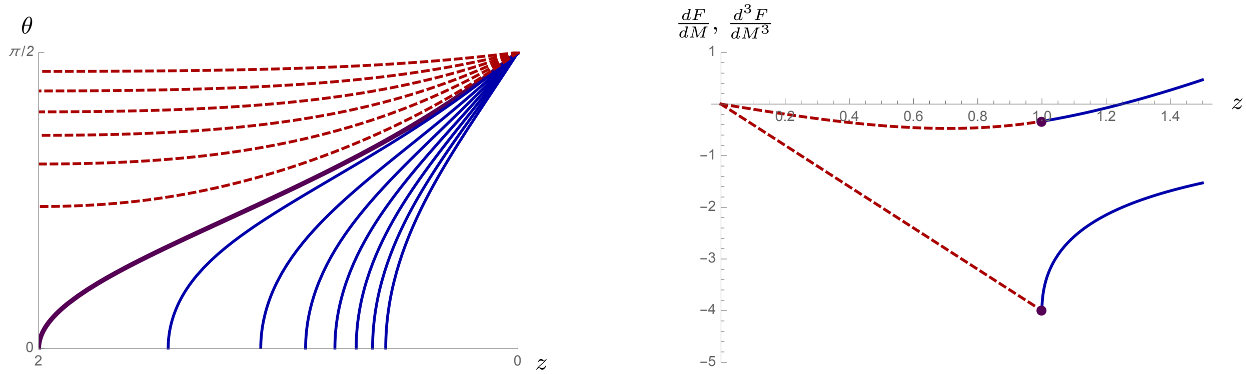
The gauge field configuration at $z=z_*$ is singular unless $f(z_*)=0$, which fixes the cap off point in terms of the mass as

$$z_* = 2(m - \sqrt{m^2-1}) \ . \tag{5b}$$

Note that these capped embeddings only exist for $m>1$. For $0\leq m<1$, we instead find embeddings that fill all of AdS. A smooth embedding in that case requires $\theta'(z=2)=0$, which translates to $c=0$. These two topologically distinct families merge, as we will see, in a continuous phase transition at $m=1$. For a plot see Fig. 1(a).

B. One-Point Functions and Free Energy

The computation of CFT one-point functions follows the standard AdS/CFT prescription, and we give the details in Sec. I of [18]. Varying the asymptotic values of θ and f independently yields the chiral condensate $\mathcal{O}_\theta$ and the scalar condensate $\mathcal{O}_f$ individually. From the localization calculation, however, we only get access to the linear combination that corresponds to varying within the family of supersymmetric embeddings.



(a) (b)

FIG. 1. The D7-brane embeddings are shown for $m \in \{0,\, 0.025,\, \ldots, 2\}$ in (a). For the dashed red curves the S^4 collapses at $z=2$, while for the solid blue curves the branes cap off at a $z_\star \in (0,2)$ with the internal S^3 collapsing. The critical embedding with both collapsing concurrently is shown as thick purple curve. The upper and lower curves in (b) show $dF/dM = V_4\langle O_s\rangle$ and d^3F/dM^3, respectively. The linear term in dF/dM is scheme dependent, and we subtracted off $2m\log\frac{\mu}{2}$ for the plot. The color/line coding reflects the embeddings from which the results are obtained holographically. The results match analytically to the matrix model calculations in (19) for the red dashed curves and (18) for the blue solid curves. The purple dots correspond to the critical embeddings/the hypermultiplets moving on to the eigenvalue distribution.

Namely, $\mathcal{O}_s \equiv \mathcal{O}_\theta + i\mathcal{O}_f$. Varying the D7-brane action with respect to the field theory mass $M = m\mu$ yields

$$\frac{\mu}{T_0}\langle\mathcal{O}_s\rangle = 3c + \frac{2m^3}{3}(1+6\alpha_1) - \frac{m}{2}(7+4\beta)\ , \qquad (6)$$

where $T_0 = T_7 V_{S^3}$ and α_1, β parametrize finite counterterms. That is, they are ambiguities in the renormalization scheme. Demanding the scheme to preserve supersymmetry on flat space/Poincaré AdS fixes $\alpha_1 = -\frac{5}{12}$ [19]. To translate T_0 in (6) to field theory quantities, we use (see e.g. the table in [20])

$$T_0 V_4/N_c^2 = \lambda\zeta/6\pi^2 = 2\mu^2\zeta/3\ , \qquad (7)$$

where V_4 denotes the volume of the unit S^4. This results in a free energy proportional to λ at strong coupling, which has long been recognized as a puzzling feature of the probe brane analysis, and the localization calculation will have to reproduce that. Note that $V_4\langle\mathcal{O}_s\rangle = dF/dM$, so (6) with (5), (7) can be readily compared to the field theory side. For a plot see Fig. 1(b).

On the matrix model side, analyses of massive large-N_c $\mathcal{N}=2$ gauge theories on S^4 have seen infinite families of phase transitions in the decompactification limit at strong coupling, as more and more resonances are excited on the eigenvalue distribution [10, 21]. In our holographic setup we see exactly one, topology changing, transition between the phases with AdS-filling branes for $m<1$ and branes capping off smoothly for $m>1$. The (quantum) critical point occurs exactly at $m=1$, where the wrapped $\mathrm{S}^3 \subset \mathrm{S}^5$ collapses concurrently with the S^4 at the origin. To determine the critical exponent we expand (6) around the critical embedding. For $m = 1+\epsilon$ with $\epsilon \ll 1$, we find

$$\frac{V_4}{\zeta\mu N_c^2}\langle\mathcal{O}_s\rangle \simeq -\frac{27+12\beta}{9} - \frac{13+4\beta}{3}\epsilon - 2\epsilon^2 + \frac{16\sqrt{2}}{15}\epsilon^{5/2} + \mathcal{O}(\epsilon^3)\ . \qquad (8)$$

The striking feature of this expansion is that we have full analytical control over extracting the critical exponents. These are distinct from the study of non-SUSY flavors in [22]. The difference can be traced back to the imaginary gauge field which gives non-trivial cancellations in the action that modify the general scaling analysis of [22].

III. LOCALIZATION WITH QUENCHED FLAVORS

Before deriving dF/dM on the field theory side, we review where the components in the matrix model originate. The localization calculation [4] begins by identifying a Grassmann scalar symmetry, Q, that is nilpotent up to gauge transformations. After adding a Q-exact term $\delta_V = tQV$ to the Lagrangian, to which the partition function is insensitive, one can take the limit $t\to\infty$. The saddle-point approximation becomes exact and the partition function reduces to an integral over the saddle points of δ_V. For $\mathcal{N}=2$ gauge theories on S^4, the saddles are parametrized by a single adjoint-valued scalar. Computing the 1-loop fluctuations about that locus exactly determines the partition function, up to instanton corrections. The latter are exponentially suppressed at large N_c [23], so we ignore them here. For $\mathcal{N}=4$ SYM, the 1-loop determinants evaluate to unity, and one ends up with a simple unitary Gaussian matrix model

$$Z = \int da^{N_c-1} \prod_{i<j} a_{[ij]}^2 \, e^{S_0} \ , \quad S_0 = -\frac{8\pi^2}{\lambda} N_c \sum_i a_i^2 \ , \quad (9)$$

where $a_{[ij]} = a_i - a_j$ labels the roots of $\mathfrak{su}(N_c)$ with weights a_i. The Vandermonde determinant $\prod_{i<j} a_{[ij]}^2$ comes from gauge fixing into the Cartan subalgebra. It provides a repulsive logarithmic interaction term for the eigenvalues.

For $\mathcal{N}=4$ SYM coupled to massive $\mathcal{N}=2$ flavors, we get an additional 1-loop factor and the matrix model becomes

$$Z = \int d^{N_c-1}a \, \frac{\prod_{i<j} a_{[ij]}^2}{\prod_i \sqrt{H_+^{N_f}(a_i) H_-^{N_f}(a_i)}} \, e^{S_0} \ , \qquad (10)$$

where $H(x \pm M) \equiv H_\pm(x)$, $H(x) = G(1+ix)G(1-ix)$ and $G(x)$ is the Barnes G-function [4]. We rearrange the integrand of (9) as a single exponential e^S, with

$$S = S_0 - N_c \sum_i \frac{\zeta}{2} \log(H_+ H_-) + \sum_{i<j} \log a_{[ij]}^2 \ . \qquad (11)$$

The quenched approximation amounts to evaluating the partition function for $1 \ll N_f \ll N_c$. Note that the sums $\sum_i$ and $\sum_{i<j}$ are $\mathcal{O}(N_c)$ and $\mathcal{O}(N_c^2)$, respectively. The calculation of the free energy can be organized according to an expansion in ζ by using $F \approx -S|_{\text{saddles}}$ and

$$\frac{S}{N_c^2} = \tilde{S}_0|_{\rho_0} + \zeta(S_1|_{\rho_0} + \delta\tilde{S}_0|_{\rho_0}) + \mathcal{O}(\zeta^2) \ , \qquad (12)$$

where $N_c^2 \tilde{S}_0 = S_0 + \sum_{i<j} \log a_{[ij]}^2$, and S_1 is the contribution of the flavors. We have denoted the solution to the Gaussian matrix model for pure $\mathcal{N}=4$ SYM in the continuum limit as ρ_0. This is the Wigner semicircle distribution

$$\rho_0(x) = \frac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2} \ , \qquad (13)$$

with maximal eigenvalue $\mu = \sqrt{\lambda}/2\pi$. Note that, since ρ_0 extremizes $\tilde{S}_0$, $\delta\tilde{S}_0|_{\rho_0} = 0$. Thus, our analysis only requires the knowledge of $S_1|_{\rho_0}$.

Since we want to compare to AdS/CFT, we also work at strong coupling, $\lambda \gg 1$. Note that $M \sim \sqrt{\lambda}$ and the typical eigenvalue contributing to the integral is of order $\mu \sim \sqrt{\lambda}$. So the arguments of H are large, validating the use of the asymptotic expansion of the log derivatives

$$H'(x_\pm)/H(x_\pm) = -x_\pm \log x_\pm^2 + 2x_\pm + \mathcal{O}(x_\pm^{-1}) \ , \quad (14)$$

where $x_\pm = x \pm M$. When $|M| < \mu$, using the semicircle distribution and large-argument expansion is only justified outside of a region of width $1/\sqrt{\lambda}$ around $x = M$, where the hypers are parametrically light. But the contribution of that region is negligible at large λ. Consequently, the flavor contribution is

$$F' = \frac{\zeta N_c^2}{2} \int_{-\mu}^{\mu} dx \, \rho_0(x) \left[4M - x_+ \log x_+^2 + x_- \log x_-^2\right] , \qquad (15)$$

where $F' = dF/dM$. Integrating explicitly in the regime where $\mu < M$, we find

$$\begin{aligned} F' = \frac{\zeta N_c^2}{3\mu^2}\Big[&-2M^3 + 2\sqrt{M^2-\mu^2}(M^2+2\mu^2) \\ &+ 3M\mu^2\Big(1 - 2\log\frac{M+\sqrt{M^2-\mu^2}}{2}\Big)\Big]. \end{aligned} \qquad (16)$$

In the matrix model, phase transitions can occur when some of the hypers become light, as demonstrated e.g. in $\mathcal{N} = 2^*$ in [21]. That is, for $M \leq \mu$ there can be resonances driving the hypers effectively massless. Zooming in on the potential phase transition point, $M = (1+\epsilon)\mu$ and expanding for $\epsilon \ll 1$, we find

$$\begin{aligned} \frac{dF/dm}{N_c^2\mu^2\zeta} &= \frac{1}{3} - \log\frac{\mu^2}{4} - \epsilon\Big(1 + \log\frac{\mu^2}{4}\Big) \\ &\quad - 2\epsilon^2 + \frac{16\sqrt{2}}{15}\epsilon^{5/2} + \mathcal{O}(\epsilon^3) \ . \end{aligned} \qquad (17)$$

This expansion reproduces the non-analytic behavior and matches exactly the coefficients of all the $\epsilon^{(2n+1)/2}$ terms that were seen on the gravitational side in (8), which is strong evidence that the topology changing phase transition is captured by the transition associated with bringing hypers on to the eigenvalue distribution.

If we set the remaining scheme dependent counterterm to $\beta = -\frac{5}{2} + \frac{3}{2}\log\frac{\mu}{2}$, we can exactly match the holographic result (6) with (5), (7) to (16) for $\mu < M$:

$$\begin{aligned} \frac{V_4}{\zeta\mu N_c^2}\langle\mathcal{O}_s\rangle &= -\frac{2m^3}{3} + \frac{2}{3}\sqrt{m^2-1}(m^2+2) \\ &+ m\left[1 + 2\log\frac{2(m-\sqrt{m^2-1})}{\mu}\right] = \frac{F'}{\zeta\mu N_c^2} \, . \end{aligned} \qquad (18)$$

Performing the same calculation of F' for $M < \mu$, where the hyper mass is on the eigenvalue distribution [18], again yields a perfect match to the holographic result:

$$V_4\langle\mathcal{O}_s\rangle = m\mu\zeta N_c^2\left(1 - \frac{2}{3}m^2 - 2\log\frac{\mu}{2}\right) = F' \ . \quad (19)$$

IV. DISCUSSION

We have studied the phase structure of $\mathcal{N}=4$ SYM coupled to massive $\mathcal{N}=2$ flavor hypermultiplets on S^4, using holography and direct QFT computations independently. The crucial ingredients to allow for localization of the path integral on the field theory side are the preservation of some supersymmetry and the formulation on a compact space. Holographically, this translates to the supersymmetry of the D7-brane embeddings we derived in [9]. We found one continuous phase transition at the same value of the flavor mass in both calculations, and analytically matching dF/dM in both phases. The remaining constant, which enters when this relation is integrated to get the free energies, is scheme dependent. So matching the free energies themselves then merely amounts to choosing compatible renormalization schemes.

Our results give strong support to the validity of the probe brane constructions used so frequently in AdS/CFT. The theories we studied are non-conformal, and the quantities we compared are not special, in the sense that they are not extrapolated from weak to strong coupling using non-renormalization theorems. Moreover, the theory described by the D3/D7 setup has a non-trivial UV fixed point only in the quenched approximation, which frequently means that extra care is needed when establishing the validity of holographic results. The fact that we found such nicely matching results therefore truly provides a non-trivial test of the dualities.

Possible directions for future research include tests for other probe brane systems like D3/D5 [24], using localization on S^4 with defect hypers [25, 26], or the computation of superconformal indices.

ACKNOWLEDGMENTS

We thank Tyson Price for contributions in the early phase of this work. The work of AK and BR was supported, in part, by the US Department of Energy under grant number DE-SC0011637. CFU is supported by *Deutsche Forschungsgemeinschaft* through a research fellowship.

[1] J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik, and I. Kirsch, Phys. Rev. **D69**, 066007 (2004), arXiv:hep-th/0306018 [hep-th].
[2] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005), arXiv:hep-th/0412141 [hep-th].
[3] A. Karch and A. O'Bannon, JHEP **09**, 024 (2007), arXiv:0705.3870 [hep-th].
[4] V. Pestun, Commun.Math.Phys. **313**, 71 (2012), arXiv:0712.2824 [hep-th].
[5] N. Bobev, H. Elvang, D. Z. Freedman, and S. S. Pufu, JHEP **1407**, 001 (2014), arXiv:1311.1508 [hep-th].
[6] A. Buchel, J. G. Russo, and K. Zarembo, JHEP **1303**, 062 (2013), arXiv:1301.1597 [hep-th].
[7] A. Buchel, JHEP **1308**, 004 (2013), arXiv:1304.5652 [hep-th].
[8] A. Karch and E. Katz, JHEP **06**, 043 (2002), arXiv:hep-th/0205236 [hep-th].
[9] A. Karch, B. Robinson, and C. F. Uhlemann, (2015), arXiv:1508.06996 [hep-th].
[10] J. Russo and K. Zarembo, JHEP **1311**, 130 (2013), arXiv:1309.1004 [hep-th].
[11] D. J. Gross and E. Witten, Phys. Rev. **D21**, 446 (1980).
[12] E. Bergshoeff and P. K. Townsend, Nucl. Phys. **B490**, 145 (1997), arXiv:hep-th/9611173 [hep-th].
[13] M. Cederwall, A. von Gussich, B. E. Nilsson, and A. Westerberg, Nucl.Phys. **B490**, 163 (1997), arXiv:hep-th/9610148 [hep-th].
[14] M. Cederwall, A. von Gussich, B. E. Nilsson, P. Sundell, and A. Westerberg, Nucl.Phys. **B490**, 179 (1997), arXiv:hep-th/9611159 [hep-th].
[15] E. Bergshoeff, R. Kallosh, T. Ortin, and G. Papadopoulos, Nucl. Phys. **B502**, 149 (1997), arXiv:hep-th/9705040 [hep-th].
[16] G. Festuccia and N. Seiberg, JHEP **06**, 114 (2011), arXiv:1105.0689 [hep-th].
[17] M. Kruczenski, D. Mateos, R. C. Myers, and D. J. Winters, JHEP **0307**, 049 (2003), arXiv:hep-th/0304032 [hep-th].
[18] See Supplemental Material at [URL], which includes Refs. [27, 28], for holographic renormalization and matrix model calculations as well as a figure of the slipping modes and free energy.
[19] A. Karch, A. O'Bannon, and K. Skenderis, JHEP **04**, 015 (2006), arXiv:hep-th/0512125 [hep-th].
[20] H.-C. Chang and A. Karch, JHEP **1401**, 180 (2014), arXiv:1307.5325 [hep-th].
[21] J. G. Russo and K. Zarembo, JHEP **1304**, 065 (2013), arXiv:1302.6968 [hep-th].
[22] A. Karch, A. O'Bannon, and L. G. Yaffe, JHEP **0909**, 042 (2009), arXiv:0906.4959 [hep-th].
[23] N. Nekrasov and A. Okounkov, (2003), arXiv:hep-th/0306238 [hep-th].
[24] A. Karch and L. Randall, JHEP **06**, 063 (2001), arXiv:hep-th/0105132 [hep-th].
[25] O. DeWolfe, D. Z. Freedman, and H. Ooguri, Phys. Rev. **D66**, 025009 (2002), arXiv:hep-th/0111135 [hep-th].
[26] J. Lamy-Poirier, (2014), arXiv:1412.0530 [hep-th].
[27] S. de Haro, S. N. Solodukhin, and K. Skenderis, Commun. Math. Phys. **217**, 595 (2001), arXiv:hep-th/0002230 [hep-th].
[28] M. Bianchi, D. Z. Freedman, and K. Skenderis, Nucl. Phys. **B631**, 159 (2002), arXiv:hep-th/0112119 [hep-th].