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Phys. Rev. Lett. **115**, 261101 — Published 23 December 2015

DOI: [10.1103/PhysRevLett.115.261101](https://doi.org/10.1103/PhysRevLett.115.261101)

Testing Einstein's Equivalence Principle with Fast Radio Bursts

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The accuracy of Einstein's Equivalence Principle (EEP) can be tested with the observed time delays between correlated particles or photons that are emitted from astronomical sources. Assuming as a lower limit that the time delays are caused mainly by the gravitational potential of the Milky Way, we prove that fast radio bursts (FRBs) of cosmological origin can be used to constrain the EEP with high accuracy. Taking FRB 110220 and two possible FRB/GRB association systems (FRB/GRB 101011A and FRB/GRB 100704A) as examples, we obtain a strict upper limit on the differences of the parameterized post-Newtonian (PPN) parameter γ values as low as $[\gamma(1.23 \text{ GHz}) - \gamma(1.45 \text{ GHz})] < 4.36 \times 10^{-9}$. This provides the most stringent limit up to date on the EEP through the relative differential variations of the γ parameter at radio energies, improving by one to two orders of magnitude the previous results at other energies based on supernova 1987A and GRBs.

PACS numbers: 04.80.Cc, 95.30.Sf, 98.70.Dk, 98.70.Rz

INTRODUCTION

The Einstein Equivalence Principle (EEP) is an important foundation of general relativity and many other metric theories of gravity. At the post-Newtonian level, the accuracy of the EEP can be tested through the numerical values of the parameterized post-Newtonian (PPN) parameters, such as the parameter γ (see, e.g., Refs. [1, 2]). Specifically, the EEP accuracy can be constrained by comparing the γ values for different kinds of particles, or for the same kind of particle with different energies, since all gravity theories satisfying the EEP predict $\gamma_1 = \gamma_2 \equiv \gamma$, where the subscripts denote two different test particles.

There are a few precise tests of the EEP using constraints on the differences of the γ values of different tested particles. Among the most famous are the measurements of the time delay of the photons and neutrinos radiated from supernova 1987A in the Large Magellanic Cloud [3, 4]. Recently it was shown [2] that the EEP can also be tested using the time delay of photons with different energies arising in cosmic transients, such as gamma-ray bursts (GRBs) [5]. With the assumption

that the time delays are mainly due to the Milky Way's gravity, it was found [2] that the value of γ is identical for photons over an energy range between eV and MeV and between MeV and GeV to within approximately 10^{-7} , which represents an improvement of at least one order of magnitude compared with the previous limits.

Recently a new type of millisecond radio burst transients, named fast radio bursts (FRBs), has attracted wide attention [7, 8]. Following the first report of an FRB by Ref. [7], a number of further FRBs have been reported, with a present total of over ten cases [8, 9]. Most of these bursts are located at high galactic latitudes and have anomalously large dispersion measures (DM). The observed event rate is predicted to be $\sim 10^{-3} \text{ gal}^{-1} \text{ yr}^{-1}$. Moreover, the higher frequency components of an FRB arrive earlier than their low frequency counterparts, the arrival time delay at a given frequency ν following a ν^{-2} law [8, 10]. Based on these typical characteristics, it has been suggested that these sources may originate at cosmological distances, corresponding to redshifts z of 0.5 to 1. If so, the isotropic total energy released in one FRB is inferred to be $\sim 10^{38-40} \text{ erg}$, and the peak radio luminosity is estimated to be $\sim 10^{42-43} \text{ erg s}^{-1}$ [8]. However, be-

cause there are no clear-cut electromagnetic counterparts at other wavelengths detected, and no progenitors have been identified, the physical nature of FRBs remains under debate. Many possible explanations for FRBs have been proposed, including magnetar flares [11], neutron star mergers [12], white dwarf mergers [13], collapsing super-massive neutron stars [14, 15], companions of extragalactic pulsars [16], asteroid collisions with neutron stars [17], quark nova [18], and dark matter-induced collapse of neutron stars [19]. All of these models considered FRBs as extragalactic burst sources. It is worth noting that some other models, which suggested a Galactic origin for FRBs, have also been proposed, e.g., galactic flare stars [20] and atmospheric phenomenon [21].

If FRBs are indeed confirmed to be cosmological origin, they will be a useful cosmic probe. For example, they can be used to determine the baryon mass density of the Universe [22], and they could also be used to constrain cosmological parameters and the dark energy [23, 24]. Here we propose that cosmic originated FRBs are also good candidates for constraining the EEP, which could further expand the scope of the tested EEP energy range out to the radio band with high accuracy.

Compared to the prospects of GRBs in constraining the EEP, the FRBs have two advantages. Firstly, the simple sharp feature of the FRB signal allow us to easily derive the observed time delay between different frequencies, and their time lags are usually shorter than that of most GRBs. The value of the time delay plays an important role in constraining the EEP, a smaller time delay leading to better constraints on the EEP. Secondly, although GRBs are multi-wavelength transients, it is hard to measure in them the arrival time lag in the radio band. A more promising way to further extend the EEP tested energy range down to the radio band is with the help of FRBs. **In addition, if a fraction of the FRBs originate from the delayed collapse of supermassive millisecond magnetars to black holes [14], it has been suggested that these sources could be associate with some GRBs [15]. In this case, the GRB location provides information additional to that of the FRB time delay.** In this paper, we extend the work of Ref. [2] by presenting stronger constraints on the EEP using FRBs.

METHOD OF TESTING THE EEP

For a cosmic transient source, the observed time delay between two different energy bands should include five terms [2]:

$$\Delta t_{\text{obs}} = \Delta t_{\text{int}} + \Delta t_{\text{LIV}} + \Delta t_{\text{spe}} + \Delta t_{\text{DM}} + \Delta t_{\text{gra}} . \quad (1)$$

Δt_{int} is the intrinsic (astrophysical) time delay between two test photons, Δt_{LIV} is the time delay caused by the

effect of Lorentz invariance violation, and Δt_{spe} represents the potential time delay due to special-relativistic effects in the case where the photons have a rest mass which is non-zero. **Δt_{DM} is the time delay contribution from the dispersion by the line-of-sight free electron content, which is non-negligible especially for low energy photons, such as the radio signals considered.** Δt_{gra} corresponds to the difference in arrival time of two photons of energy E_1 and E_2 , caused by the gravitational potential $U(r)$ integrated from the emission site to the Earth, which reads

$$\Delta t_{\text{gra}} = \frac{\gamma_1 - \gamma_2}{c^3} \int_{r_o}^{r_e} U(r) dr , \quad (2)$$

where r_e and r_o are locations of source and observation, γ is the PPN parameter. For the purposes of this work, both Δt_{LIV} and Δt_{spe} are negligible, we thus ignore them in our analysis (see Ref. [2], for more explanations). Leaving out the negligible components, and assuming that $\Delta t_{\text{int}} > 0$, we have

$$\Delta t_{\text{obs}} - \Delta t_{\text{DM}} > \frac{\gamma_1 - \gamma_2}{c^3} \int_{r_o}^{r_e} U(r) dr . \quad (3)$$

For a cosmic transient, in principle, $U(r)$ has contributions from the gravitational potential of the Milky Way $U_{\text{MW}}(r)$, the intergalactic potential $U_{\text{IG}}(r)$ between the transient host galaxy and the Milky Way, and the potential of the host galaxy $U_{\text{host}}(r)$. The potential models for $U_{\text{IG}}(r)$ and $U_{\text{host}}(r)$ are poorly known, but it is very likely that the effect of these two terms is much larger than if we simply assumed that the potential is just $U_{\text{MW}}(r)$ extended to the distance of the transient. Adopting the Keplerian potential for our galaxy, we have

$$\gamma_1 - \gamma_2 < (\Delta t_{\text{obs}} - \Delta t_{\text{DM}}) \left(\frac{GM_{\text{MW}}}{c^3} \right)^{-1} \ln^{-1} \left(\frac{d}{b} \right) , \quad (4)$$

where $M_{\text{MW}} \simeq 6 \times 10^{11} M_{\odot}$ is the Milky Way mass [25], d is the distance from the source to the Earth, and b is the impact parameter of the light rays relative to the Milky Way center [26].

TESTS OF THE EEP USING FRBS

As mentioned above, if FRBs are proven to be of cosmological origin and if their distances can in the future be accurately measured, FRBs will be a new powerful tool for obtaining EEP constraints, while extending the tested energy range down to the radio band.

The single-dish telescope used to detect all but one of the currently known FRBs localizes these sources to about 0.25 degrees [8], and hence electromagnetic counterparts, if any, such as GRBs,

may be critical to determine distances and thereafter analyze source energetics and estimate event rates [15, 27].

So far, there are two main methods to estimate the distances of FRBs. The first method (Method 1) is to directly estimate the redshifts of the FRBs through their DM values (e.g., Ref. [8]). In this method, some delicate assumptions are adopted, which can result in a large uncertainty for the source distance. For example, to estimate the DM value, $\Delta t_{\text{obs}} \approx \Delta t_{\text{DM}}$ is assumed. However, if the main contribution to Δt_{obs} is from Δt_{gra} instead of Δt_{DM} , this method would severely overestimate the source distance. On the other hand, to connect the cosmic distances of FRBs with their DM values, the contributions of the DM dispersion from the source environment and host galaxy are assumed to be negligible compared to the intergalactic medium (IGM) component. If this is not the case, the source distance would also be overestimated. Finally, to calculate the distance with the IGM DM value, one needs to assume that the average IGM DM value in all directions for a given z is well defined by known cosmological parameters [22, 23]. The uncertainty of the relevant cosmological parameters could cause either under- or over- estimation of the source distance.

The second method (Method 2) is to make use of the redshifts of possible FRB/GRB association systems. In this method, the FRB distance can be precisely estimated when the associated GRB has a direct redshift measurement (from the GRB afterglows or the GRB host galaxies). If not, some empirical luminosity relations, such as the Amati relation [32], may be used to give a rough range of pseudo-redshifts for the FRB/GRB system (e.g., Ref. [22]).

Here we take the following two examples, FRB 110220 (one of the brightest bursts, with the clearest frequency-dependent delays) and two potential FRB/GRB association systems (FRB/GRB 101011A and FRB/GRB 100704A), and use these to constrain the EEP.

FRB 110220

In a recent search for pulsars, four FRBs have been discovered by the 64-m Parkes multibeam radio telescope [8]. FRB 110220 is one of the brightest bursts, with a flux ~ 2.5 Jy (at 1.5 GHz), and it was detected at $T_0 = 01 : 55 : 48.957$ UT, February 20th, 2011, with coordinates (J2000) R.A. = $22^{\text{h}}34^{\text{m}}$, Dec. = $-12^{\circ}24'$. **From the frequency-dependent delay of FRB 110220 (see Figure 2 of [8]), one can easily identify the arrival time delay $\Delta t_{\text{obs}} \sim 1$ s for photons ranging in frequency from about 1.5 GHz to 1.2 GHz.**

With Method 1, this source is inferred to be at a redshift $z_{\text{infer}} \sim 0.81$.

With the above information, from Equation (4) a severe limit on the EEP is

$$[\gamma(1.2 \text{ GHz}) - \gamma(1.5 \text{ GHz})] < 2.52 \times 10^{-8} \quad (5)$$

for FRB 110220, which is almost 10^2 times tighter than the constraints from supernova 1987A, and is as good as the results on GRBs from Ref. [2]. Note that this is a relatively conservative upper limit: we assume $\Delta t_{\text{obs}} \approx \Delta t_{\text{DM}}$ when estimating the redshift (which is inferred from the characteristic observational feature of FRBs, i.e., that the arrival time delay at a given frequency ν follows a ν^{-2} law), but we take $\Delta t_{\text{obs}} \gg \Delta t_{\text{DM}}$ for a stringent limit on the EEP. For completeness, we also tested two more cases by assuming $\Delta t_{\text{DM}} = 0.001\Delta t_{\text{obs}}$ and $\Delta t_{\text{DM}} = 0.999\Delta t_{\text{obs}}$. As shown in Figure 1, much more severe constraints would be achieved ($\sim 10^{-11}$) if the effects of the dispersion process represent 99.9% of Δt_{obs} . Even if they represent only 0.1% of Δt_{obs} , the limits we derived here are still about $\sim 10^{-8}$.

To account for the source distance uncertainty, we also test the results by varying the source distance from 1 Mpc (the edge distance of the Local Group) to $3z_{\text{infer}}$. As shown in Figure 1, we find that even if the distance estimation for FRBs have large uncertainties, the implications of the FRB tests of the EEP are not greatly affected, e.g., the constraint results vary within one order of magnitude.

FRB/GRB systems

Recently, two possible associations of FRBs with GRB 101011A and GRB 100704A have been proposed by Ref. [28]. If such FRB/GRB association systems are commonly discovered, the combination of redshifts measured from GRBs and DMs measured from FRBs not only opens a new window to study cosmology [22, 23], but also makes them an interesting tool for EEP constraints (more on this below).

For these two FRB/GRB association systems, one can in principle use their location information from the GRB observations. GRB 101011A was detected and localized by *Swift*/BAT at $T_0 = 16 : 58 : 35$ UT, October 11th, 2010, with coordinates (J2000) R.A. = $03^{\text{h}}13^{\text{m}}12^{\text{s}}$, Dec. = $-65^{\circ}59'08''$ [29]. At $T_0 = 03 : 35 : 06.10$ UT on 04 July 2010, the *Fermi* Gamma-Ray Burst Monitor triggered and located GRB 100704A [30], which was also detected by *Swift*/BAT with coordinates (J2000) R.A. = $08^{\text{h}}54^{\text{m}}33^{\text{s}}$, Dec. = $-24^{\circ}12'55''$ [31]. Unfortunately, neither of these two GRBs had a redshift mea-

surement. Ref. [22] used the Amati relation to estimate the redshifts of these two FRB/GRB association systems: $z \geq 0.246$ for GRB 101011A and $z \geq 0.166$ for GRB 100704A. Ref. [28] observed these two systems over a 220 MHz bandwidth, with a highest frequency of $\nu_h = 1.45$ GHz and lowest frequency of $\nu_l = 1.23$ GHz. The delay times between these two frequencies are $\Delta t_{\text{obs}} = 0.149$ s for FRB/GRB 100704A and $\Delta t_{\text{obs}} = 0.438$ s for FRB/GRB 101011A.

With the inferred redshifts for the two FRB/GRB systems (here we adopt $z_{\text{infer}} = 0.246$ for GRB 101011A and $z_{\text{infer}} = 0.166$ for GRB 100704A) and assuming $\Delta t_{\text{obs}} \gg \Delta t_{\text{DM}}$, a stringent limit on the EEP from Equation (4) is

$$[\gamma(1.23 \text{ GHz}) - \gamma(1.45 \text{ GHz})] < 1.24 \times 10^{-8} \quad (6)$$

for GRB 101011A, and

$$[\gamma(1.23 \text{ GHz}) - \gamma(1.45 \text{ GHz})] < 4.36 \times 10^{-9} \quad (7)$$

for GRB 100704A.

We also tested these two results by assuming $\Delta t_{\text{DM}} = 0.001\Delta t_{\text{obs}}$ and $\Delta t_{\text{DM}} = 0.999\Delta t_{\text{obs}}$, and by varying the source distance from the edge distance of the Local Group to $3z_{\text{infer}}$. We find that considering these uncertainties, the limits on the EEP stay at the level of $\sim 10^{-8}$, and much more severe constraints could be achieved if it turns out that the effects of dispersion process dominate the observed time delay. Note also that the EEP test energy range could in principle be extended further by using the measured time difference between the GRB itself and the FRB, although this would involve additional astrophysical uncertainties, which we do not consider here.

SUMMARY AND DISCUSSION

The accuracy of the EEP can be characterized by constraints on the differences in PPN parameters, such as the parameter γ , for different kinds of particles, or for the same kind of particle with different energies. Following the method of Ref. [2], we prove that FRBs, if cosmological, can be used to test the accuracy of the EEP, leading to 1-2 orders of magnitudes stricter limits than previously.

From the arrival time delay $\Delta t \sim 1$ s for photons ranging in frequency from about 1.2 GHz to 1.5 GHz, assuming that the observed time delay is caused mainly by the gravitational potential of the Milky Way and adopting the inferred redshift either based on DM measurement or based on associated GRB observations, we obtain stringent limits on the differences of the γ values for three FRB cases:

$[\gamma(1.2 \text{ GHz}) - \gamma(1.5 \text{ GHz})] < 2.52 \times 10^{-8}$ for FRB 110220, $[\gamma(1.23 \text{ GHz}) - \gamma(1.45 \text{ GHz})] < 1.24 \times 10^{-8}$ for FRB/GRB 101011A and $[\gamma(1.23 \text{ GHz}) - \gamma(1.45 \text{ GHz})] < 4.36 \times 10^{-9}$ for FRB/GRB 100704A.

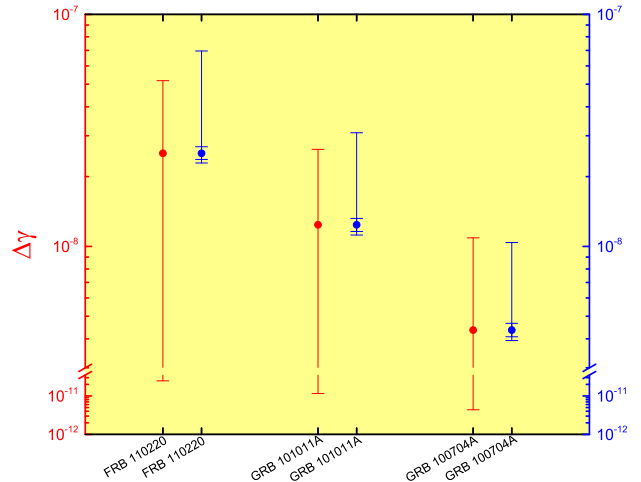


FIG. 1: Limits on the differences of the γ values for three FRB observations. The dots represent the conservative upper limits on $\Delta\gamma$ by taking the inferred redshifts from Method 1 or 2, and assuming $\Delta t_{\text{obs}} \gg \Delta t_{\text{DM}}$. The red lines show the test results from considering different Δt_{DM} contributions. From top to bottom, the red error bars correspond to the case of $\Delta t_{\text{DM}} = 0.001\Delta t$ and $\Delta t_{\text{DM}} = 0.999\Delta t$, respectively. The constraints on $\Delta\gamma$ from varying the values of redshifts are presented as the blue lines. From top to bottom, the blue error bars correspond to the cases of 1 Mpc, $0.5z$, $2z$, and $3z$, respectively.

Previously, by analysing the photons and neutrinos from supernova 1987A, Ref. [4] set a severe limit on γ differences of 0.34% for optical photons (eV) and neutrinos (MeV), and a more precise limit of 1.6×10^{-6} for two neutrinos with different energies (7.5 – 40 MeV). Very recently, Ref. [2] proposed that this EEP parameter can also be tested with the time delays between correlated photons from GRBs. Compared with previous limits, their constraint on the accuracy of the EEP of $\sim 10^{-7}$ represented an improvement of at least one order of magnitude, extending also the tested energy range to the eV-MeV and MeV-GeV range. In the present paper, using the sharp features of the FRB radio signals, we have extended the energy range over which the EEP is tested to the radio band at comparable or higher levels of accuracy, and we have obtained the most stringent limit to date on the EEP, namely $\sim 10^{-8}$, which represents an improvement of one to two orders of magnitude over the results previously obtained at other energies using supernova 1987A and GRBs.

Note that this is a conservative upper limit: the inclusion of contributions from the neglected potentials in

Equation (1) could only make these limits even more stringent. **On the other hand, if the time delay between different frequencies is mainly contributed by the dispersion process by the line-of-sight free electron content, much more severe constraints could be achieved (e.g., $\Delta\gamma \sim 10^{-11}$) if the effects of dispersion process represent 99.9% of Δt_{obs}).** It is worth noting also that large uncertainties in the source distance would not significantly affect our conclusions, as long as the FRBs originate outside of the Local Group.

The impact of the results presented in this work is expected to increase significantly as more FRBs are observed, and if their origin is more firmly established. Based on current observations, the FRB event rate is estimated to be as high as $\sim 10^4 \text{ sky}^{-1} \text{ day}^{-1}$ [8]. The current low detection rate of FRBs may be due to the lack of either the necessary high time resolution or a wide enough field of view in the current telescopes. Fortunately, the upcoming radio transient surveys such as the Square Kilometer Array, with much wider field of view and higher sensitivity, will be able to discover and precisely localize more and more FRBs. With more abundant observational information in the future, we will have a better understanding of the physical nature of FRBs, and we will also know better how to use them as a probe of the cosmic web, the host galaxies, the intergalactic medium and to test fundamental physics, as discussed in this Letter.

Acknowledgements: We are grateful to the anonymous referees for insightful comments, and to K. W. Bannister for helpful communications. This work is partially supported by the National Basic Research Program (“973” Program) of China (Grants 2014CB845800 and 2013CB834900), the National Natural Science Foundation of China (grants Nos. 11322328 and 11433009), the One-Hundred-Talents Program, the Youth Innovation Promotion Association (2011231), the Strategic Priority Research Program “The Emergence of Cosmological Structures” (Grant No. XDB09000000) of the Chinese Academy of Sciences, and NASA NNX13AH50G.

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