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Cooling Atomic Gases With Disorder

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Cold atomic gases have proven capable of emulating a number of fundamental condensed matter phenomena including Bose-Einstein condensation, the Mott transition, Fulde-Ferrell-Larkin-Ovchinnikov pairing and the quantum Hall effect. Cooling to a low enough temperature to explore magnetism and exotic superconductivity in lattices of fermionic atoms remains a challenge. We propose a method to produce a low temperature gas by preparing it in a disordered potential and following a constant entropy trajectory to deliver the gas into a non-disordered state which exhibits these incompletely understood phases. We show, using quantum Monte Carlo simulations, that we can approach the Néel temperature of the three-dimensional Hubbard model for experimentally achievable parameters. Recent experimental estimates suggest the randomness required lies in a regime where atom transport and equilibration are still robust.

Introduction: The interplay of disorder and interactions is a central problem in condensed matter physics, both from the viewpoint of materials like the heavy fermions [1, 2], high-temperature superconductors [3], and manganites [4], and also because of intriguing theoretical issues such as the fate of Anderson localization in the presence of interactions, especially in two dimensions [5, 6]. Ultracold atomic gases offer the opportunity to emulate these fundamental issues using optical speckle [7, 8], impurities [9], or a quasiperiodic optical lattice [10, 11] to introduce randomness. In the bosonic case, the competition between strong interactions and strong disorder has been studied in the context of the elusive Bose glass phase [7, 9, 11], while for fermions, a recent experiment has explored disorder-induced localization in the three-dimensional (3D) Hubbard model of strongly-interacting fermions [12].

In this paper, we explore the thermodynamics of interacting, disordered systems and suggest that, in addition to studies of the many-body phenomena noted above, preparing a gas in a random potential might be exploited to cool the atoms. Specifically, we show using an unbiased numerical method that one can lower the temperature and access the regime with long-range magnetic order by adiabatically decreasing the randomness in the chemical potential or hopping energies of the Hubbard Hamiltonian. The achievement of new quantum phases in cold atom experiments largely depends on the reduction of the entropy per particle. The success of our proposal requires that the gas would have to be cooled (eg evaporatively) *after* the disorder is in place. We will return in the conclusions to a discussion of how our approach can be implemented in practice.

Results for the double occupancy and

antiferromagnetic structure factor lend physical insight into this effect. We also present arguments, partially based on recent experiments, that our suggestion is achievable in practice.

We consider the disordered Hubbard Hamiltonian,

$$H = - \sum_{\langle ij \rangle \sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + U \sum_i (n_{i\uparrow} - \frac{1}{2})(n_{i\downarrow} - \frac{1}{2}) - \sum_i \mu_i (n_{i\uparrow} + n_{i\downarrow}) \quad (1)$$

whose emulation [13, 14] with optical lattices is possible using two hyperfine species of fermionic atoms. Here $c_{i\sigma}^\dagger (c_{i\sigma})$ is the creation (destruction) operator for a fermion at spatial site i and spin (or hyperfine state) σ . We consider a cubic lattice of N sites, and hopping t_{ij} between near neighbors $\langle ij \rangle$. The hopping, and the on-site repulsion U , can be tuned with the lattice depth and the Feshbach resonance [13], allowing for the successful exploration of the Mott transition [15–17].

Disorder is introduced via a spatially random chemical potential μ_i or hopping t_{ij} . We choose uniform distributions $\mu_0 - \Delta_\mu < \mu_i < \mu_0 + \Delta_\mu$ or $t_0 - \Delta_t < t_{ij} < t_0 + \Delta_t$, and set the mean of the hopping energy $t_0 = 1$ as the energy scale. For most of this paper we choose $\mu_0 = 0$, which makes the lattice half-filled (average density $n = 1$). However, we also gain insight into the effects of a confining potential, in which the chemical potential increases as one moves spatially away from the trap center, by presenting data for different densities.

Our computational method, determinant quantum Monte Carlo (QMC) [18, 19], treats disorders and interactions on an equal, exact footing, and provides a solution to the Hubbard Hamiltonian on lattices of finite spatial size, when the sign problem isn't too

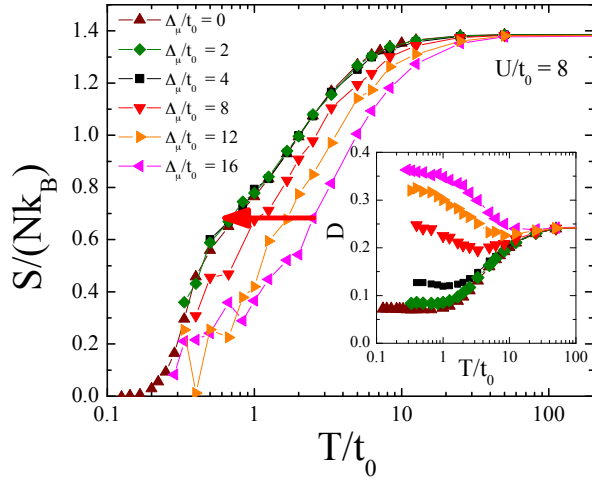


FIG. 1: (Color online) Entropy per site S/N as a function of T/t_0 for different site disorder strengths Δ_μ at $U/t_0 = 8$. S is largely independent of disorder strength for $\Delta_\mu/t_0 = 2, 4 \lesssim U/t_0 = 8$. For larger randomness, $S(T)$ decreases with Δ_μ so that if disorder is turned off adiabatically, the temperature T decreases, as indicated by the horizontal arrow. The inset shows the double occupancy $D(T)$. Large disorder Δ_μ changes the sign of the slope dD/dT from mostly positive, to mostly negative. Here, and in all subsequent figures, unless otherwise indicated, the lattice size is 6^3 , $n = 1$, and the Trotter discretization is $\Delta\tau = 1/(20t_0)$. Up to 300 disorder realizations are used in the disorder averages.

serious [20–24]. We focus on the disorder dependence of the entropy $S(T)$, obtained via a thermodynamic integration of the energy [25] down from $T = \infty$. We also report results for the (site-averaged) double occupancy $D = 1/N \sum_i \langle n_{i\uparrow} n_{i\downarrow} \rangle$, and the structure factor $S_{\mathbf{q}} = \sum_{\mathbf{r}} e^{i\mathbf{q} \cdot \mathbf{r}} c(\mathbf{r})$ at $\mathbf{q} = (\pi, \pi, \pi)$; S_π , where $c(\mathbf{r}) = \langle c_{i+\mathbf{r}\downarrow}^\dagger c_{i+\mathbf{r}\uparrow}^\dagger c_{i\uparrow}^\dagger c_{i\downarrow} \rangle$ are spin-spin correlation functions.

Results: The effect of site disorder on $S(T)$ is shown in Fig. 1 at $U/t_0 = 8$, where the Néel transition temperature (T_N) in the homogeneous 3D Hubbard model attains its maximal value [26]. $S(T)$ is largely unaffected by disordered site energies until Δ_μ becomes comparable to U . This is a consequence of the fact that for temperatures less than the repulsion U , the Hubbard model has the character of a Mott insulator in which U blocks transport of Fermions away from singly occupied sites. Such a Mott state is immune to the effects of small disorder $\Delta_\mu/U \lesssim 1$. Our calculated entropy $S(T)$ and double occupancy $D(T)$ (see the inset of Fig. 1) confirm this picture. However, when $\Delta_\mu/U \gtrsim 1$ the entropy curves shift systematically to higher T , reflecting a disorder-driven decrease in S at constant T . The reduction in S can be viewed as the transfer of weight in the specific heat $C(T)$ to a higher temperature: Disorder suppressing the peak in $C(T)$ associated with local magnetic ordering at the exchange energy scale $T \sim J = 4t_0^2/U$, and increasing

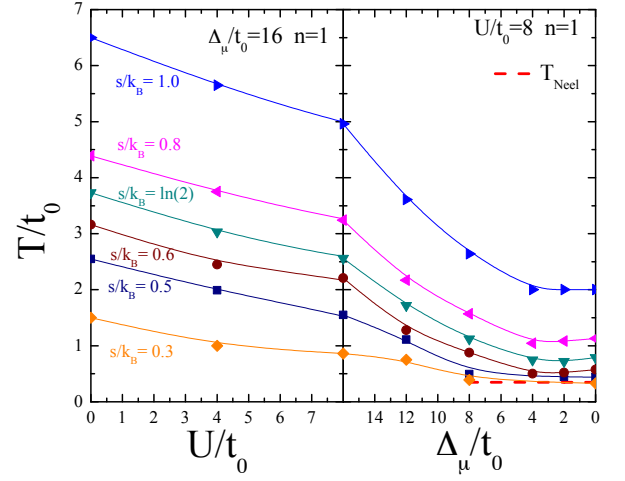


FIG. 2: (Color online) Adiabats of the disordered 3D Hubbard model along a path which combines an increase of the interaction strength from $U/t_0 = 0$ to $U/t_0 = 8$ at fixed $\Delta_\mu/t_0 = 16$ followed by a reduction of the site disorder. T/t_0 decreases along both trajectories, and, in particular, by about a factor of three at fixed $S/(Nk_B) \equiv s/k_B = \ln 2$ along the second path. For $s/k_B = 0.5$, the same reduction brings T down to near T_N .

$C(T)$ at a higher T that scales like Δ_μ due to excitations arising from the transfer of charge between sites of different local μ_i [27]. It is expected that at very low temperatures, the disorder increases the degeneracy of the low lying states, and hence the entropy. However, our results indicate that in the temperature range of interest, $T \gtrsim T_N$ (the Néel temperature), disorder reduces S .

The family of $S(T)$ curves in Fig. 1 indicates that if Δ_μ is switched to zero at constant entropy, the temperature T decreases, in analogy to Pomeranchuk cooling which occurs in a non-disordered lattice when the ratio of repulsion to hopping U/t_0 is increased adiabatically [40]. For the case of site disorder, the double occupancy shows a negative slope $dD/dT < 0$ as seen in the inset of Fig. 1. At high enough temperatures $T \gtrsim t_0, U, \Delta_\mu$, up and down spin fermions are uncorrelated, and D factorizes, $D = \langle n_{i\uparrow} n_{i\downarrow} \rangle \rightarrow \langle n_{i\uparrow} \rangle \langle n_{i\downarrow} \rangle$ ($= 1/4$ at half-filling). In the clean limit, as T is lowered, the on-site repulsion eliminates double occupancy, and D falls. At finite U the presence of quantum fluctuations leads to a finite double occupancy even as $T \rightarrow 0$. Disordered site energies reduce the penalty for double occupancy from U to $U_{\text{eff}} = U - |\mu_i - \mu_j|$ so that as Δ_μ grows, U_{eff} becomes negative. The low T phase consists predominantly of doubly occupied and empty sites so that in the limit $\Delta_\mu/t_0 \gg 1$, D approaches $\frac{1}{2}$.

From Fig. 1 we can infer the behavior of T as Δ_μ/t_0 is lowered adiabatically at fixed U/t_0 . Optical lattice experiments, however, typically involve an increase of U/t_0 from zero to its final value. Figure 2 presents the adiabatic curves of a combined protocol in which the

interaction is increased from $U/t_0 = 0$ to $U/t_0 = 8$ in the presence of fixed disorder $\Delta_\mu/t_0 = 16$, followed by the suppression of the disorder to $\Delta_\mu/t_0 = 0$. Data are shown for different values of the starting entropy S/k_B . Figure 2 contains the central observation of our paper: a significant decrease in temperature results from following these adiabats. The substantial cooling in the second part of the path, at fixed U/t_0 , is implicit in Fig. 1. A reduction in T/t_0 also occurs in the initial turning on of the interaction, more so in the presence of disorder than occurs in the clean system [41, 42]. Our QMC results indicate that beginning at temperatures $T/t_0 \lesssim 2.5$ at $\Delta_\mu = 16t_0$ would be sufficient to reach T_N by the time the clean limit is reached. However, an important question arises: Can the trapped system in the presence of disorder be cooled down to an initial temperature $T/t_0 \sim 1.5$, or possibly even lower, close to what is initially needed for the clean system to reach the Neél phase ($T_N/t_0 \sim 0.35$) [42]. Current cooling capabilities have achieved a final temperature of $T/t_0 = 0.5$ ($1.4T_N$) for $U/t_0 \sim 11$ at the trap center [43]. We provide several suggestions concerning its feasibility in our concluding remarks.

Since random μ_i and t_{ij} occur together with optical speckles [44, 45], we also explore the case of bond disorder. Figure 3 shows $S(T)$ for nonzero Δ_t (and $\Delta_\mu = 0$). Significant disorder-induced cooling occurs. It is notable that $\Delta_t/t_0 \sim 1$ is sufficient to produce an effect on the entropy, whereas the scale of random site energies required to change S is much larger (Fig. 1). This is a consequence of the fact that random hopping immediately leads to a range of exchange energies $J_{ij} \sim 4t_{ij}^2/U$ which reduces the moment ordering. Random μ_i also smear J_{ij} but, since they are added to U in the energy denominator, initially have only a small effect. Random hopping thus offers cooling at lower temperature (entropy) scales for $\Delta_t \sim t_0$ than does random chemical potential, without requiring a ‘threshold value’, $\Delta_\mu > U$. Unlike for the chemical potential disorder, the basic structure of $D(T)$ remains unaltered for the clean system [27].

To provide some insight into possible effects of the inhomogeneous densities resulting from a confining potential, we show the entropy as a function of density for the clean system and for chemical potential disorder $\Delta_\mu/t_0 = 16$ and hopping disorder $\Delta_t/t_0 = 4$ in the inset of Fig. 3. Although there is some structure to the curves, entropy is systematically lowered for all densities as disorder is introduced. Thus disorder cooling is not a special feature of half-filling, but likely occurs for a broad range of densities.

We note that there are important questions of principle which would arise in a full treatment of a trap [41, 42]. QMC calculations for clean systems employed a set of homogeneous simulations, combined with the local density approximation (LDA), to understand

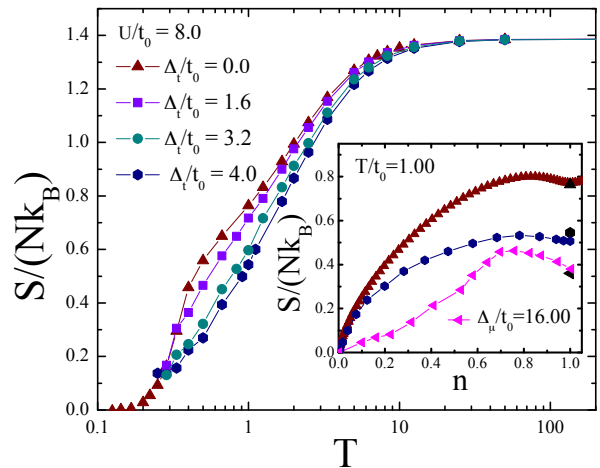


FIG. 3: (Color online) Entropy per site versus temperature for hopping disorder. Here, disorder cooling is strongest at lower entropies $s \sim 0.5$. The inset shows the entropy as a function of density of the clean system for $\Delta_\mu/t_0 = 16$ and $\Delta_t/t_0 = 4$ at fixed $U/t_0 = 8$ and $T/t_0 = 1$. Here, the entropy is obtained using $s(\mu, T) = \int_{-\infty}^{\mu} d\mu' \frac{\partial n}{\partial \mu'}|_{\mu}$ [46], except for the three data points in black (darker shade) at $n = 1$, which are obtained via integrating over β .

how the density, double occupancy, and entropy are inhomogeneously distributed in a system with smoothly varying chemical potential. This is a considerably harder task in the presence of disorder, because the implementation, and indeed even the validity, of the LDA is much less straightforward with a rapidly varying μ_i or t_{ij} . In fact, the LDA has the curious feature that thermodynamic properties are insensitive to the specific geometric organization of the sites with the different chemical potentials: The local entropy s_{μ_i} is unaltered for any two systems with the same collection $\{\mu_i\}$ whether they are randomly distributed or ordered spatially in some pattern, a patently unphysical result.

Further Analysis: Observing the onset of long-range antiferromagnetic (AF) correlations is a central goal of the field. To see the development of these correlations as the disorder is turned off, we show in Fig. 4 the structure factor S_π as a function of T for different site (top panel) and bond (bottom panel) disorder strengths. $\Delta_\mu > U$ completely destroys the sharp rise in S_π , which occurs here on a 6^3 lattice at a value close to the bulk $T_N/t_0 \sim 0.35$ for $U/t_0 = 8$. The suppression of magnetic order is a consequence of the destruction of the local moments $m^2 = \langle (n_{i\uparrow} - n_{i\downarrow})^2 \rangle = 1 - 2D$ at half-filling (see the inset of Fig. 1). S_π is also suppressed by Δ_t despite the fact that it has only a small effect on m^2 [27]. The likely mechanism for the destruction of AF order in this case is the introduction of fluctuations in the near-neighbor exchange $J_{ij} \sim 4t_{ij}^2/U$. As a consequence, of this anisotropy, singlets can form on the bonds with large

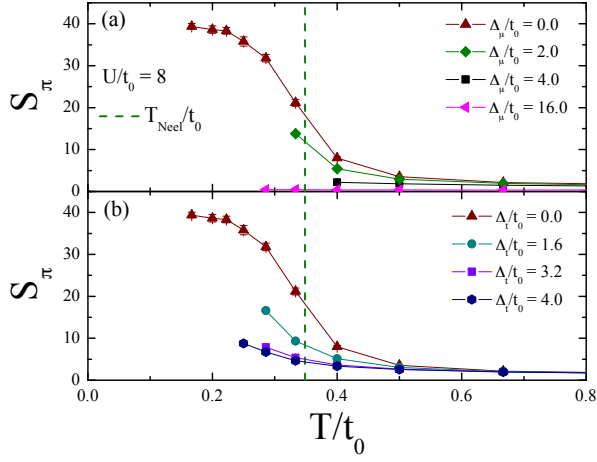


FIG. 4: (Color online) Antiferromagnetic structure factor S_π as a function of T as the site (top) and bond (bottom) disorder is varied. Site disorder drives S_π to zero for $\Delta_\mu \gtrsim U$ by destroying the magnetic moments $m^2 = 1 - 2D$, whereas singlet formation on bonds with large $J_{ij} \sim t_{ij}^2/U$ is induced by sufficiently large Δ_t and also destroys the AF long-range order.

J_{ij} . When many pairs of sites are effectively removed from the lattice, order is lost. Although both bond and site disorder reduce S_π , it is important to emphasize that low T is reached by turning the disorder *off*, so that the terminal state is the sought after regime of large AF correlations.

Equilibration is crucial to the viability of disorder cooling. Recent experiments by the DeMarco group [12] and theory [47] provide evidence that the requisite Δ_μ lie well below the threshold where randomness drives atomic velocities to zero: Measurements of mass transport show that the center-of-mass velocity only vanishes above $\Delta_\mu/t_0 \sim 21.7 \pm 1.6$ for $U/t_0 = 3.8$ and $\Delta_\mu/t_0 \sim 31.7 \pm 4.2$ for $U/t_0 = 9.1$. The implications of these results for disorder cooling are considered in Fig. 5, which shows the final temperature $T_f(T_i, \Delta_\mu)$ which would result from starting at initial temperature T_i and disorder Δ_μ , and turning off randomness adiabatically. Figure 5 complements Fig. 2 and provides another way of analyzing the lowering of T_f starting from states at T_i with Δ_μ beyond U/t_0 and adiabatically following a path to $\Delta_\mu = 0$. The reduction in temperature, $T_i - T_f$, can be as large as $0.65t_0$ for $\Delta_\mu/t_0 = 16$ and $U/t_0 = 4$, starting at $T_i/t_0 = 1$ and $1.35t_0$ for $T_i/t_0 = 2$. The many-body localization (MBL) critical disorder strengths for $U/t_0 = 4.0, 8.0$ and 12.0 (denoted by horizontal arrows in Fig. 5) lie above the range which provides substantial cooling. These comparisons provide considerable support to the likelihood that equilibration will still occur in the regime where disorder-induced cooling is effective.

Implementation and Concluding Remarks: The scheme proposed here, on its own, is not sufficient to achieve new quantum phases, and must be accompanied

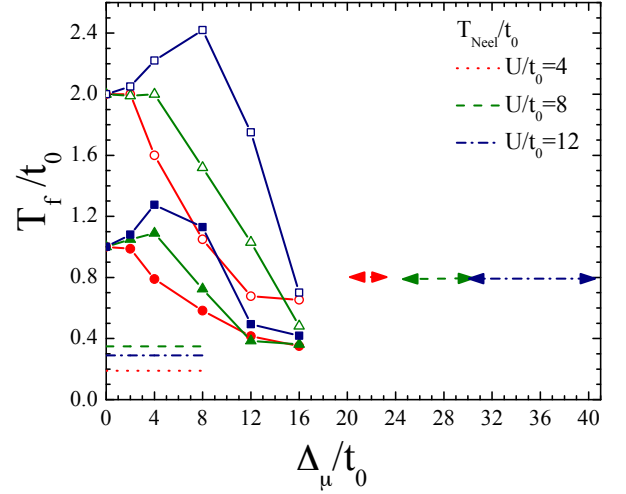


FIG. 5: (Color online) Final temperature T_f/t_0 resulting from adiabatically turning off disorder in a system with initial disorder strength Δ_μ/t_0 , shown on the horizontal axis, and two different initial temperatures $T_i/t_0 = 1, 2$. As Δ_μ increases beyond U/t_0 , T_f decreases. The Neel temperatures $T_N/t_0 = 0.19, 0.35, 0.29$ for $U/t_0 = 4, 8, 12$, respectively, are shown as dashed horizontal lines. The horizontal arrows are estimates for the onsets of MBL for (from left to right) $U/t_0 = 4.0, 8.0, 12.0$, obtained by linearly interpolating $\Delta_c/12t_0$ vs $U/12t_0$ in Fig. 3 of Ref. [12].

by an additional scheme to reduce the entropy in the initial disordered lattice. Since turning on disorder heats the gas, this energy must be removed before attempting to cool more deeply using our method. There have been no direct attempts to cool in a disordered lattice, but several schemes are promising. One such method is sympathetic cooling by another atomic species [48] or spin-state of the same species [49, 50] that by proper choice of lattice wavelength or polarization is unaffected by the lattice. Another approach is to implement a compensated lattice, where the overall confinement created by the infrared lattice beams is compensated by overlapping blue-detuned beams [51]. By tuning the intensity of the blue-detuned beams the threshold for evaporation can be brought near the chemical potential, resulting in very low temperatures [43]. While this scheme has only been implemented in a clean lattice, it seems plausible that it can work in any situation where there is sufficient mobility.

A second approach is to mask the disorder in such a way that it is applied only to a small spatial subregion of the entire gas. Through thermal contact, atoms in this region could be cooled by the larger reservoir region outside the disordered volume. If the clean gas is then discarded, one again has the starting point of a disordered gas at the same initial T as a clean one. Complex optical potentials to perform these roles can be created using phase-imprinting spatial light modulators [52, 53] or micro-mirror devices [54].

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