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Quantum Phase Transition and Protected Ideal Transport in a Kondo Chain

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We study the low energy physics of a Kondo chain where electrons from a one-dimensional band interact with magnetic moments via an anisotropic exchange interaction. It is demonstrated that the anisotropy gives rise to two different phases which are separated by a quantum phase transition. In the phase with easy plane anisotropy, Z_2 symmetry between sectors with different helicity of the electrons is broken. As a result, localization effects are suppressed and the dc transport acquires (partial) symmetry protection. This effect is similar to the protection of the edge transport in time-reversal invariant topological insulators. The phase with easy axis anisotropy corresponds to the Tomonaga-Luttinger liquid with a pronounced spin-charge separation. The slow charge density wave modes have no protection against localizatioin.

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Introduction. One-dimensional systems present an ideal platform for formation of charge density waves (CDW) [1]; the transport in clean systems is almost ideal [2]. However, for realistic interactions and at low temperatures, even a weak disorder pins the CDW suppressing the charge transport [3]. The ideal transport can be protected by symmetries: a well-known example is the edge transport in two-dimensional time-reversal invariant topological insulators (TIs)[4–7]. The topologically nontrivial state of the bulk and time-reversal symmetry leads to a lock-in relation between the chirality and the spin of edge modes making them helical [8]. As a result, the electron backscattering must be accompanied by a spin-flip; hence the edge transport becomes immune to effects of potential disorder. Other processes which can suppress the ideal transport include scattering by magnetic impurities [9] or inelastic processes due to interactions [10–14]. All of them become ineffective at low temperatures. The presence of (almost) ballistic edge transport has been confirmed in state-of-the-art experiments [15–18]. Hence it is accepted that the ballistic transport is protected by time-reversal symmetry and this protection is removed when this symmetry is broken [19, 20].

Helical boundary modes can exist in noninteracting systems due to topological nontriviality of the bulk [21]. *In this Letter*, we show that helical modes may emerge in interacting systems as a result of spontaneous symmetry breaking. As an illustration, we study a model of Kondo chain [22–26] consisting of band one-dimensional electrons interacting with local spins; the Hamiltonian of this system is:

$$\hat{H} = -t\sum_n \hat{c}^\dagger_{n+1}\hat{c}_n + \sum_m J_a\,\hat{c}^\dagger_m\,\hat{\sigma}^a\hat{S}^a(m)\,\hat{c}_m + H.c. \quad (1)$$

$\hat{c}^{\mathrm{T}}_n \equiv (\hat{c}_\uparrow(n), \hat{c}_\downarrow(n))$ are electron operators at lattice site n; $\hat{\sigma}^a$ are Pauli matrices ($a = x, y, z$); $\hat{S}^a(m)$ are components of the spin-s operator located on lattice site m; t denotes the overlap integral. Sites $\{m\}$ constitute some (not necessarily regular) subset of sites $\{n\}$. We concentrate on the regime of sufficiently high density of spins where the Kondo effect is suppressed and the physics is determined mostly by the exchange (RKKY) interaction [27]. The band is far from half filling, the spins are quantum and the coupling constants are much smaller than the bandwidth, $sJ_a \ll t$. We will consider the coupling which is isotropic in the XY-plane: $J_x = J_y \equiv J_\perp$.

Summary of the results: The low energy (LE) behavior of model (1) includes two distinct regimes corresponding to the easy axis (EA), $J_z > J_\perp$, and the easy plane (EP), $J_z < J_\perp$, anisotropy. In the first case, all quasiparticle (fermionic) excitations are gapped. The transport is carried by gapless collective modes. The CDW couples to a potential disorder which is able to pin it and to block the charge transport. The SU(2) symmetric point, $J_z = J_\perp$, is the point of quantum phase transition into a phase with spontaneously broken helicity. In the EP phase at $T = 0$, quasiparticles with a given helicity acquire a gap and the other helical branch remains gapless. The charge transport is carried by the gapless helical electrons and by the slow collective excitations (spin-fermion waves). If the spin U(1) symmetry is respected, the long range helical ordering makes single-particle backscattering of the gapless modes impossible as in the noninteracting TIs. This leads to suppression of localization effects: the localization radius becomes parametrically large and the dc transport acquires a (partial) symmetry protection in finite but long samples.

Continuum limit: The LE physics must be described in a continuum limit. This requires to single out smooth modes. We linearize the electron spectrum and expand operators $\hat{c}$ in smooth chiral modes:

$$\hat{c}_{\uparrow\downarrow}(n) = \mathrm{e}^{-\mathrm{i}k_F\xi_0 n}\hat{R}_{\uparrow\downarrow}(x) + \mathrm{e}^{\mathrm{i}k_F\xi_0 n}\hat{L}_{\uparrow\downarrow}(x),\ x = n\xi_0\,; \quad (2)$$

ξ_0 is the lattice constant. The Lagrangian of the band electrons becomes

$$\mathcal{L}_{\mathrm{e}} = \Psi^\dagger\left[\hat{I}\otimes(\hat{I}\partial_\tau - \mathrm{i}\hat{\tau}^z v_F\partial_x)\right]\Psi, v_F = 2t\xi_0\sin(k_F\xi_0); \quad (3)$$

τ is the imaginary time; the first space in the tensor product is the spin one, the Pauli matrices $\hat{\tau}^a$ act in the chiral space; $\hat{I} = \mathrm{diag}(1,1)$; $\{v_F, k_F\}$ are the Fermi velocity and momentum; $\Psi^{\mathrm{T}} = \left(R^{\mathrm{T}}, L^{\mathrm{T}}\right)$ is the 4-component fermionic field.

Contrary to Ref.[22], where the effects of forward scattering at $J_z \sim t$ (of the Kondo physics) were considered, we suggest that the LE physics in the dense limit with $J_a \ll t$ (dominated by the RKKY interaction) is governed by backscattering of the fermionic modes. It is described by

$$\mathcal{L}_{\mathrm{bs}} = \rho_s \sum_{a=x,y,z} J_a \sum_m \mathrm{e}^{2\mathrm{i}k_F\xi_0 m} R^\dagger S^a(m)\hat{\sigma}^a L + H.c. \quad (4)$$

ρ_s denotes the dimensionless spin density. $\mathcal{L}_{\mathrm{bs}}$ is expected to lead to opening of the spectral gaps thus reducing the energy of the electrons. The resulting physics is quite different from that of Ref.[22].

We can eliminate the oscillatory factors in (4) by absorbing them into the spin configurations which amounts to separation of fast and slow spin variables [28]. The standard parametrization of the spin by azimuthal and polar angles, $\mathbf{S} = s\{\sin(\theta)\cos(\psi), \sin(\theta)\sin(\psi), \cos(\theta)\}$, with the integration measure $\mathcal{D}\{\Omega_S\} = \sin(\theta)\mathcal{D}\{\theta\}\mathcal{D}\{\psi\}$ [29] is not convenient for our purposes. Therefore, we change to the rotating orthonormal basis $\mathbf{e}_{1,2,3}$ with $\mathbf{e}_3 = \mathbf{S}/s$ and decompose the new spin vector, Fig.1: $\vec{\mathcal{S}} = \vec{\mathcal{S}}_\perp + \vec{\mathcal{S}}_\parallel$,

$$\frac{\vec{\mathcal{S}}_\parallel}{s} \equiv \mathbf{e}_3 \sin\alpha_\parallel; \; \frac{\vec{\mathcal{S}}_\perp}{s} \equiv [\mathbf{e}_1\cos\alpha_\perp + \mathbf{e}_2 \sin\alpha_\perp]\cos\alpha_\parallel; \quad (5)$$

$\alpha_\perp = 2k_F\xi_0 m + \alpha(x)$. The orthonormality can be resolved by choosing

$$\mathbf{e}_1 = \{-\cos(\theta)\cos(\psi), -\cos(\theta)\sin(\psi), \sin(\theta)\}, \quad (6)$$
$$\mathbf{e}_2 = \{\sin(\psi), -\cos(\psi), 0\}. \quad (7)$$

The integration measure for $\alpha, \alpha_\parallel$ will be $\mathcal{D}\{\Omega_\alpha\} = \cos(\alpha_\parallel)\mathcal{D}\{\alpha_\parallel\}\mathcal{D}\{\alpha\}$, the total measure reads $\mathcal{D}\{\Omega\} = \mathcal{D}\{\Omega_\alpha\}\mathcal{D}\{\Omega_S\}$. This does not result in overcounting the degrees of freedom since we will find a scale separation with two fast (massive $\alpha_\parallel, \theta$) and two slow (massless α, ψ) angles [30]. Verification of the scale separation and stability of the chosen spin configuration will confirm self-consistency of our approach.

Inserting the new parametrization in Eq.(4) and keeping only the non-oscillatory terms, we find LE Lagrangian $\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{e}} + \mathcal{L}^{(\mathrm{sl})}_{\mathrm{bs}} + \mathcal{L}_{\mathrm{WZ}}$ where

$$\mathcal{L}^{(\mathrm{sl})}_{\mathrm{bs}} = \frac{\tilde{s}\rho_s}{2} R^\dagger \Big\{ J_\perp \left[\mathrm{e}^{\mathrm{i}\psi}\sin^2\left(\frac{\theta}{2}\right)\hat{\sigma}^- - \mathrm{e}^{-\mathrm{i}\psi}\cos^2\left(\frac{\theta}{2}\right)\hat{\sigma}^+\right] + J_z\sin(\theta)\hat{\sigma}^z \Big\} L\mathrm{e}^{-\mathrm{i}\alpha} + H.c.; \quad \tilde{s} \equiv s\cos(\alpha_\parallel); \quad (8)$$

$\mathcal{L}_{\mathrm{WZ}}$ is the topological Wess-Zumino term [31, 32]:

$$\mathcal{L}_{\mathrm{WZ}} = \mathrm{i}s\rho_s\xi_0^{-1}\sin(\alpha_\parallel)[\partial_\tau\alpha + \cos(\theta)\partial_\tau\psi]. \quad (9)$$

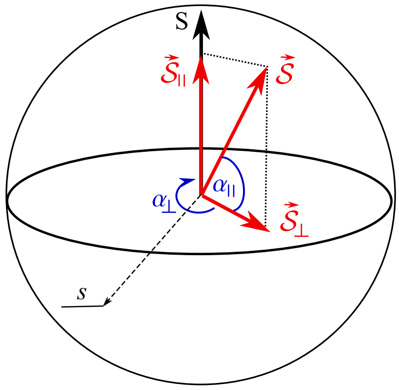

FIG. 1. Transformation from the frame of the vector $\mathbf{S}$ to that of $\vec{\mathcal{S}}$. Angles $\alpha_{\parallel,\perp}$ define the modulus of the "transverse" component $\vec{\mathcal{S}}_\perp$ and its rotation around the "longitudinal" component $\vec{\mathcal{S}}_\parallel$, respectively.

The fermionic gaps become maximal at $\alpha_\parallel = 0$ and $\theta = 0, \pi/2, \pi$. Thus, we expect three extrema of the action whose stability depends on $J_\perp/J_z$.

EA anisotropy, $J_z > J_\perp$: The term $O(J_z)$ dominates and opens the gap in all fermionic modes. This can be shown after removing the angles α, ψ from the backscattering term (8) by using the Abelian bosonization [33, 34]: we bosonize the fermions and shift bosonic phases:

$$\tilde{\Phi}_c = \Phi_c - \alpha/2, \; \tilde{\Theta}_s = \Theta_s - \psi/2\,; \quad (10)$$

$\{\Phi_c, \Theta_s\}$ are the charge and the (dual) spin phases; $\partial_x\Phi$ is coupled to a charge source field [35]: $\mathcal{L}_h = h_c\partial_x\Phi_c$. The shift (10) generates $h_c\partial_x\alpha/2$ in $\mathcal{L}_h$. Finally, we can return to the fermionic variables:

$$\mathcal{L}^{(\mathrm{sl})} \simeq \mathcal{L}_{\mathrm{e}} + \mathcal{L}^{(\mathrm{sl})}_{\mathrm{bs}}|_{\alpha,\psi=0} + \sum_{2\Phi=\alpha,\psi} \mathcal{L}_{\mathrm{TL}}(\Phi, v_F) + \mathcal{L}_{\mathrm{WZ}}; \quad (11)$$

$\mathcal{L}_{\mathrm{TL}}(\Phi, v) = [(\partial_\tau\Phi)^2 + (v\,\partial_x\Phi)^2]/\pi v$ is the Lagrangian of the Tomonaga-Luttinger Liquid (TLL) [36].

For fixed values of $\{\theta, \alpha_\parallel\}$, the fermionic spectrum consists of the four Dirac modes with the masses:

$$m^2_\pm = (\tilde{s}\rho_s/2)^2\left(\sqrt{J^2_\perp\cos^2\theta + J^2_z\sin^2\theta} \pm J_\perp\right)^2. \quad (12)$$

Integrating out the gapped fermions, we get the contribution to the ground state energy:

$$E_{\mathrm{GS}} = -\frac{\xi_0}{2\pi v_F}\sum_{\chi=\pm} m^2_\chi \ln[t/|m_\chi|] + o(J^2_\perp, J^2_z). \quad (13)$$

If $J_z > J_\perp$, E_{GS} has minima at $\theta = \pi/2, \alpha_\parallel = 0$; small fluctuations around the minima read:

$$\delta E_{\mathrm{ea}}/\mathcal{E} \approx (J^2_z - J^2_\perp)\cos^2(\theta) + (J^2_z + J^2_\perp)\sin^2(\alpha_\parallel), \quad (14)$$

$\mathcal{E} \equiv \ln(t/J)\,(s\rho_s)^2\xi_0/4\pi v_F$ [37]. Using Eqs.(9,14) and integrating over the Gaussian fluctuations of the angles, we find parameters of $\mathcal{L}_{\rm TL}(\alpha)$ which are renormalized due to the coupling of the spin wave to the gapped fermions [38]. The LE Lagrangian for the EA anisotropy is [39]:

$$\mathcal{L}_{\rm ea} = \frac{\mathcal{L}_{\rm TL}(\psi, v_F)}{4} + \frac{\mathcal{L}_{\rm TL}(\alpha, v_\alpha)}{K_\alpha} + \mathcal{L}_h^{(\rm ea)};\ \frac{v_\alpha}{v_F} = \frac{K_\alpha}{4} \ll 1. \tag{15}$$

$\mathcal{L}_{\rm ea}$ corresponds to two U(1)-symmetric TLL models with the slow charge, α, and the fast spin, ψ, bosonic modes.

Breaking Z_2 symmetry: If $J_z \gg J_\perp$, then $m_+ \simeq m_-$, all fermionic modes have the gap $\sim J_z$. Mass m_- progressively shrinks towards the SU(2) symmetric point of the quantum phase transition where $m_- = 0$; one subsystem of the helical fermions becomes gapless and our approach looses its validity. We leave a description of the SU(2) symmetric point for future studies and consider instead the strong EP anisotropy $J_z \ll J_\perp$.

EP anisotropy, $J_z \ll J_\perp$: We put $J_z \to 0$ and rewrite Eq.(8) as a sum of helical contributions:

$$\mathcal{L}_{\rm bs}^{(\rm H1)} = \tilde{s}\rho_s J_\perp R_\uparrow^\dagger \cos^2(\theta/2)\, \mathrm{e}^{-\mathrm{i}(\psi+\alpha)} L_\downarrow + H.c. \tag{16}$$

$$\mathcal{L}_{\rm bs}^{(\rm H2)} = -\tilde{s}\rho_s J_\perp R_\downarrow^\dagger \sin^2(\theta/2)\, \mathrm{e}^{\mathrm{i}(\psi-\alpha)} L_\uparrow + H.c. \tag{17}$$

If $\theta \simeq \pi/2$, both helical sectors have gaps though the coupling constant $J_\perp$ is effectively decreased, $\sin^2(\theta/2) \simeq \cos^2(\theta/2) \simeq 1/2$. If $\theta \simeq 0, \pi$, only one helical sector acquires the gap $m = m_+\big|_{J_z, \alpha_\parallel, \theta=0}$, and $J_\perp$ is not suppressed, either $\sin^2(\theta/2) \simeq 1$ or $\cos^2(\theta/2) \simeq 1$. Since the contribution of the gapped fermions to $E_{\rm GS}$ is negative and quadratic in the gap, Eq.(13), $\theta = \pi/2$ yields maximum of the energy and two (degenerate) minima are $\theta = 0, \pi$. Thus, the Z_2 symmetry between the helical subsystems is spontaneously broken. This confirms a quantum phase transition at $J_z = J_\perp$ [40].

We consider the configuration $\theta \simeq 0$ where only $\mathcal{L}_{\rm bs}^{(\rm H1)}$ yields the femionic gap [41]. One can estimate that contributions of the gapped and the gapless fermions to fluctuations of $E_{\rm GS}$ are of order $\sim (J_\perp^2/v_F)\sin^2(\theta/2)$ and $\sim (J_\perp^2/v_F)\sin^4(\theta/2)$, respectively. The latter is subleading, it is beyond our accuracy and must be neglected. Thus, $\mathcal{L}_{\rm bs}^{(\rm H2)}$ is irrelevant for the effective LE theory and must be neglected too. The combination $\psi - \alpha$ becomes redundant; ψ in the combination $\psi + \alpha$, Eqs.(9,16), can be absorbed in α: $\psi + \alpha \to \alpha$ [42]. Now, we proceed very similar to the EA case: (a) eliminate the shifted spin phase α from $\mathcal{L}_{\rm bs}^{(\rm H1)}$ by doing the transformation

$$\tilde{\Phi}_c = \Phi_c + \alpha/2,\ \tilde{\Theta}_s = \Theta_s - \alpha/2; \tag{18}$$

(b) integrate out massive helical fermions and obtain the fermionic energy close to its minima:

$$\delta E_{\rm ep}/\mathcal{E} \simeq J_\perp^2[\sin^2(\theta/2) + \sin^2(\alpha_\parallel)/2]\,; \tag{19}$$

(c) integrate out small quadratic fluctuations of angles around the stationary value; (d) bosonize gapless helical fermions by using the Abelian phase $\Phi_{\rm H}$. These steps yield the effective Lagrangian for the EP case [38, 39]:

$$\mathcal{L}_{\rm ep} = \frac{\mathcal{L}_{\rm TL}(\Phi_{\rm H}, v_F)}{2} + \frac{\mathcal{L}_{\rm TL}(\alpha, v'_\alpha)}{K'_\alpha} + \mathcal{L}_h^{(\rm ep)};\ \frac{v'_\alpha}{v_F} = \frac{K'_\alpha}{4} \ll 1. \tag{20}$$

Similar to $\mathcal{L}_{\rm ea}$, $\mathcal{L}_{\rm ep}$ corresponds to two U(1)-symmetric TLL models with the fast, $\Phi_{\rm H}$, and the slow, α, bosonic modes. However, as we discuss below, the effective theories with- and without the helical symmetry have different transport properties if a disorder is added.

We note that Eqs.(16,20) are similar to their counterparts describing a helical edge mode in the TI with an array of the Kondo impurities [19, 20]. In our case, however, this helical mode has emerged as a result of spontaneous symmetry breaking.

Density correlation functions and disorder effects: The source terms, $\mathcal{L}_h^{(\rm ea)} = h_c\,\partial_x\alpha/2$; $\mathcal{L}_h^{(\rm ep)} = h_c(\partial_x\Phi_{\rm H} + \partial_x\alpha/2)$, generate the charge density-density correlation function: $\mathcal{C}_{\rm ea} \propto \langle\partial_x\alpha\partial_x\alpha\rangle$, $\mathcal{C}_{\rm ep} \propto \big(\langle\partial_x\Phi_{\rm H}\partial_x\Phi_{\rm H}\rangle + \langle\partial_x\alpha\partial_x\alpha\rangle/4\big)$. $\mathcal{C}_{\rm ea,ep}$ with Lagrangians $\mathcal{L}_{\rm ea,ep}$ correspond to the ideal metallic transport. In the EA case, it is supported by the slow CDW with the small compressibility K_α. $\mathcal{C}_{\rm ep}$ contains the contribution from the helical quasiparticles with the bare velocity and from the slow collective wave with the small compressibility K'_α [43].

Coupling of backscattering spinless impurities to the fermions is described by: $V_{\rm dis}[g] = g(x)\Psi^\dagger(I \otimes \tau^\dagger)\Psi + H.c.$; g is the smooth $2k_F$-component of the scalar random potential. We use the model of the Gaussian white noise: $\langle g^{1,2}\rangle_{\rm dis} = 0; \langle g(x_1)g^*(x_2)\rangle_{\rm dis} = \mathcal{D}\delta(x_1 - x_2)$, assuming that the disorder is weak, $\mathcal{D} \ll (m_\pm, m)v_F$, and gaps are unchanged.

After shifts Eq.(10,18), g acquires the phase factor: $g \to g \times \mathrm{e}^{\mathrm{i}\alpha/2}$. Thus, the backscatterering impurities are coupled to all gapless charge carriers. To figure out whether this may lead to localization, we perform the disorder averaging and integrate out the massive fermions [44]. The relevant terms appear only in $\mathcal{D}^2$-order and have a different form in EA and EP phases. In the first case, $\mathcal{D}^2$ couples directly to $\exp(\mathrm{i}\alpha)$; in the EP phase, it couples to $R_\sigma^+ L_{-\sigma}\exp(\mathrm{i}\alpha)$. The latter fact is related to impossibility of single particle backscattering in the phase with broken helicity. The power counting indicates the parametric difference in the localization radius in different phases: $L_{\rm ea}^{(\rm loc)}/L_{\rm ep}^{(\rm loc)} \sim K_\alpha(\mathcal{D}/v_F m)^{4/3} \ll 1$ with $L_{\rm ep}^{(\rm loc)} \sim (v_F/m)\,(v_F m/\mathcal{D})^2$.

Localization blocks the dc transport if a sample size is large: $L \gg L^{(\rm loc)}$. Thus, the ballistic transport in the phase with broken helicity acquires the symmetry protection up to the parametrically large scale $L_{\rm ep}^{(\rm loc)}$. This conclusion holds true as long as the U(1) symmetry in the spin sector is respected. Breaking the U(1) spin symmetry allows the direct backscattering of fermions and removes protection of the ideal transport in the EP phase (cf. localization of the helical edge modes of the TIs after introducing an anisotropy in the XY-plane [19]).

Finite temperature effects (clean case): Previous calculations are done for zero temperature. They can be generalized for $T \neq 0$ provided that $T \ll m_{\pm}, m$. Finite temperature restores broken helical symmetry at $J_z < J_\perp$ since thermal fluctuations produce domains with opposite helicity. When the spin configuration interpolates between the phases with different helicity there is an energy increase of the order of the difference between the energy in the unstable state (with $\theta \simeq \pi/2$) and the ground state energy (with $\theta \simeq 0, \pi$). Thus, we can estimate the energy of the domain wall as $E_{\rm wall} \sim m^2 \xi_0 / v_F$, cf.Eq.(13). The maximal number of the domain walls is Lm/v_F. If $T \ll E_{\rm wall}$, it becomes exponentially suppressed: $N_{\rm wall} \sim Lm/v_F \exp(-E_{\rm wall}/T)$. If $N_{\rm wall} > 1$, the walls appear and block the quasiparticle transport since the electrons with a given helicity are massless only in one domain and massive in the other (neighboring) one. Hence the electrons are reflected from domain boundaries. An influence of the domain wall on the phase α is reduced to a jump in the Luttinger parameter K'_α which cannot affect the dc conductance [45]. Thus, the dc transport in the phase with the broken helicity remains ballistic at finite temperatures [46].

Validity: The effective LE theory, Eqs.(15,20), is valid at energies below the smallest fermionic gap, m_- and m for the EA and the EP anisotropy, respectively. Since m_- vanishes at the SU(2) symmetric point, the approach fails in the vicinity of the quantum critical point. Quickly oscillating contributions $\propto e^{\pm 2ik_F x}$, which we neglected, are generically unable to change the physics at the large distances: If the Kondo chain is close to incommensurability the quickly-oscillating exponentials can be treated as random variables [23]. However, in the most interesting case of the broken helicity, the amplitude of the oscillating terms is suppressed in the vicinity of the classical spin configuration, $\theta \simeq 0$, as $\sim (\xi_0 J_\perp^2 / v_F) \sin^4(\theta/2)$ [see the discussion of the derivation of Eq.(20)] which is squared after averaging over the random fluctuations, i.e., becomes negligible.

Conclusions: We have demonstrated that the dc charge transport in the Kondo chain model (1) with the U(1) symmetry of spins remains ballistic in long samples, $L < L_{\rm ep}^{\rm (loc)}$, in the presence of the potential disorder when the anisotropy of the exchange interaction is of the easy plane type. Due to the spontaneous breaking of the Z_2 symmetry, the charge carriers are quasiparticles possessing a particular helicity (whose spin and chirality are locked) and composite spin-fermion collective modes. In the presence of the U(1) spin symmetry, all gapless modes are protected from simple backscattering by the mechanism similar to that in noninteracting TIs. We emphasize that the symmetry protected transport in our model results from interaction many-body effects instead of the coupling to the non-interacting and topologically non-trivial bulk. In the case of the easy axis anisotropy, the helical symmetry is respected. The quasiparticles are fully gapped and transport is carried solely by the collective modes, slow CDWs, which do not posses the symmetry protection.

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[27] This statement means that the spin density is sufficiently high so that the Kondo effect is cut by the gap generated by backscattering terms in $\hat{H}$, cf. Refs.[19, 20]. It holds true when the gap exceeds the Kondo temperature.

[28] This approach is supported by numerical studies of the Kondo chain of classical spins which indicate that the phase diagram of the system reflects helical spin configurations [47]. The physics, which is governed by the helical effects in spins, is also studied in Refs.[48–50].

[29] The topological Wess-Zumino term (the Berry phase), $\mathcal{L}_{\mathrm{WZ}}$, must be added to the Lagrangian [31].

[30] We note that the procedure of separation of slow and fast variables is standard and is routinely used in RG calculations: see, for example, Chapter 1 in Ref.[31].

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[32] Derivation of Eq.(9) is explained in Suppl.Mat. No.1.

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[34] Alternatively, one can do gauge transformations of the fermionic fields and obtaine an anomaly from the Jacobian [51].

[35] Two remaining fields can be integrated out.

[36] While deriving Eq.(11), we have neglected the gradient coupling of the fermions and the spin phases, $\Psi^\dagger[\partial_{x,\tau}(\alpha,\psi)]\Psi$, which is justified for energies below the fermion gaps.

[37] We do not distinguish between J_z and $J_\perp$ in the logarithm.

[38] Expressions for the Luttinger parameters K_α and K'_α are given Suppl.Mat. No.3.

[39] While deriving Eqs.(15) and (20), we have neglected in $\mathcal{L}_{\mathrm{WZ}}$ the subleading terms $\propto \sin(\alpha_\parallel)\cos(\theta)\partial_\tau\psi$ and $\propto \sin(\alpha_\parallel)\sin^2(\theta)\partial_\tau\psi$, respectively. This is justified for energies below the fermion gaps.

[40] We note that helicity is given by the product of the spin projection and the electron velocity, $H = \mathrm{sign}(\sigma v)$. Hence, helical symmetry is discrete and it can be spontaneously broken in one dimension at $T = 0$. The corresponding order parameter is related to the average $\mathcal{C}_a = \langle S_b(x)S_c(x+\xi_0)\epsilon_{abc}\rangle$: in the phase with the broken symmetry $\mathcal{C}_z \propto \sin(2k_F\xi_0)$ and $\mathcal{C}_z = 0$ otherwise.

[41] The second minima $\theta \simeq \pi$ can be considered similarly and it yields the same effective Lagrangian $\mathcal{L}_{\mathrm{ep}}$, Eq.(20).

[42] One can arrive at the same conclusion even faster using the standard parametrization of $\mathbf{S}$, see Suppl.Mat. No.2.

[43] The small compressibility of the bosonic modes leads to suppression of the Drude weight which reflects the coupling of the spin waves to the gapped fermions [19].

[44] All essential details of this calculus are presented in Suppl.Mat. No.4.

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