Quantum Speed Limits for Leakage and Decoherence

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Quantum speed limits for leakage and decoherence

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We introduce state-independent, non-perturbative Hamiltonian quantum speed limits for population leakage and fidelity loss, for a gapped open system interacting with a reservoir. These results hold in the presence of initial correlations between the system and the reservoir, under the sole assumption that their interaction and its commutator with the reservoir Hamiltonian are norm-bounded. The reservoir need not be thermal and can be time-dependent. We study the significance of energy mismatch between the system and the local degrees of freedom of the reservoir which directly interact with the system. We demonstrate that, in general, by increasing the system gap we may reduce this energy mismatch, and consequently drive the system and the reservoir into resonance, which can accelerate fidelity loss, irrespective of the thermal properties or state of the reservoir. This implies that quantum error suppression strategies based on increasing the gap are not uniformly beneficial. Our speed limits also yield an elementary lower bound on the relaxation time of spin systems.

Quantum speed limits (QSLs) answer the fundamental question of how fast a quantum system can evolve, and have numerous applications, e.g., in quantum computation, control, and metrology. Traditionally, they characterize the minimum amount of time required for a quantum state of a closed quantum system to evolve to an orthogonal state. Mandelstam & Tamm (MT) [1] first showed that this time is lower bounded by the inverse of the standard deviation of the Hamiltonian. Margolus & Levitin (ML) [2] gave a different bound involving the inverse of the mean of the Hamiltonian, and the two bounds were subsequently unified [3]. These results led to numerous applications and extensions which go beyond the traditional QSLs, and consider, e.g., the minimum time for optimal control, or for implementing a unitary gate in quantum computation [4–20].

In this Letter we are concerned with QSLs for open quantum systems [21], a question that has attracted significant recent attention [22–25]. While earlier work focused on generalizing the MT or ML-bounds to the open system setting, here we present state-independent, non-perturbative Hamiltonian QSL bounds for population leakage and fidelity loss, for a gapped open system interacting with a reservoir. The assumptions behind the results we present here are also different and independent from those behind previous such bounds. First, we make the (often natural) assumption that the system's initial state is restricted to an energy sector which is separated from the rest of the spectrum by a nonzero gap ΔE, e.g., the ground subspace in various quantum information processing applications. Second, our bounds are independent of the state of the system or reservoir, and in particular, remain valid in the presence of initial correlations between the system and the reservoir. Third, our bounds are obtained purely at the Hamiltonian level. Thus, unlike most other open system QSL bounds [22–25], we do not assume that the system's evolution is governed by a master equation or a completely positive channel.

Our key result is given in Eqs. (6) and (7) below, and comprises fundamental QSL bounds on decoherence and leakage times, expressed in terms of ΔE and the bounded norms of the interaction Hamiltonian and its commutator with the reservoir Hamiltonian, without assuming that the reservoir Hamiltonian is norm-bounded. Note that applying the traditional closed system QSL bounds to the system and reservoir together in general yields bounds which are rather loose and independent of the gap ΔE [26].

Given the very general assumptions behind our QSLs, they have a wide range of applicability similar to the previously known QSLs, including relaxation in many-body spin systems and limitations of control via a remote controller. The primary application on which we focus is quantum error suppression, where the goal is to slow down the loss of fidelity relative to some desired system state, e.g., in the context of quantum information processing tasks [27, 28]. A common strategy to achieve fidelity enhancements is to use or generate energy gaps (e.g., [29–35]). Therefore, after deriving our QSLs we study the dependence of the speed of decoherence and leakage on ΔE. This enables us to find a general lower bound on the timescale for leakage. As expected, we find that in the $\Delta E \to \infty$ limit the probability of leakage at any finite time goes to zero, and moreover, that if the error detection condition [36] holds then in this limit the state retains its fidelity and remains unaffected by the environment. However, we demonstrate that such protection is not guaranteed when ΔE is finite. Namely, by analyzing a spin system model, we show that increasing the gap can in fact accelerate fidelity loss and decoherence, essentially because of a resonance between the system and the reservoir. This means that protection via increasing energy gaps can be counterproductive [37].

Technical results.—Consider a system S coupled to a reservoir R with the total Hamiltonian $H_{\text{tot}}(t) = H_{\text{S}} + H_{\text{R}}(t) + H_{\text{I}}$ where $[H_{\text{S}}, H_{\text{R}}(t)] = 0$ and the interaction satisfies $\|H_{\text{I}}\| < \infty$ (we use the operator norm $\|\cdot\|$, i.e., the largest singular value; we also use $\hbar = 1$ units throughout). An important class of examples are spin-bath models [38]. We denote the time-dependent joint system-reservoir state evolving under $H_{\text{tot}}(t)$ by $\rho_{\text{SR}}(t)$ and the reduced state of the system at time t by $\rho(t) = \mathrm{Tr}_{\mathrm{R}}[\rho_{\mathrm{SR}}(t)]$. Let $\mathcal{C}$ be the subspace of the system Hilbert space spanned by the eigenstates of H_{S} whose energies lie in the interval $\mathcal{I} \subseteq \mathbb{R}$, which includes at least one eigenvalue of H_{S}. Let $P_{\mathcal{C}}$ be the projector onto $\mathcal{C}$, and $Q_{\mathcal{C}} \equiv I - P_{\mathcal{C}}$ be the projector onto the orthogonal subspace $\mathcal{C}^{\perp}$. Thus $[P_{\mathcal{C}}, H_{\mathrm{S}}] = 0$. Let δE denote the energy spread in $\mathcal{C}$, i.e., the difference between the minimum and maximum eigenvalues of H_{S} in $\mathcal{I}$. Let ΔE denote the gap between the energy levels of H_{S} inside and outside $\mathcal{C}$ (i.e., if λ_1 and λ_2 are two distinct eigenvalues of H_{S} such that $\lambda_1 \in \mathcal{I}$ but $\lambda_2 \notin \mathcal{I}$, then $|\lambda_1 - \lambda_2| \geq \Delta E$). Initially we assume $\Delta E > 2\|H_{\mathrm{I}}\|$, which guarantees that there is a separation between the system energies inside $\mathcal{I}$ and the rest of the spectrum, even in the presence of the interaction. This simplification is relevant because we are mostly interested in the large ΔE limit. Later, when we arrive at Eq. (11), we present the general form of the result which relaxes this condition, and results in tighter bounds for small t, even when $\Delta E < 2\|H_{\mathrm{I}}\|$. Before we introduce our bounds, we define an important inverse timescale for open system dynamics, that will make repeated appearances:

$$\Omega(t) \equiv \frac{2\|[H_{\mathrm{I}}, H_{\mathrm{R}}(t)]\|}{\Delta E - 2\|H_{\mathrm{I}}\|} \,. \tag{1}$$

We proceed to present and interpret our main results. All our results are given rigorous proofs in the Supplementary Material (SM) [39]. Unless stated otherwise, throughout we assume that the system state is initialized in $\mathcal{C}$, i.e., $\rho(0) = P_{\mathcal{C}}\rho(0)P_{\mathcal{C}}$.

Leakage.—Leakage is the process whereby the system state develops support in $\mathcal{C}^{\perp}$, which we quantify in terms of the leakage probability $p_{\text{leak}}(t) \equiv \mathrm{Tr}[\rho(t)Q_{\mathcal{C}}]$. Our first general result is an upper bound on $p_{\text{leak}}(t)$, proved in the SM [39]:

$$p_{\text{leak}}(t) \leq \left(\frac{4\|H_{\mathrm{I}}\|}{\Delta E} + \int_0^t ds\, \Omega(s)\right)^2 \xrightarrow{\Delta E \to \infty} 0 \,. \tag{2}$$

To explain this bound, note that the terms $\|H_{\mathrm{I}}\|/\Delta E$ and $\int_0^t ds\, \Omega(s)$ correspond to two different sources of leakage: $\|H_{\mathrm{I}}\|/\Delta E$ determines how much $\mathcal{C}$ is rotated by the interaction H_{I}. The rotated eigenstates of the perturbed Hamiltonian can cause leakage relative to the eigenstates of the original Hamiltonian. Of course, this also happens in the closed systems, and this is why this term does not vanish for $H_{\mathrm{R}}(t) = 0$, where the total system Hamiltonian becomes $H_{\mathrm{S}} + H_{\mathrm{I}}$. Since $\|H_{\mathrm{I}}\|/\Delta E$ is time-independent, it remains small and insignificant in the limit where the gap is large. The term $\int_0^t ds\, \Omega(s)$ is more interesting. In particular, in the case of time-independent $H_{\mathrm{R}}(t) = H_{\mathrm{R}}$, where the total energy of the system and reservoir is a conserved quantity, $\|[H_{\mathrm{R}}, H_{\mathrm{I}}]\|$ can be interpreted as the maximum rate of change of energy of reservoir. Then, in the special case where $\mathcal{C}$ is the bottom (top) energy sector, Ω^{-1} can be interpreted as the minimum time the reservoir needs to transfer (absorb) the required energy to move the system from $\mathcal{C}$ to $\mathcal{C}^{\perp}$ (see the SM [39]).

Fidelity.—We compare the instantaneous "actual" state $\rho(t)$ and the "ideal" system state $\rho_{\text{id}}(t) = e^{-itH_{\mathrm{S}}}\rho(0)e^{itH_{\mathrm{S}}}$ using their Uhlmann fidelity [40, 41] $F[\rho(t), \rho_{\text{id}}(t)] \equiv \|\sqrt{\rho(t)}\sqrt{\rho_{\text{id}}(t)}\|_1$ ($\|\cdot\|_1$ is the trace norm) and their Bures angle $\Theta(t) \equiv \arccos\left(F[\rho(t), \rho_{\text{id}}(t)]\right)$, a generalization to mixed states of the angle between two pure states [42]. Let $P_0 \equiv P_{\mathcal{C}} \otimes I_{\mathrm{R}}$. We define the *induced splitting* by H_{I} on $\mathcal{C}$ as

$$\mathrm{IS}(P_0 H_{\mathrm{I}} P_0) \equiv \min_{K_{\mathrm{R}} \in \mathrm{Herm}(\mathcal{H}_{\mathrm{R}})} \|P_0 H_{\mathrm{I}} P_0 - P_{\mathcal{C}} \otimes K_{\mathrm{R}}\| \,, \tag{3}$$

where the minimization is over the Hermitian operators acting on the reservoir Hilbert space $\mathcal{H}_{\mathrm{R}}$. This quantity can be interpreted as the strength of the effective interaction between the code subspace and the reservoir in the lowest order of perturbation theory. It exists because the reservoir can couple to different states in the subspace $\mathcal{C}$ in different ways and this generally leads to decoherence, or, in special cases, to a modification of the system Hamiltonian, a potentially beneficial effect [43] (see the SM [39]). This term can be nonzero only when $\mathcal{C}$ is at least two-dimensional. We can now state our second general result, an infidelity upper-bound:

$$\sin\frac{\Theta(t)}{2} = \frac{1}{\sqrt{2}}\sqrt{1 - F[\rho(t), \rho_{\text{id}}(t)]} \leq \frac{2\|H_{\mathrm{I}}\|}{\Delta E} + \int_0^t ds\, \Omega(s) + t\left[\frac{\mathrm{IS}(P_0 H_{\mathrm{I}} P_0)}{2} + \frac{2\|H_{\mathrm{I}}\|(\delta E + \|H_{\mathrm{I}}\|)}{\Delta E}\right] . \tag{4}$$

Related bounds have been obtained in Ref. [34]. While bound (4) holds for $\Delta E > 2\|H_{\mathrm{I}}\|$ and states initialized in $\mathcal{C}$, our third general result is a simple universal QSL bound which does not require either one of these assumptions:

$$\sin\frac{\Theta(t)}{2} \leq \frac{t(\lambda_{\max} - \lambda_{\min})}{4} \leq \frac{t\|H_{\mathrm{I}}\|}{2} \,, \tag{5}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues of H_{I}, respectively. This bound formalizes the standard intuition that the minimum relaxation time of an interacting system is determined by the inverse of the couplings. However, as we will show in an explicit example, our QSL bounds (2) and (4) can lead to much stronger bounds on the relaxation time.

Quantum speed limits.—The bounds we have presented directly lead to QSLs on open-system quantum evolution, as we show next. For simplicity, in the following we assume that $H_{\mathrm{R}}(t) = H_{\mathrm{R}}$.

Let $\tau_{\text{leak}}^{\mathcal{C}}$ denote the smallest time at which the probability of leakage from $\mathcal{C}$ exceeds a constant threshold $p_0 \in (0, 1)$. Then, it follows from bound (2) that in the large-gap limit (i.e., $\|H_{\mathrm{I}}\|/\Delta E \ll p_0^{1/2}$) this timescale is lower-bounded by $\frac{\Delta E}{2\|[H_{\mathrm{I}}, H_{\mathrm{R}}]\|}p_0^{1/2}$. We can find a different lower bound on $\tau_{\text{leak}}^{\mathcal{C}}$ using bound (5) together with the fact that $F[\rho(t), \rho_{\text{id}}(t)] \leq \sqrt{1 - p_{\text{leak}}(t)}$ (see the SM [39]). Let $\tau_{\min}$ be the smallest time at which $F[\rho(t), \rho_{\text{id}}(t)]$ drops below the threshold $(1 - p_0)^{1/2}$ for an arbitrary initial state. This threshold convention guarantees $\tau_{\text{leak}}^{\mathcal{C}} \geq \tau_{\min}$. Then bound (5) implies

$\tau_{\rm min} \geq c(p_0)\|H_{\rm I}\|^{-1}$, where $c(p_0) = [2(1-(1-p_0)^{1/2}]^{1/2}$, and hence

$$\tau^{\mathcal{C}}_{\rm leak} \geq \max\left\{ c(p_0)\|H_{\rm I}\|^{-1} \ , \ p_0^{1/2}\frac{\Delta E}{2\|[H_{\rm I}, H_{\rm R}]\|}\right\} \ . \qquad (6)$$

Similarly, we can define $\tau^{\mathcal{C}}_{\rm fid}$ to be the smallest time at which $F[\rho(t), \rho_{\rm id}(t)]$ drops below the threshold $(1-p_0)^{1/2}$. For this choice of threshold we always have $\tau^{\mathcal{C}}_{\rm fid} \leq \tau^{\mathcal{C}}_{\rm leak}$. If we further assume the same large-gap limit and also that $\mathrm{IS}(P_0 H_{\rm I} P_0) = 0$ and $\delta E = 0$, which is a relevant assumption in the context of error suppression, we find using bound (4) that

$$\tau^{\mathcal{C}}_{\rm fid} \geq c(p_0) \max\left\{ \|H_{\rm I}\|^{-1} \ , \ \frac{\Delta E}{4(\|[H_{\rm I}, H_{\rm R}]\| + \|H_{\rm I}\|^2)}\right\} \ . \qquad (7)$$

Equations (6) and (7) constitute our key new QSL bounds. It follows from the definitions of the various timescales we have introduced, together with our result in the bound (5), that

$$\tau^{\mathcal{C}}_{\rm leak} \geq \tau^{\mathcal{C}}_{\rm fid} \geq \tau_{\rm min} \geq c(p_0)\|H_{\rm I}\|^{-1} \ . \qquad (8)$$

The above bounds on $\tau^{\mathcal{C}}_{\rm leak}$, $\tau^{\mathcal{C}}_{\rm fid}$ and $\tau_{\rm min}$ are all first-order in $\|H_{\rm I}\|^{-1}$. On the other hand, any master equation derived under the Born-Markov approximation (BMA) is necessarily second-order in the coupling strength [21]. Therefore, these QSL time scales, or more generally any open-system behavior which occurs on a timescale of order $\|H_{\rm I}\|^{-1}$, such as the resonance phenomenon discussed below, cannot be described under the BMA.

Quantum error suppression.—One of the main applications of these bounds is in the context of quantum error suppression. $\mathcal{C}$ is then the *code subspace* and one is usually interested in the case where it is a degenerate eigensubspace of $H_{\rm S}$ (i.e., $\delta E = 0$). In this case $\rho_{\rm id}(t) = \rho(0)$, whence $F[\rho(t), \rho_{\rm id}(t)]$ is simply the fidelity between the initial state and the state at time t. The fidelity can degrade even if the gap is large compared to the interaction, i.e., if $\|H_{\rm I}\|/\Delta E \leq \epsilon \ll 1$. In this limit bound (4) implies that the rate of fidelity loss is upper bounded by $\Omega'(t) = 2\sqrt{2}\Omega(t) + \sqrt{2}\,\mathrm{IS}(P_0 H_{\rm I} P_0) + \mathcal{O}(\epsilon)\|H_{\rm I}\|$. This result has a simple interpretation: fidelity loss can happen either because of leakage, whose speed is bounded by $\Omega(t)$, or because of the effect of the reservoir on $\mathcal{C}$, whose strength is given by $\mathrm{IS}(P_0 H_{\rm I} P_0)$. In the limit $\Delta E \to \infty$ the rate of fidelity loss is determined just by the induced splitting $\mathrm{IS}(P_0 H_{\rm I} P_0)$, and if this quantity vanishes then for any finite time t, $F[\rho(t), \rho_{\rm id}(t)] \to 1$.

Therefore, the special case where $\mathrm{IS}(P_0 H_{\rm I} P_0) = 0$ is particularly important for error suppression. To illuminate it, consider the decomposition of the interaction term as $H_{\rm I} = \sum_\alpha S^\alpha \otimes B^\alpha$, where $\{S^\alpha\}$ and $\{B^\alpha\}$ are, respectively, independent system and reservoir operators. Then, using Eq. (3), we find that $\mathrm{IS}(P_0 H_{\rm I} P_0) = 0$ iff $P_{\mathcal{C}} S^\alpha P_{\mathcal{C}} \propto P_{\mathcal{C}}$ for all S^α. This is also known as the quantum error detection (QED) condition [28, 36]. Thus the induced splitting quantifies the deviation from the QED conditions. $\mathrm{IS}(P_0 H_{\rm I} P_0) = 0$ can be the result of symmetries of the interaction as in a decoherence-free subspace [44, 45], or it can be engineered using QED codes (e.g., [32]). Using bound (4) we can go beyond this special case and study the effectiveness of a particular error suppressing scheme in the case where the perfect QED condition does not hold (see also Ref. [46]).

Role of the reservoir and system parameters.—One of the interesting aspects of bounds (2)-(4) is that they are independent of the reservoir state, and the reservoir Hamiltonian enters only via $\|[H_{\rm I}, H_{\rm R}(t)]\|$. This means that even if the reservoir is infinitely large and $\|H_{\rm R}(t)\|$ or $\|dH_{\rm R}(t)/dt\|$ are unbounded, as long as $\|[H_{\rm I}, H_{\rm R}]\|$ remains small and bounded, the leakage can be a slow process, depending on the ratio $\Delta E/\|[H_{\rm I}, H_{\rm R}]\|$. This happens, in particular, when the interaction with the reservoir is quasilocal, i.e., the system degrees of freedom (DOFs) interact only weakly with the distant reservoir DOFs. To be concrete, consider the decompositions $H_{\rm I} = \sum_{i\in \rm R} H_{\rm I}^{(i)}$ and $H_{\rm R} = \sum_{i\in \rm R} H_{\rm R}^{(i)}$, where each term $H_{\rm I}^{(i)}$ and $H_{\rm R}^{(i)}$ acts non-trivially only on a local DOF i in the reservoir. Then $\|[H_{\rm I}, H_{\rm R}(t)]\| \leq 2\sum_{i\in \rm R}\|H_{\rm I}^{(i)}\|\,\|H_{\rm R}^{(i)}(t)\|$. In many physical scenarios this sum, and hence $\|[H_{\rm I}, H_{\rm R}(t)]\|$, is bounded and small while $\|H_{\rm R}(t)\|$ is unbounded and contains long-range interactions. E.g., the reservoir may contain bosonic DOFs, for which $\|H_{\rm R}(t)\|$ is infinite. But, as long as these bosonic DOFs do not directly interact with either the system or the DOFs which directly couple to the system (i.e., those with $H_{\rm I}^{(i)} \neq 0$), $\|[H_{\rm I}, H_{\rm R}(t)]\|$ can be small. This remains true even if information propagates arbitrarily fast through the reservoir and the Lieb-Robinson velocity [47] is unbounded.

On the other hand, if $\|[H_{\rm I}, H_{\rm R}(t)]\|$ is large and comparable to $\Delta E\|H_{\rm I}\|$, then our QSL bounds suggest that the timescales for fidelity loss and leakage error can be as small as $\|H_{\rm I}\|^{-1}$, even in the large ΔE limit. As we explicitly show below, the bounds are attainable when $\|[H_{\rm I}, H_{\rm R}(t)]\| \simeq \Delta E\|H_{\rm I}\|$.

It is also interesting to note that bounds (2)-(4) are independent of the state of the reservoir. This implies that even in the limit of infinitely high temperature T, leakage can still be a very slow process, depending on the ratio $\Delta E/\|[H_{\rm I}, H_{\rm R}]\|$. This is a consequence of the assumption that both $\|H_{\rm I}\|$ and $\|[H_{\rm I}, H_{\rm R}]\|$ are bounded, and it does not hold, e.g., in the case of bosonic reservoirs with the standard spin-boson coupling to the system. On the other hand, even at $T = 0$ all information in the system state can be erased by the reservoir in a time of order $\|H_{\rm I}\|^{-1}$, the shortest timescale over which the reservoir can have any influence on the system. Thus, the time it takes the reservoir to affect the evolution of the system is not necessarily related to T.

A model of resonance.—To study the dependence of the time scales for leakage and fidelity loss on the system and reservoir parameters, we present an illustrative example. This is a simple model that exhibits the phenomenon of resonance between the system and reservoir, and is relevant, e.g., also in the context of state transfer via spin chains [48]. The system is a single qubit ($k = 1$) with Hamiltonian $\Delta E_1 \sigma_1^z/2$, and gap ΔE_1. The reservoir can have an arbitrarily large number of DOFs and may contain bosonic modes. The only assumptions we make about the structure of the reservoir are (i) the only reservoir DOF which directly couples to the system is another qubit ($k = 2$), and (ii) the interaction between the reservoir qubit and other reservoir DOFs, denoted by $h_{2,\text{rest}}(t)$, is bounded. The total Hamiltonian is

$$H_{\text{tot}}(t) = \sum_{k=1}^{2} \frac{\Delta E_k}{2} \sigma_k^z + J\vec{\sigma}_1 \cdot \vec{\sigma}_2 + h_{2,\text{rest}}(t) + H_{\text{rest}}(t)\ , \quad (9)$$

where $H_{\text{rest}}(t)$ is an arbitrary Hamiltonian that acts trivially on qubits 1 and 2.

The system qubit is initially in a σ_1^z eigenstate, and the reservoir is in an arbitrary initial state. It turns out that the system's behavior differs strongly between the *resonance* ($|\Delta E_1 - \Delta E_2| \ll |J|$) and *out-of-resonance* ($|\Delta E_1 - \Delta E_2| \gg |J|$) regimes. To demonstrate this it is useful to transform to the rotating frame defined by $|\phi\rangle \mapsto \exp[i\Delta E_2 t(\sigma_1^z + \sigma_2^z)]\,|\phi\rangle$. Both the leakage probability of the system qubit and the Heisenberg Hamiltonian are invariant under this unitary transformation. Thus, the new total Hamiltonian in the rotating frame can be obtained from $H_{\text{tot}}(t)$ by replacing $\Delta E_k \mapsto \Delta E_k - \Delta E_2$, $k = 1, 2$ and $h_{2,\text{rest}}(t) \mapsto \exp[i\Delta E_2 \sigma_2^z)] h_{2,\text{rest}}(t) \exp[-i\Delta E_2 \sigma_2^z)]$. Therefore, the system's energy gap in this rotating frame is $\Delta E_1 - \Delta E_2$.

In the resonance regime leakage can occur in a time of $\mathcal{O}(|J|^{-1})$, the fastest time allowed by the fundamental QSL bound (5). This happens, e.g., already in the case of single qubit reservoir, i.e., $h_{2,\text{rest}}(t) = H_{\text{rest}}(t) = 0$, for which H_{tot} can easily be diagonalized. Under the resonance condition the states of the system and reservoir qubits are then swapped in a time of $\mathcal{O}(|J|^{-1})$, so the fidelity with the initial state is lost. On the other hand, using our QSL bound (6), we find that to have leakage with probability of $\mathcal{O}(1)$ in the out-of-resonance regime, the minimum required time is lower bounded as

$$\tau_{\text{leak}} \geq c|J|^{-1} \max\{1, \frac{|\Delta E_1 - \Delta E_2|}{\max_t \|h_{2,\text{rest}}(t)\|}\}\ , \quad (10)$$

representing a potentially drastic increase in the time required for leakage relative to the minimum time $c|J|^{-1}$ (where c is a constant of order one) obtained from more standard QSL bounds in the form of Eq. (5).

This model has several interesting general implications: (i) Increasing the system gap can *increase* fidelity loss because the system may become resonant with reservoir DOFs; (ii) The relevant parameter which determines the speed of leakage and fidelity loss is not the system gap but the energy mismatch between the system DOFs and the local reservoir DOFs, i.e., those that couple *directly* to the system. If they are in resonance with the system gap, then the reservoir can be insensitive to the gap, and leakage can happen in a time of order $\tau_{\min} \sim \|H_{\text{I}}\|^{-1}$, i.e., as fast as allowed by the fundamental QSL bound (5). On the other hand, if this energy mismatch is large then relaxation is slow, even if the remote DOFs of the reservoir are in resonance with the system. (iii) Our QSL bounds are attainable in the regime where $\|[H_{\text{I}}, H_{\text{R}}]\| \sim \Delta E\|H_{\text{I}}\|$. (iv) Applying these bounds in different rotating frames can lead to different independent constraints.

Beyond the $\Delta E > 2\|H_I\|$ assumption.—Finally, we discuss how the large-gap assumption $\Delta E > 2\|H_{\text{I}}\|$, used in deriving our previous bounds, can be relaxed. The key idea is to transform to a rotating frame in which ΔE becomes larger. Let $Q_{\mathcal{C}}^{+}$ ($Q_{\mathcal{C}}^{-}$) be the projector onto the subspace spanned by the eigenstates of H_{S} whose eigenvalues are greater (less) than those in $\mathcal{I}$, and transform to the frame defined by $|\Phi\rangle \mapsto e^{-itF(Q_{\mathcal{C}}^{+} - Q_{\mathcal{C}}^{-})}\,|\Phi\rangle$, where $F \in \mathbb{R}$. As we prove in the SM [39], the new gap between $\mathcal{C}$ and $\mathcal{C}^{\perp}$ becomes $\Delta E + F$. Consequently, bounds (2)-(7) all remain valid for any $F > 2\|H_{\text{I}}\| - \Delta E$, after the substitutions

$$\Delta E \mapsto \Delta E + F\ , \qquad H_{\text{R}}(t) \mapsto H_{\text{R}}(t) - F(Q_{\mathcal{C}}^{+} - Q_{\mathcal{C}}^{-})\ . \quad (11)$$

Moreover, using this generalization, we find that in the large F limit, bound (4) implies that $\sin\frac{\Theta(t)}{2} \leq 9\|H_{\text{I}}\|t$, which is the same as bound (5), up to a constant. Thus, by varying F from 0 to ∞ we can find a family of bounds which gradually changes from (4) to (5), and find the strongest bound for fixed given values of the parameters.

Conclusions.—In this work we introduced state-independent QSLs on leakage and fidelity loss in a Hamiltonian open system framework. The reservoir Hamiltonian $H_{\text{R}}(t)$ only enters our bounds via $\|[H_{\text{I}}, H_{\text{R}}(t)]\|$, implying that only local reservoir modes play a role in our QSLs. Another important conclusion concerns the common claim that increasing the system's energy gap ΔE always results in better protection from coupling to the reservoir. The intuitive basis for this claim is the idea that a large gap suppresses thermal excitations by the Boltzmann factor $e^{-\Delta E/kT}$. Under the BMA, the claim can be justified provided the spectral density of the reservoir is monotonically decreasing [49]. However, this condition is often violated, e.g., as in the case of an Ohmic bath. Our results, which are derived without approximations, demonstrate that this intuition is not always correct. Increasing ΔE can result in a resonance with the reservoir, causing the fidelity to drop on a timescale independent of ΔE, even if $T = 0$ and the reservoir is in a pure state. These results demonstrate the utility of state-independent QSL bounds for open system dynamics, and raise new questions about the efficacy of energy gaps in protecting quantum information.

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