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Matching pion–nucleon Roy–Steiner equations to chiral perturbation theory

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We match the results for the subthreshold parameters of pion–nucleon scattering obtained from a solution of Roy–Steiner equations to chiral perturbation theory up to next-to-next-to-next-to-leading order, to extract the pertinent low-energy constants including a comprehensive analysis of systematic uncertainties and correlations. We study the convergence of the chiral series by investigating the chiral expansion of threshold parameters up to the same order and discuss the role of the $\Delta(1232)$ resonance in this context. Results for the low-energy constants are also presented in the counting scheme usually applied in chiral nuclear effective field theory, where they serve as crucial input to determine the long-range part of the nucleon–nucleon potential as well as three-nucleon forces.

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INTRODUCTION

Chiral symmetry of QCD, the invariance of the QCD Lagrangian under chiral rotations of the quark fields in the chiral limit of vanishing quark masses, is a powerful tool to elucidate the properties of strong interactions at low energies, where QCD becomes non-perturbative. This chiral symmetry is known to be broken spontaneously and explicitly, with the appearance of almost massless pseudo-Goldstone bosons, the pions. By expanding systematically around the chiral limit of vanishing quark/pion masses, one obtains an expansion in momenta and quark masses, with non-analytic terms predicted and the effects of high-energy physics incorporated in low-energy constants (LECs). These LECs appear in different physical processes, so that once fixed in one process, they can be used to predict others. In particular, one can derive low-energy theorems that relate different observables, at a given order in the chiral expansion. This approach, Chiral Perturbation Theory (ChPT), was pioneered in the meson sector in [1–3], and manifold extensions have been worked out over the last decades. In particular, it has been extended to the single-baryon sector, see [4–6], with pion–nucleon (πN) scattering as one of the most fundamental applications [5, 7–15].

However, as first pointed out in [16–18], constraints from chiral symmetry are by no means limited to systems with at most one nucleon: once so-called nucleon–nucleon (NN) reducible contributions are separated, the remaining irreducible parts of the NN potential again permit a chiral expansion, despite the non-perturbative nature of the NN interactions. In this way, Nuclear Chiral Effective Field Theory (ChEFT), the extension of ChPT to the multi-nucleon sector, has been developed as a powerful tool for a systematic, model-independent approach to nuclear forces, see [19, 20] for recent reviews. One particularly valuable feature of ChEFT concerns the prediction of a hierarchy between two- and multi-nucleon forces, with the NN interactions starting at leading order (LO), three-nucleon forces are predicted to enter at next-to-next-to-leading ($\mathrm{N^2LO}$) order, and even higher forces are accordingly suppressed [21]. Further, the LECs appearing in the expansion relate different processes. In fact, the LECs that appear in πN scattering determine the long-range part of the NN potential and the three-nucleon force. Accordingly, if sufficiently precise information on πN scattering were available, the required input could be immediately used in multi-nucleon applications. This is the aim of the present Letter.

Such improved input for the πN LECs is becoming increasingly urgent, since with higher orders in ChEFT being worked out the uncertainties in the πN LECs are starting to significantly contribute to the error budget in some observables, see e.g. [22]. In the past, several strategies have been pursued: extractions from πN scattering data, either in terms of phase shifts [11, 23] or cross sections [24], determinations from NN observables [25, 26], or a combination of both [27]. Moreover, in [9] the matching with a reconstructed dispersive πN amplitude was performed in the subthreshold region where ChPT is expected to converge best, but the extrapolation from the physical region still required input from πN data (similarly, while starting from the subthreshold region, the LECs are determined from fits to phase shifts in [13]). Given that the long-range contributions are entirely determined by πN physics, πN scattering provides the cleanest access and offers, at least for most LECs, also the highest sensitivity for their extraction. However, such a program has been hampered by inconsistencies in the low-energy πN data base, as exemplified by contradicting partial-wave analyses, the Karlsruhe–Helsinki [28, 29] and the GWU/SAID solutions [30].

In $\pi\pi$ scattering, a similar situation prevailed until the consequent use of Roy equations [31], a combination of constraints from analyticity, unitarity, and crossing symmetry in the form of coupled integral equations for the partial waves. This significantly advanced the knowledge of the low-energy $\pi\pi$ phase shifts [32, 33]. Indeed, the matching to ChPT then allowed for a very precise determination of the pertinent $\pi\pi$ LECs [34]. Meanwhile, Roy-equation techniques have been extended to other processes [35, 36], in particular, a similar program has been pursued for πN scattering based on Roy–Steiner (RS) equations [37–40], making use of a high-accuracy extraction of the πN scattering lengths from pionic atoms as an additional constraint [41–45]. In this Letter, we work out the consequences of our RS solution for the πN LECs by matching the RS and the ChPT representation of the πN amplitude in the subthreshold region. The main advantages of such an approach are the following: first, the πN amplitude in the subthreshold region is a polynomial in the Mandelstam variables (apart from the Born terms), so that the chiral series is expected to converge best there. In contrast to [9], we do not need additional input from the physical region, as in our case the subthreshold parameters follow from the RS solution alone. Second, the matching amounts to equating the subthreshold parameters from [40, 46] with their chiral expansion, which reduces the determination of the LECs to an algebraic problem. Third, we can use the comprehensive error analysis performed in [40, 46], which translates to a full covariance matrix for the extracted LECs.

SUBTHRESHOLD PARAMETERS

We start by specifying conventions for the process

$$\pi^a(q) + N(p) \to \pi^b(q') + N(p'), \tag{1}$$

with pion isospin labels a, b and Mandelstam variables

$$s = (p+q)^2, \qquad t = (p'-p)^2, \qquad u = (p-q')^2, \tag{2}$$

fulfilling $s+t+u = 2m_N^2 + 2M_\pi^2$. We parameterize the scattering amplitude as

$$\begin{aligned} T^{ba}(\nu,t) &= \delta^{ba} T^+(\nu,t) + \frac{1}{2}[\tau^b,\tau^a] T^-(\nu,t), \\ T^I(\nu,t) &= \bar{u}(p')\bigg\{ D^I(\nu,t) - \frac{[\not{q}',\not{q}]}{4m_N} B^I(\nu,t)\bigg\} u(p), \end{aligned} \tag{3}$$

where $\nu = (s-u)/(4m_N)$, the isospin index $I = \pm$ refers to isoscalar/isovector amplitudes, m_N and M_π to the nucleon and pion mass, and τ^a denotes isospin Pauli matrices. Throughout, the amplitudes with a definite $I = \pm$ index are understood to be related to the $\pi^\pm p \to \pi^\pm p$ charge channels according to

$d^+_{00}\ [M_\pi^{-1}]$	$-1.36(3)$	$d^-_{00}\ [M_\pi^{-2}]$	$1.41(1)$
$d^+_{10}\ [M_\pi^{-3}]$	$1.16(2)$	$d^-_{10}\ [M_\pi^{-4}]$	$-0.159(4)$
$d^+_{01}\ [M_\pi^{-3}]$	$1.16(2)$	$d^-_{01}\ [M_\pi^{-4}]$	$-0.141(5)$
$d^+_{20}\ [M_\pi^{-5}]$	$0.196(3)$	$b^-_{00}\ [M_\pi^{-2}]$	$10.49(11)$
$d^+_{11}\ [M_\pi^{-5}]$	$0.185(3)$	$b^-_{10}\ [M_\pi^{-4}]$	$1.00(3)$
$d^+_{02}\ [M_\pi^{-5}]$	$0.0336(6)$	$b^-_{01}\ [M_\pi^{-4}]$	$0.21(2)$
$b^+_{00}\ [M_\pi^{-3}]$	$-3.45(7)$		

TABLE I: Subthreshold parameters from the RS analysis [40, 46].

$$X^\pm \equiv \frac{1}{2}\big(X_{\pi^-p\to\pi^-p} \pm X_{\pi^+p\to\pi^+p}\big), \tag{4}$$

for $X \in \{D, B, \ldots\}$, and the nucleon and pion mass are identified with the masses of the proton and the charged pion, respectively, see [40] and [47–50] for a discussion of the pertinent isospin-breaking corrections. As mentioned above, once the Born terms are subtracted, the amplitude in the subthreshold region becomes a polynomial in ν and t. A particularly convenient representation is provided by the subthreshold expansion

$$\begin{aligned} \bar{D}^\pm(\nu,t) &= \begin{pmatrix} 1 \\ \nu \end{pmatrix} \sum_{n,m=0}^{\infty} d^\pm_{mn} \nu^{2m} t^n, \\ \bar{B}^\pm(\nu,t) &= \begin{pmatrix} \nu \\ 1 \end{pmatrix} \sum_{n,m=0}^{\infty} b^\pm_{mn} \nu^{2m} t^n, \end{aligned} \tag{5}$$

where the upper/lower entry corresponds to $I = \pm$, and the Born-term-subtracted amplitudes are defined as

$$\bar{X}^\pm(\nu,t) = X^\pm(\nu,t) - X^\pm_{\text{pv}}(\nu,t), \quad X \in \{D, B\}, \tag{6}$$

with

$$\begin{aligned} B^\pm_{\text{pv}}(\nu,t) &= g^2\bigg(\frac{1}{m_N^2 - s} \mp \frac{1}{m_N^2 - u}\bigg) - \frac{g^2}{2m_N^2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ D^\pm_{\text{pv}}(\nu,t) &= \frac{g^2}{m_N}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \nu B^\pm_{\text{pv}}(\nu,t), \end{aligned} \tag{7}$$

where g denotes the πN coupling constant.

For the matching to ChPT at N^{3}LO (complete one-loop order) we need the 13 subthreshold parameters listed in Table I. The solution of the RS equations is obtained by minimizing a χ^2-like function, defined as the difference between left- and right-hand side of the equations on a grid of points, with respect to the subtraction constants and the low-energy phase shifts. Most of the subthreshold parameters listed in Table I already appear as subtraction constants of the RS system, and thus follow as output from the RS solution, while the remaining ones,

d_{20}^+, d_{11}^+, and d_{02}^+, are calculated from sum rules afterwards. The uncertainty estimates include a number of effects: first, the RS equations are valid only in a finite energy range below the so-called matching point and only a finite number of partial waves are included explicitly in the solution. We varied the input for the matching condition as well as for the energy region above the matching point and higher partial waves, both regarding different partial-wave analyses and truncations of the partial-wave expansion. Furthermore, we varied the input for the πN coupling constant within $g^2/(4\pi) = 13.7(2)$ [44, 45] and investigated the sensitivity to the parameterization of the low-energy phase shifts used in the solution. Second, we observed that the RS equations are more sensitive to some subthreshold parameters than others. To account for this effect, we generated a set of solutions corresponding to different starting values of the χ^2-minimization, while imposing sum rules for the higher subthreshold parameters, and took the observed distribution as an additional source of uncertainty. Third, we propagated the errors in the scattering lengths, which crucially enter as constraints in the minimization, to the results for the subthreshold parameters. Taking everything together we obtain a 13×13 covariance matrix that encodes uncertainties and correlations of the 13 subthreshold parameters relevant for the matching to ChPT.

CHIRAL EXPANSION

The chiral expansion for the subthreshold parameters is spelled out explicitly in [12], in particular

$$\begin{aligned} d_{00}^+ &= -\frac{2M_\pi^2(2c_1-c_3)}{F_\pi^2} + \frac{g_A^2\big(3+8g_A^2\big)M_\pi^3}{64\pi F_\pi^4} \\ &\quad + M_\pi^4\Bigg\{\frac{16\bar{e}_{14}}{F_\pi^2} + \frac{3g_A^2\big(1+6g_A^2\big)}{64\pi^2F_\pi^4 m_N} - \frac{2c_1-c_3}{16\pi^2F_\pi^4}\Bigg\}, \\ d_{00}^- &= \frac{1}{2F_\pi^2} + \frac{4M_\pi^2(\bar{d}_1+\bar{d}_2+2\bar{d}_5)}{F_\pi^2} + \frac{g_A^4 M_\pi^2}{48\pi^2F_\pi^4} \\ &\quad - M_\pi^3\Bigg\{\frac{8+12g_A^2+11g_A^4}{128\pi F_\pi^4 m_N} - \frac{4c_1+g_A^2(c_3-c_4)}{4\pi F_\pi^4}\Bigg\}, \end{aligned} \tag{8}$$

where c_i, $\bar{d}_i$, and $\bar{e}_i$ denote the NLO, N^{2}LO, N^{3}LO πN LECs, respectively, F_π the pion decay constant, and g_A the axial coupling of the nucleon. The conventions for the $\bar{e}_i$ correspond to the general classification [10] and the c_i have been redefined to absorb a quark-mass renormalization, see [11, 23]. Finally, the expressions in (8) follow the standard counting in the single-nucleon sector, where the expansion parameter is given by $\mathcal{O}(p) = \{p, M_\pi\}/\Lambda_\mathrm{b}$, for momenta p and the breakdown-scale $\Lambda_\mathrm{b} \sim \Lambda_\chi \sim 4\pi F_\pi \sim m_N \sim M_\rho \sim 1\,\mathrm{GeV}$. In contrast, the breakdown-scale in few-nucleon applications is typically lower, $\Lambda_\mathrm{b} \sim 0.6\,\mathrm{GeV}$, so that relativistic corrections are often counted as $\{p, M_\pi\}/m_N = \mathcal{O}(p^2)$ [17].

	NLO	N^{2}LO	N^{3}LO	N^{3}LO^{NN}
c_1	$-0.74(2)$	$-1.07(2)$	$-1.11(3)$	$-1.10(3)$
c_2	$1.81(3)$	$3.20(3)$	$3.13(3)$	$3.57(4)$
c_3	$-3.61(5)$	$-5.32(5)$	$-5.61(6)$	$-5.54(6)$
c_4	$2.17(3)$	$3.56(3)$	$4.26(4)$	$4.17(4)$
$\bar{d}_1+\bar{d}_2$	—	$1.04(6)$	$7.42(8)$	$6.18(8)$
$\bar{d}_3$	—	$-0.48(2)$	$-10.46(10)$	$-8.91(9)$
$\bar{d}_5$	—	$0.14(5)$	$0.59(5)$	$0.86(5)$
$\bar{d}_{14}-\bar{d}_{15}$	—	$-1.90(6)$	$-13.02(12)$	$-12.18(12)$
$\bar{e}_{14}$	—	—	$0.89(4)$	$1.18(4)$
$\bar{e}_{15}$	—	—	$-0.97(6)$	$-2.33(6)$
$\bar{e}_{16}$	—	—	$-2.61(3)$	$-0.23(3)$
$\bar{e}_{17}$	—	—	$0.01(6)$	$-0.18(6)$
$\bar{e}_{18}$	—	—	$-4.20(5)$	$-3.24(5)$

TABLE II: Results for the πN LECs at NLO, N^{2}LO, and N^{3}LO (standard and NN counting only differ at N^{3}LO, except for NLO in c_4, which in the NN scheme becomes 2.44(3)). The results for the c_i, $\bar{d}_i$, and $\bar{e}_i$ are given in units of GeV^{-1}, GeV^{-2}, and GeV^{-3}, respectively.

As a consequence, in this counting one would drop the $1/m_N$ suppressed terms in (8). In this Letter, we consider both counting schemes, which we will refer to as standard and NN counting in the following. The full set of subthreshold parameters can be easily inverted for the LECs, for the result at different chiral orders see Table II (masses, F_π, and g_A are taken from [51]). The errors as propagated from the subthreshold parameters are tiny compared to the shifts observed between chiral orders: clearly, the dominant uncertainty now resides in the chiral expansion. For completeness, we also quote the N^{3}LO correlation coefficients, see Table III. Note that this table contains the correlation matrices for the standard and the NN counting and therefore appears asymmetric.

In general, the values for the LECs are expected to be $\mathcal{O}(1)$, e.g. $c_i \sim g_A/\Lambda_\mathrm{b}$ [6] and significant departures indicate the presence of additional degrees of freedom. In the case of c_{2-4} the main origin of their enhancement is well understood: while other resonances do contribute as well, it is primarily the presence of the $\Delta(1232)$ resonance that makes these LECs take unnaturally large values [52–54]. Following [54], we extract the Δ contributions to the individual subthreshold parameters from the corresponding tree-level Δ-exchange diagrams and convert the result to the LECs. For the numerical analysis we use the Δ coupling constant $g_{\pi N\Delta} = 1.2$ [29], which lies right in the middle of the range 1.05 (extracted from the Δ width [55]) and $3g_A/(2\sqrt{2}) = 1.35$ (predicted by large N_c [56]). Keeping the full pion-mass dependence, we obtain the values shown in the first column of Table IV, while the second column corresponds to the leading expansion in M_π and $m_\Delta - m_N$. Only in the latter case

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
c_1	1	0.18	0.58	0.06	−0.42	0.71	0.04	0.47	−0.59	0.33	−0.21	−0.11	−0.21
c_2	−0.20	1	−0.64	−0.01	0.67	−0.36	−0.27	−0.55	0.56	−0.59	0.59	0.21	0.47
c_3	0.58	−0.86	1	0.04	−0.86	0.91	0.16	0.87	−0.97	0.68	−0.60	−0.24	−0.46
c_4	0.06	−0.03	0.04	1	0.18	−0.22	0.03	−0.31	−0.02	0.07	−0.08	−0.61	−0.63
$\bar{d}_1+\bar{d}_2$	−0.42	0.83	−0.86	0.18	1	−0.83	−0.40	−0.94	0.88	−0.77	0.74	0.23	0.34
$\bar{d}_3$	0.68	−0.63	0.90	−0.25	−0.83	1	0.05	0.93	−0.94	0.53	−0.47	−0.07	−0.17
$\bar{d}_5$	0.04	−0.28	0.16	0.03	−0.40	0.03	1	0.18	−0.14	0.40	−0.29	−0.18	−0.29
$\bar{d}_{14}-\bar{d}_{15}$	0.47	−0.73	0.87	−0.31	−0.94	0.93	0.18	1	−0.91	0.64	−0.61	−0.03	−0.21
$\bar{e}_{14}$	−0.60	0.77	−0.97	−0.02	0.88	−0.94	−0.13	−0.91	1	−0.70	0.65	0.23	0.43
$\bar{e}_{15}$	0.33	−0.72	0.68	0.07	−0.77	0.52	0.40	0.64	−0.69	1	−0.97	−0.28	−0.65
$\bar{e}_{16}$	−0.21	0.67	−0.60	−0.08	0.74	−0.45	−0.29	−0.61	0.65	−0.97	1	0.29	0.60
$\bar{e}_{17}$	−0.11	0.25	−0.24	−0.61	0.23	−0.05	−0.18	−0.03	0.23	−0.28	0.29	1	0.19
$\bar{e}_{18}$	−0.20	0.55	−0.46	−0.63	0.34	−0.14	−0.29	−0.20	0.42	−0.65	0.60	0.19	1

TABLE III: Correlation coefficients at N^{3}LO in standard (upper-right triangle) and NN (lower-left triangle) counting.

c_1^Δ	0.0	0.0	$\bar{d}_1^\Delta+\bar{d}_2^\Delta$	1.9	1.9	$\bar{e}_{14}^\Delta$	−0.4	0.0
c_2^Δ	1.6	2.2	$\bar{d}_3^\Delta$	−0.9	−1.9	$\bar{e}_{15}^\Delta$	−2.6	−3.2
c_3^Δ	−2.1	−2.2	$\bar{d}_5^\Delta$	−0.4	0.0	$\bar{e}_{16}^\Delta$	1.4	3.2
c_4^Δ	1.2	1.1	$\bar{d}_{14}^\Delta-\bar{d}_{15}^\Delta$	−2.9	−3.7	$\bar{e}_{17}^\Delta$	0.3	0.0
						$\bar{e}_{18}^\Delta$	1.1	1.6

TABLE IV: Δ contributions to the πN LECs, in GeV units, for the full Δ-exchange diagrams (first column) and to leading order in M_π and $m_\Delta - m_N$ (second column).

one recovers the relation $c_2^\Delta = -c_3^\Delta = 2c_4^\Delta$ [53].

While the Δ can indeed explain a significant portion of the physical values of the c_i, its effect is too small to explain the large numbers for the $\bar{d}_i$ that appear at N^{3}LO (except for $\bar{d}_5$). The origin of this large shift can be traced back to the terms proportional to $g_A^2(c_3 - c_4) \sim -16\,\text{GeV}^{-1}$ in d_{00}^- in (8) (and similarly in d_{10}^-, d_{01}^-, and b_{00}^+). These terms mimic loop diagrams with Δ degrees of freedom. Our results show that if the Δ is not included explicitly, such contributions lead to a substantial renormalization of the LECs. Indeed, if we drop the $c_3 - c_4$ loop terms, the $\bar{d}_i$ are reduced to $\bar{d}_{1+2,3,14-15} = (2.2, -3.9, -2.6)\,\text{GeV}^{-2}$, in good agreement with the expectations from Table IV.

THRESHOLD PARAMETERS

With the LECs determined by matching to the subthreshold expansion, it is important to check how well the chiral series converges in other kinematic regions. A prime test case is provided by the S-wave scattering lengths $a_{0+}^\pm$: they are known very precisely from pionic atoms [44, 45]. In the isospin conventions (4) their values are $a_{0+}^+ = -0.9(1.4)$ and $a_{0+}^- = 85.4(9)$ (always in units of $10^{-3}M_\pi^{-1}$), and the problem is still purely algebraic. The fourth-order expressions for their chiral expansion were first given in [57], in our conventions they read

$$\begin{aligned}
a_{0+}^+ &= \frac{M_\pi^2}{4\pi F_\pi^2(m_N+M_\pi)}\bigg\{\frac{3g_A^2 m_N M_\pi}{64\pi F_\pi^2} - \frac{g_A^2 M_\pi^2}{16 m_N^2} \\
&\quad - \frac{1}{4}\big[g_A^2 + 8m_N(2c_1 - c_2 - c_3)\big] \\
&\quad + M_\pi^2\big[-16c_1c_2 + \bar{d}_{18}g_A + 16m_N(\bar{e}_{14}+\bar{e}_{15}+\bar{e}_{16})\big] \\
&\quad - \frac{M_\pi^2\big[8 - 3g_A^2 + 2g_A^4 + 4m_N(2c_1-c_3)\big]}{64\pi^2F_\pi^2}\bigg\}, \\
a_{0+}^- &= \frac{m_N M_\pi}{8\pi F_\pi^2(m_N+M_\pi)}\bigg\{1 + \frac{M_\pi^2}{8\pi^2F_\pi^2} + \frac{g_A^2M_\pi^2}{4m_N^2} \\
&\quad + 8M_\pi^2(\bar{d}_1+\bar{d}_2+\bar{d}_3+2\bar{d}_5)\bigg\}. \qquad (9)
\end{aligned}$$

Fixing the only new LEC, $\bar{d}_{18}$, from the Goldberger–Treiman discrepancy,

$$\bar{d}_{18} = \frac{g_A}{2M_\pi^2}\bigg(1 - \frac{gF_\pi}{m_N g_A}\bigg) = -0.44(24)\,\text{GeV}^{-2}, \qquad (10)$$

we obtain the following results

$$\begin{aligned}
a_{0+}^+ &= \{-23.8, 0.2, -7.9\}, \{-14.2, 0.2, -1.4\}, \\
a_{0+}^- &= \{79.4, 92.9, 59.4\}, \{79.4, 92.2, 69.2\}, \qquad (11)
\end{aligned}$$

where the first/second array refers to the standard/NN counting and the three entries to NLO, N^{2}LO, N^{3}LO. It is not surprising that the chiral expansion in the isoscalar sector is slow, after all its LO vanishes. Unexpectedly, a similarly slow convergence is also found for a_{0+}^-, whose low-energy theorem at LO is tantalizingly close to the full answer, while the agreement in both counting schemes deteriorates when going to fourth order. The largest part of this discrepancy can be attributed to the Δ loops discussed above, i.e. for a_{0+}^- the largest portion of the $c_3 - c_4$ terms does cancel between $\bar{d}_1 + \bar{d}_2$ and $\bar{d}_3$, but the cancellation is incomplete and the remainder spoils the agreement with the pionic-atom value.

This example shows that in a theory without explicit Δ degrees of freedom the LECs determined in a particular kinematic region do not necessarily ensure convergence in the full low-energy domain. However, especially when going to higher orders, including the Δ explicitly becomes extremely challenging, so that in practice the Δ-less approach can be pushed to higher orders and it remains to be seen if in the end the Δ-full or Δ-less theory proves more efficient. We argue here that for Δ-less applications in NN scattering matching at the subthreshold point is the preferred choice: the two-pion-exchange diagrams can be reconstructed from πN scattering by means of Cutkosky rules [58] (see [59, 60] for recent applications of this approach), with spectral functions involving πN amplitudes either directly evaluated at or weighted towards zero pion center-of-mass momenta [58], which translates to $s = m_N^2 - M_\pi^2$. Moreover, for physical values of the momentum transfer t in NN scattering the Cauchy kernels in the spectral integrals become largest for $t = 0$. Since the corresponding combination of (s, t) is much closer to subthreshold $(m_N^2 + M_\pi^2, 0)$ than threshold $((m_N + M_\pi)^2, 0)$ kinematics, we conclude that the LECs to be applied in nuclear forces should be extracted from the subthreshold point in πN scattering instead of the physical region. In the present Letter we have presented such an extraction based on a comprehensive analysis of low-energy πN scattering in the framework of RS equations. The corresponding LECs clearly defined at a given chiral order will be valuable for assessing the uncertainties from the long-range part of the nuclear force in future ChEFT calculations [61, 62].

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