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Dynamical detection of topological phase transitions in short-lived atomic systems

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We demonstrate that dynamical probes provide direct means of detecting the topological phase transition (TPT) between conventional and topological phases, which would otherwise be difficult to access because of loss or heating processes. We propose to avoid such heating by rapidly quenching in and out of the short-lived topological phase across the transition that supports gapless excitations. Following the quench, the distribution of excitations in the final conventional phase carries signatures of the TPT. We apply this strategy to study the TPT into a Majorana-carrying topological phase predicted in one-dimensional spin-orbit-coupled Fermi gases with attractive interactions. The resulting spin-resolved momentum distribution, computed by self-consistently solving the time-dependent Bogoliubov-de Gennes equations, exhibit Kibble-Zurek scaling and Stückelberg oscillations characteristic of the TPT. We discuss parameter regimes where the TPT is experimentally accessible.

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Systems of ultracold atoms provide one of the most versatile platforms for realizing many-body quantum phases of matter. In fact, several quantum phases and phase transitions such as the superfluid-Mott transition [1–6] have been realized in such systems. Yet, many of the most interesting phases or phase transitions in such systems are yet to be observed. One of the most glaring examples is the elusive antiferromagnetic Néel order [7, 8] in the fermionic Hubbard model, which is believed to be a precursor of superconductivity in the model. Another example is the recently-proposed family of phases based on the realization of spin-orbit coupling (SOC) by artificial gauge fields [9–13], which includes topological insulators [14–16], topological superfluids (TSFs) [17–22], and fractional quantum Hall phases [23]. A generic obstruction to the observations of many of these phases is heating due to spontaneous emission from applied laser fields. The heating problem makes it difficult to cool into the equilibrium thermal state of many of these topological phases. To study these phases, one can also prepare a gapped non-topological state and ramp the Hamiltonian to drive the system from the non-topological to topological state. However, the properties of the short-lived topological phase are difficult to probe while it is subject to thermal fluctuations.

In this Letter, we propose a dynamical solution to the problem of studying the short-lived topological phase by starting the system in its long-lived non-topological phase and driving it into the topological phase and back. The rapid nature of this process obviates heating; this is expected to make our proposal easily implementable in experiments. The process involves crossing the quantum phase transition between the phases, which supports gapless excitations. Driving through the gapless phase

transition produces excitations in the gapped phase via the Landau-Zener (LZ) transitions [24, 25] with a defect density that demonstrates Kibble-Zurek (KZ) scaling [26–35]. More interestingly, our dip-in-dip-out strategy, where the system is driven through the phase transition and back, leads to the Stückelberg interference phenomenon [36, 37] between the two LZ transitions which in turn results in oscillations of the momentum and energy distribution of the excitations with the ramp rate. In many cases the unique ramp-rate dependence of the excitations' momentum distributions can be measured via standard time-of-flight techniques. This provides an experimentally viable test for the dynamical fingerprints of the topological phase transition (TPT), whose equilibrium properties would otherwise be hard to access.

While this general idea applies to many phase transitions in ultracold bosonic and fermionic systems [34, 38–40], we focus on phase transitions whose dynamical properties are well-understood [20–22, 40–48]. In particular, we apply this idea to the proposed TSFs [20–22] in systems of ultracold atoms which host the Majorana modes [49–53]. Two of the key ingredients [54] for realization of TSFs, namely controllable Zeeman coupling and fermionic Cooper-pairing are readily available in cold atomic systems. The recent realization of synthetic SOC in cold atoms [9–13] provides the third critical ingredient for realizing topological superfluidity thus opening up the possibility of observing topological phases in ultracold atomic setting. In addition, the challenges of spatial and energy resolved spectroscopy are easily resolved [19, 55]. Despite the advantages of these proposals, the detection of TSFs in cold atomic systems is made difficult by the low temperature scales involved combined with the heating associated with SOC.

For the 1D spin-orbit-coupled Fermi gases (SOCFGs) studied here, the TPT is accessed by raising the Zeeman field past a critical value [17–19, 54]. Using the self-consistent time-dependent Bogoliubov-de Gennes equation (td-BdGE) formalism, we calculate the spin-resolved momentum distribution (SRMD) of the SOCFGs as it is ramped across the TPT through our dip-in-dip-out protocol described earlier. We find that the dynamics of the SRMD reflect both Stückelberg interference phenomenon and KZ scaling behavior for appropriate experimentally accessible ramp rates. We demonstrate that these oscillations and the scaling behavior persist at finite initial temperature and are robust features of the TPT separating the conventional and topological phases of the Fermi superfluids (SFs). While a gap closing is not by itself unique to TSFs, a closing of the gap of non-degenerate Bogoliubov quasiparticles spectrum at zero momentum [56] is a yet experimentally unobserved smoking gun signature for a TPT.

We study 1D fermionic atoms with SOC and attractive s -wave interactions. The SOC is generated by a pair of counterpropagating Raman lasers, with recoil wavevector k_r , energy $E_r = \hbar^2 k_r^2 / 2m$, and characteristic time scale $t_r = \hbar / E_r$, giving the SOC strength $\alpha = \hbar^2 k_r / m$. These lasers couple two hyperfine atomic states representing the pseudospins $\sigma = \uparrow, \downarrow$ (for example, $|\uparrow\rangle \equiv |f = 9/2, m_F = -7/2\rangle$ and $|\downarrow\rangle \equiv |f = 9/2, m_F = -9/2\rangle$ in ^{40}K atoms [57]). The transverse Zeeman potential strength Ω_R , set by the Raman coupling strength [9], is varied in time to drive the TPT. Here we consider varying Ω_R linearly from 0 to Ω_{Rf} in a time t_{ramp} , and back in the same time: a piecewise linear ramp protocol of duration $2t_{\text{ramp}}$, see blue curve in Fig. 1(a). Because our protocol starts with Raman lasers off ($\Omega_R = 0$), it is straightforward to experimentally realize a long-lived conventional SF as the initial state [58]; as we will see below, t_{ramp} is much less than the system's lifetime (either limited by the spontaneous emission of the Raman lasers or inelastic scattering from the Feshbach resonances).

The system's Hamiltonian in the Nambu basis $\Psi_k(t) = [\psi_{k\uparrow}(t), \psi_{k\downarrow}(t), \psi_{-k\downarrow}^\dagger(t), -\psi_{-k\uparrow}^\dagger(t)]^\top$ is $H(t) = \frac{1}{2} \int dk \Psi_k(t)^\dagger \mathcal{H}_{\text{BdG},k}(t) \Psi_k(t)$, where $\psi_{k\sigma}$ ($\psi_{k\sigma}^\dagger$) denote the annihilation (creation) operators for fermions with momentum k and spin σ . The Bogoliubov-de Gennes (BdG) Hamiltonian is [19, 59–61]

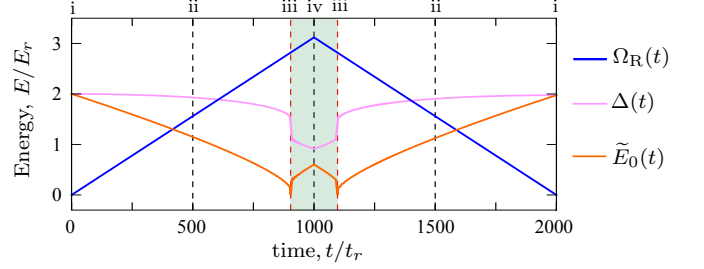
$$\mathcal{H}_{\text{BdG},k}(t) = \xi_k(t)\tau_z + \alpha k\tau_z\sigma_z + \frac{\Omega_R(t)}{2}\sigma_x + \Delta(t)\tau_x, \quad (1)$$

where σ and τ are vectors of Pauli operators acting on spin and particle-hole space respectively. Here $\xi_k(t) = \hbar^2 k^2 / 2m - \mu(t)$ combines the kinetic energy and the chemical potential $\mu(t)$, which is determined self-consistently to keep the number of atoms fixed.

The mean-field pairing potential

$$\Delta(t)e^{i\vartheta(t)} = g_{1\text{D}} \int \langle \psi_{k\uparrow}(t)\psi_{-k\downarrow}(t) \rangle dk \quad (2)$$

(a) Time Profiles



(b) BdG Spectra

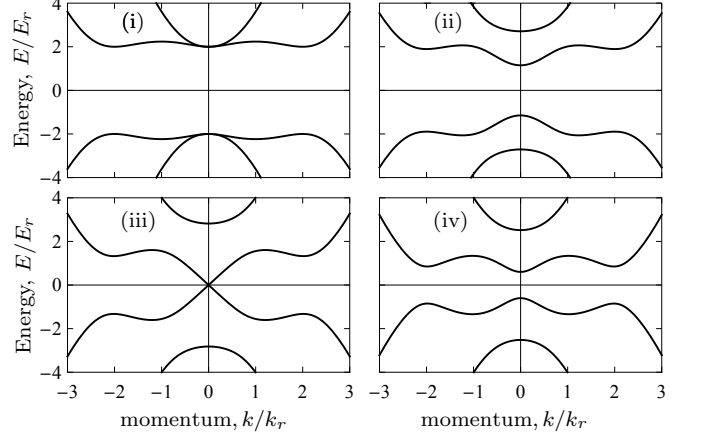


FIG. 1. (Color online) (a) Time profiles of $\Omega_R(t)$, $\Delta(t)$, and $\tilde{E}_0(t)$ for $t_{\text{ramp}} = 1000t_r$. The dashed lines denote the times whose instantaneous band diagrams are plotted in (b). The red dashed lines mark the critical times when TPT happens and the shaded region corresponds to the topological regime. Plots are obtained from numerically solving the td-BdGE [Eq. (9)] self-consistently [Eq. (10)] with initial parameters: $\mu(0) = 0$, $\Delta(0) = 2E_r$, $\alpha = 2$ and $\Omega_R(0) = 0$ for $t_{\text{ramp}} = 1000t_r$. (b) Quasiparticle spectra at different Zeeman potentials Ω_R . From top to bottom, the energy bands are labeled by $E_{2,k}$, $E_{1,k}$, $E_{-1,k}$ and $E_{-2,k}$. The parameters are: (i) $\Omega_R = 0$, $\Delta = 2E_r$, $\mu = 0$, (ii) $\Omega_R = 1.56E_r$, $\Delta = 1.93E_r$, $\mu = -0.02E_r$, (iii) $\Omega_R = 2.8E_r$, $\Delta = 1.4E_r$, $\mu = -0.14E_r$, and (iv) $\Omega_R = 3.12E_r$, $\Delta = 0.91E_r$, $\mu = -0.3E_r$.

is also self-consistently determined, where $\langle \dots \rangle$ denotes averaging with respect to the initial thermal distribution. The attractive effective 1D coupling constant $g_{1\text{D}} < 0$ can be controlled by Feshbach tuning the 3D scattering length [62–64]. In Eq. 1, we used the transformed basis where $\psi_{k\sigma}(t) \rightarrow \psi_{k\sigma}(t) \exp[i\vartheta(t)/2]$, giving a real pairing potential: $\Delta(t) \exp[i\vartheta(t)] \rightarrow \Delta(t)$.

The instantaneous quasiparticle excitation spectrum of the BdG Hamiltonian [cf. Fig. 1(b)] consists of four bands, $E_{n,k} = \text{sgn}(n)\epsilon_{(-1)^n,k}$ where $n = \pm 1, \pm 2$ and

$$\epsilon_{\pm,k}^2(t) = \frac{\Omega_R(t)^2}{4} + \Delta(t)^2 + \xi_k(t)^2 + \alpha^2 k^2 \quad (3)$$

$$\pm 2\sqrt{\xi_k(t)^2 \left[\alpha^2 k^2 + \frac{\Omega_R(t)^2}{4} \right] + \Delta(t)^2 \frac{\Omega_R(t)^2}{4}}.$$

Since $\mathcal{H}_{\text{BdG},k}$ respects particle-hole symmetry, the spectrum is symmetric around $E = 0$. As shown in Fig. 1(b), the instantaneous energy spectrum is gapped for $k \neq 0$, however, for $k = 0$ the gap closes when $\epsilon_{-,0}(t) = \Omega_R(t)/2 - \sqrt{[\Delta(t)^2 + \mu(t)^2]} = 0$. Such a gap closing without change in the symmetry of the ground state (which remains SF for all Ω_R) signifies a TPT [54, 59, 60] between topological [$\epsilon_{-,0}(t) > 0$] and conventional SF phases [$\epsilon_{-,0}(t) < 0$]. For $\Omega_R = 0$, the positive and negative bands are doubly degenerate at $k = 0$; any non-zero Ω_R lifts this degeneracy.

To study the dynamics around the TPT, we propose to prepare conventional SFs [$\epsilon_{-,0}(t) < 0$] at non-zero temperature T . We then drive the system through the TPT by changing Ω_R according to our ramp protocol with $\Omega_{Rf} > 2\sqrt{\Delta_f^2 + \mu_f^2}$ (where the subscript f denotes the quantities at time $t = t_{\text{ramp}}$) such that the ramp crosses the TPT (cf. Fig. 1).

We first analytically study the dynamics, considering the simple case of slow ramps at $T = 0$. In this limit, excitations occur near $k = 0$ and at the transition times $t = t_{c(1,2)}$, given by the roots of $\Omega_R(t_c) = 2\sqrt{\Delta(t_c)^2 + \mu(t_c)^2}$, where the Fermi gas changes from conventional to TSF and vice versa. For $\hbar^2 k^2/2m \ll \alpha k$, we approximate

$$\mathcal{H}_{\text{BdG},k}(t) \approx \alpha k \tau_z \sigma_z - \mu(t) \tau_z + \frac{\Omega_R(t)}{2} \sigma_x + \Delta(t) \tau_x. \quad (4)$$

In this limit, excitations occur only between the $E_{1,k}$ and $E_{-1,k}$ bands [cf. Fig. 1(b)]. At $k = 0$, the eigenenergies are $\pm \tilde{E}_0(t)$ where $\tilde{E}_0(t) = |\sqrt{\Delta(t)^2 + \mu(t)^2} - \Omega_R(t)/2|$ with eigenstates $\tilde{\phi}_0^+(t) = (\cos[\theta(t)/2], \sin[\theta(t)/2]) \otimes (1, 1)/\sqrt{2}$ and $\tilde{\phi}_0^-(t) = (-\sin[\theta(t)/2], \cos[\theta(t)/2]) \otimes (1, -1)/\sqrt{2}$, where $\tilde{\phi}_0^\pm(t)$ corresponds to positive and negative bands [with pseudospin $|\pm\rangle \equiv (|\uparrow\rangle \pm |\downarrow\rangle)/\sqrt{2}$] and $\cos \theta(t) \equiv \mu(t)/(\sqrt{\Delta(t)^2 + \mu(t)^2})$. In the subspace of these eigenstates, the effective low-energy Hamiltonian near $k = 0$ is

$$\tilde{\mathcal{H}}_{\text{BdG},k}(t) = \tilde{\alpha}(t) k \eta_x + \tilde{E}_0(t) \eta_z, \quad (5)$$

where $\tilde{\alpha}(t) = \alpha \sin \theta(t)$, $\eta_x = \tilde{\phi}_0^+(t)[\tilde{\phi}_0^-(t)]^\dagger + \text{h.c.}$, $\eta_z = \tilde{\phi}_0^+(t)[\tilde{\phi}_0^+(t)]^\dagger - \tilde{\phi}_0^-(t)[\tilde{\phi}_0^-(t)]^\dagger$, and $2\eta_y = -i[\eta_z, \eta_x]$. Equation (5) is a two-parameter driven Hamiltonian [61] with instantaneous energy eigenvalues $\pm \tilde{E}_k(t)$ where $\tilde{E}_k(t) = \sqrt{\tilde{E}_0(t)^2 + \tilde{\alpha}(t)^2 k^2}$.

We analyze the dynamics of the TPT using $\tilde{\mathcal{H}}_{\text{BdG},k}(t)$ where the single-particle state of the system at time t is given by

$$\tilde{\phi}_k(t) = b_k^+(t) \begin{pmatrix} w_k^+(t) \\ \text{sgn}(k) w_k^-(t) \end{pmatrix} + b_k^-(t) \begin{pmatrix} -\text{sgn}(k) w_k^-(t) \\ w_k^+(t) \end{pmatrix}, \quad (6)$$

with the initial condition $b_k^-(0) = 0$ and $b_k^+(0) = 1$. These two-component vectors are expressed in the basis $\tilde{\phi}_0^\pm$

with $w_k^\pm(t) = \sqrt{[1 \pm \tilde{E}_0(t)/\tilde{E}_k(t)]/2}$. The Schrödinger equation for the system then leads to $i\hbar \partial_t \tilde{b}_k(t) = \tilde{\mathcal{H}}_{\text{BdG},k}(t) \tilde{b}_k(t)$ where $\tilde{b}_k(t) = [b_k^+(t), b_k^-(t)]^\top$.

We make further analytical progress by ignoring the self-consistency condition so that the system can be treated as a collection of two-level system for each $(k, -k)$ pair and use the adiabatic-impulse approximation [37, 65–69] which describes such periodic dynamics accurately for low frequency and/or large amplitude drives. Within this approximation, excitations are produced only near the critical gap-closing times $t_{c(1,2)}$ when the system enters the impulse regime; otherwise the dynamics occur adiabatically in each band and the system accumulates dynamical phase $U(t_f, t_i) = \exp[-i\eta_z \int_{t_i}^{t_f} dt \tilde{E}_k(t)/\hbar]$. In the former regime, near the gap-closing times $t_{c(1,2)}$, excitations are produced and the evolution operator is [37]

$$N = \sqrt{1 - p_k} [i \sin(\varphi_{S,k}) - \eta_z \cos(\varphi_{S,k})] - i\eta_y \sqrt{p_k}, \quad (7)$$

where $p_k = \exp(-2\pi\delta_k)$ is the probability of excitation formation in each passage through the critical point [24, 25] with $\delta_k = (\alpha k)^2/(2\hbar|d\tilde{E}_0(t)/dt|_{t_c})$, and $\varphi_{S,k} = \pi/4 + \delta_k(\ln \delta_k - 1) + \arg \Gamma(1 - i\delta_k)$ is the Stokes phase originating from the interference of the parts of the system wavefunction in the instantaneous ground and excited states at $t = t_{c(1,2)}$. These results give the probability of defect formation

$$P_k^{\text{ex}} = 4p_k(1 - p_k) \sin^2 \Phi_{\text{St},k} \quad (8)$$

at $t = 2t_{\text{ramp}}$, where $\Phi_{\text{St},k} = \zeta_{2k} + \varphi_{S,k}$ is the Stückelberg phase and $\zeta_{2k} = \int_{t_{c1}}^{t_{c2}} dt \tilde{E}_k(t)/\hbar$ is the dynamical phase factor accumulated during passage between the two crossings of the gap-closing points [37, 67, 69]. Since the excitations occur near $k \sim 0$ where $E_{\pm 1,k}$ band approximately corresponds to spin $|\pm\rangle$ (along x -direction), P_k^{ex} is directly related to changes in the SRMD $\delta n_{k\pm}$ measured along the pseudospin x -direction. Furthermore, within these approximations, $|d\tilde{E}_0(t)/dt|_{t_{c(1,2)}} = \Omega_{Rf}/(2t_{\text{ramp}})$, and it can be shown that P_k^{ex} is a function of $k\sqrt{t_{\text{ramp}}}$ only. Thus, the integrated change of the SRMD $\delta \tilde{n}_\pm = \int dk \delta n_{k\pm}$ displays KZ scaling $\sim \sqrt{t_{\text{ramp}}}$ of defect density for a system dynamically evolved through the TPT. We now show that these properties persist even when the self-consistency conditions for $\Delta(t)$ and $\mu(t)$ are imposed and at non-zero T (see Fig. 2).

We solve for the dynamics of the single-particle density matrix $\rho_k^{ab}(t) = \langle \Psi_k^a(t) \Psi_k^b(t) \rangle$ self-consistently and at finite initial temperature, where a, b denote the indices of elements in Nambu basis (see page 2). The density matrix obeys the equation of motion

$$i\hbar \partial_t \rho_k(t) = [\mathcal{H}_{\text{BdG},k}(t), \rho_k(t)], \quad (9)$$

subject to the self-consistency conditions (see SM [69] for

the derivation)

$$\begin{aligned}\Delta(t) &= \frac{g}{4} \int dk \text{Tr}[\rho_k(t)\tau_x], \\ \mu(t) &= \frac{g}{4\Delta(t)} \int dk \text{Tr}[\rho_k(t)\Lambda_k(t)],\end{aligned}\quad (10)$$

where $\Lambda_k(t) = (\xi_k + \alpha k \sigma_z)\tau_x - \Delta(t)\tau_z$. Our system begins in the thermal state

$$\rho_k(t) = \sum_{E_{nk} < 0} f_{nk} \chi_{nk}(t) \chi_{nk}^\dagger(t) + (1 - f_{nk}) \tilde{\chi}_{n,-k}(t) \tilde{\chi}_{n,-k}^\dagger(t),$$

where $f_{nk} = [\exp(E_{nk}(0)/k_B T) + 1]^{-1}$ is the Fermi function of the initial Hamiltonian, and k_B is Boltzmann's constant. The wavefunction $\chi_{nk}(t)$ with its particle-hole conjugate $\tilde{\chi}_{nk}(t) = \tau_y \sigma_y \chi_{-n-k}^*(t)$ begin as eigenfunctions of the initial Hamiltonian and evolve according to $i\hbar \partial_t \chi_{nk}(t) = \mathcal{H}_{\text{BdG},k}(t) \chi_{nk}(t)$. Figure 1(a) shows the resulting time profiles of the pairing potential obtained from solving the td-BdGE (see SM [69] for the time dependence of all parameters and remarks on the numerical simulation).

We numerically solved the td-BdGE for the change in the SRMD $\delta n_{k\pm} = \text{Tr} \{ [\rho_k(2t_{\text{ramp}}) - \rho_k(0)] [(1 + \tau_z)(1 \pm \sigma_x)] \} / 4$. Figure 2 shows that δn_{k-} still exhibits Stückelberg oscillations even with inclusion of the self-consistency conditions and at $T > 0$. Furthermore, for $t_{\text{ramp}} \gg \hbar/\Delta_f$, we still see $\delta n_{k\pm} \sim k\sqrt{t_{\text{ramp}}}$ (see SM [69] for explicit demonstration of the scaling) and the integrated change in SRMD $\delta \tilde{n}_{\pm} = \int dk \delta n_{k\pm}$ therefore scales with $\sqrt{t_{\text{ramp}}}$, thus showing the robustness of such interference phenomenon in the present system. We verified that these features appear only if $\Omega_{\text{Rf}} > 2\sqrt{\Delta_f^2 + \mu_f^2}$, where the ramp takes the system through the TPT; thus both the KZ scaling and the presence of Stückelberg oscillations mark the TPT. In our calculation, we ignored the effect of phase fluctuation as this effect can be suppressed by coupling an array of 1D SOCFGs [71–74].

The parameters used for the plots in Fig. 2 are realistic for 1D SOCFG experiments. For experiments with ^{40}K , the Raman laser beams, coupling the $|\uparrow\rangle \equiv |9/2, -7/2\rangle$ and $|\downarrow\rangle \equiv |9/2, -9/2\rangle$ states, have laser wavelength $\lambda_r = 768.86$ nm giving the recoil energy $E_r = \hbar \times 8.445$ kHz, and time $t_r = \hbar/E_r \approx 20$ μs [57]. The single-body decay time due to photons scattering from the Raman lasers is about 60 ms [57] and the lifetime owing to three-body recombination is about 200 ms [75]. We consider SOCFGs with Fermi energy $E_F = E_r$. The 1D Fermi gas criterion is satisfied when $E_F < \hbar\omega_{\perp}$; for the lateral trapping frequency $\omega_{\perp}/2\pi = 5 \times 10^4$ Hz which corresponds to characteristic harmonic oscillator length $d_{\perp} = \sqrt{\hbar/m\omega_{\perp}} \approx 1345a_0$ where a_0 is the Bohr radius, the parameters used in the calculation for the plots in Fig. 2 correspond to linear density $\tilde{n} \approx 5$ μm^{-1} and 1D

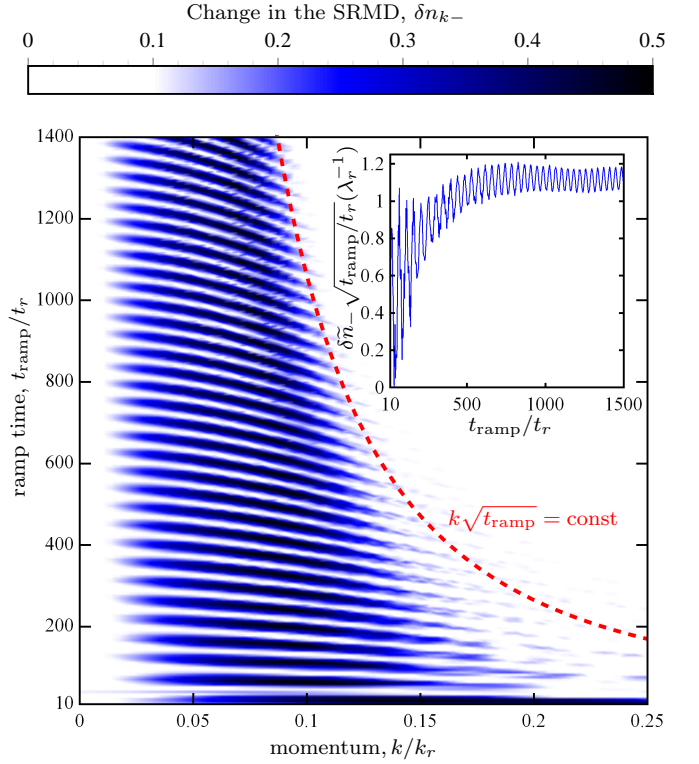


FIG. 2. (Color online) Change in the SRMD δn_{k-} for spin $|- \rangle = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$ as a function of t_{ramp}/t_r and k/k_r . For large t_{ramp} , the width of the oscillation envelopes scale with $1/\sqrt{t_{\text{ramp}}}$ as shown by the red dashed line. δn_{k-} is symmetric with respect to $k = 0$; thus for illustration purpose, we only plot δn_{k-} for $k \geq 0$. Note that $\delta n_{k+} = -\delta n_{k-}$. Inset: integrated change in SRMD $\delta \tilde{n}_{-} = \int dk \delta n_{k-}$ as a function of t_{ramp}/t_r exhibiting oscillations, with amplitude of the oscillations at large t_{ramp} scaling like $\sqrt{t_{\text{ramp}}}$ as can be read off directly from the y axis. The plots are obtained by numerically solving Eqs. (9) and (10) with initial conditions: $\mu(0) = 0$, $\Delta(0) = 2E_r$ and $\Omega_{\text{R}}(0) = 0$ for a temperature $k_B T = 0.1E_F$ (which is below the critical temperature $T_c = 0.19T_F$ [76, 77]) and $\Omega_{\text{Rf}} = 3.12E_r$.

interaction strength $g_{1\text{D}} \approx -0.73E_r\lambda_r$ (or 3D scattering length $a_{3\text{D}} \approx -2870a_0$ [64]). For these values, Fig. 2 shows that the Stückelberg oscillations and KZ scaling behavior of the SRMD can be observed within the experimentally limiting single-body decay time ($\approx 3000t_r$), thus is feasible experimentally.

Our dip-in-dip-out protocol is quite general and can be gainfully used for observing features related to quantum phase transitions between long-lived and short-lived phases of ultracold bosonic and fermionic atoms. In addition, it provides a route to escaping the heating problem which is one of the major obstacles in measuring properties of such systems in or near their short-lived phases. Moreover, our work also shows that such a protocol applied to ultracold atom systems, including the one we analyzed in details, may provide us with test beds for

observation of both KZ scaling [28–35] and Stückelberg interference phenomenon [78–80].

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