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New Physics in Astrophysical Neutrino Flavor

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Astrophysical neutrinos are powerful tools to investigate the fundamental properties of particle physics through their flavor content. In this paper, we perform the first general new physics study on ultra high energy neutrino flavor content by introducing effective operators. We find that at the current limits on these operators, new physics terms cause maximal effects on the flavor content, however, the flavor content at Earth is confined to a region related to the assumed initial flavor content. Furthermore, we conclude that a precise measure of the flavor content at Earth will provide orders of magnitude improvement on new physics bounds. Finally, we discuss the current best fits of flavor content of the IceCube data and their interplay with new physics scenarios.

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Introduction — The existence of extra-terrestrial ultra high energy neutrinos has been confirmed by the Ice-Cube neutrino observatory [1, 2], opening the possibility to study ultra high energy particle production mechanisms as well as new neutrino physics [3, 4]. The nature of these neutrinos from 35 TeV to 2 PeV is still a puzzle, at the moment there are many astrophysical and beyond the standard model candidate sources [5–10] that may produce these neutrinos. Currently, there is no statistically significant spatial correlation between observed neutrinos and potential sources [11, 12].

Even though the sources of these neutrinos remains unknown, it is still possible to find evidence of new physics. The vacuum neutrino propagation Hamiltonian is linearly proportional to the neutrino square mass differences and inversely proportional to the neutrino energy. For astrophysical ultra high energy neutrinos this operator is suppressed allowing to look for extremely tiny new physics effects otherwise cannot be seen. In the standard oscillation scenario, for any given initial flavor composition, the final composition, after the propagation, lies in a small region on the flavor triangle close to $(\phi_e:\phi_\mu:\phi_\tau)=(1:1:1)$. The flavor content of the astrophysical neutrinos has been studied in [1, 2, 13–17]. These analyses find flavor content is statistically consistent with the standard oscillations expectations. Future data will clarify the IceCube astrophysical event flavor composition.

In this paper, we perform the first general new physics study of the astrophysical neutrino flavor content by introducing effective operators in the standard three neutrino scenario with unitary evolution. This is the far most general approach to study new physics in astrophysical neutrino flavors, and this approach covers many exotic particle physics models. There are few cases we do not consider in this paper. First, the model we work are limited within lepton number conservation, and we do not consider models such as the neutrino-antineutrino oscillations [18, 19]. Second, we do not consider neutrino decay model which violates unitary evolution and was discussed elsewhere [20]. Similarly, we also do not consider models with sterile neutrino states [21]. The sterile neutrino mixing matrix elements are known to be miniscule comparing with the active neutrino mixing elements [21–25], and the contribution to the transition probability due to the sterile neutrinos is suppressed by the sterile-active matrix element to the fourth power.

Ultra High Energy Astrophysical Neutrino Oscillations — Neutrinos change lepton flavors as they propagate macroscopic distances. This is due to the fact that the neutrino propagation eigenstates are not the eigenstates of the charged current weak interaction. In presence of a dense medium the decoherent scattering interactions are important [26], but in this paper we assume vacuum propagation.

In general the relation between the propagation eigenstates $|\nu_i\rangle$, and the flavor eigenstates $|\nu_{\alpha}\rangle$, is given by a unitary transformation V(E),

$$|\nu_{\alpha}\rangle = \sum_{i} V_{\alpha i}(E)|\nu_{i}\rangle .$$
 (1)

For astrophysical neutrinos, the propagation distance is much longer than the oscillation length, and in this limit the oscillation from flavor state $|\nu_{\alpha}\rangle$ to a flavor state $|\nu_{\beta}\rangle$ can be averaged,

$$\bar{P}_{\nu_{\alpha} \to \nu_{\beta}}(E) = \sum_{i} |V_{\alpha i}(E)|^{2} |V_{\beta i}(E)|^{2} ,$$
 (2)

where the probability depends only on the mixing matrix elements $|V_{\alpha i}(E)|$, which is in general energy dependent.

Using the probability given in this equation and the flux at production ϕ_{α}^{p} , we can calculate the neutrino flux at Earth, $\phi_{\beta}^{\oplus}(E)$, for a flavor β . It is more convenient to define the energy averaged flavor composition as

$$\bar{\phi}_{\beta}^{\oplus} = \frac{1}{|\Delta E|} \int_{\Delta E} \sum_{\alpha} \bar{P}_{\nu_{\alpha} \to \nu_{\beta}}(E) \phi_{\alpha}^{p}(E) dE , \qquad (3)$$

where we assume E^{-2} power law for the production flux and $\Delta E = [10 \text{ TeV}, 10 \text{ PeV}]$. Note, however, that our main results are largely insensitive to the spectral index. We also assume that all flavors have the same energy dependence at the source.

In astrophysics charged pion decay from proton-proton collisions is one of the preferred neutrino production channels. In this scenario the initial flavor composition is $(\phi_e:\phi_\mu:\phi_\tau)=(1:2:0)$. Other scenarios such as rapid muon energy loss produce (0:1:0), neutron decay dominated sources produce (1:0:0) are of interest, while compositions such as (0:0:1) are not expected in the standard particle astrophysics scenarios. In order to plot the flavor content in a flavor triangle we introduce the flavor fraction, $\alpha_\beta^\oplus = \bar{\phi}_\beta^\oplus/\sum_\gamma \bar{\phi}_\gamma^\oplus$. For the vacuum propagation, the Hamiltonian of the

For the vacuum propagation, the Hamiltonian of the standard neutrino oscillation only depends on the neutrino mass term,

$$H = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^{\dagger} = \frac{1}{2E} U M^2 U^{\dagger} , (4)$$

where E is neutrino energy, $\Delta m_{ij}^2 = m_i^2 - m_j^2$, and U is the standard lepton mixing matrix U. Throughout this paper we assume the normal mass ordering. We also performed the same study by assuming the inverted mass ordering, however, differences are minor and mass ordering does not affect any of our main conclusions.

The current measurements of the standard neutrino oscillation experiments allows us to determine the astrophysical neutrino flavor content at detection given an assumption of the neutrino production. In Fig. 1 we show allowed regions of the flavor content at Earth, where we use the standard mixing angles and their errors from the global fits [27] in order to produce probability density distributions for the flavor content. Since the CP-phase is not strongly constrained by neither terrestrial [28, 29] nor astrophysical [30] neutrinos, we assume a flat distribution from 0 to 2π . Note that for simplicity we use the larger of the asymmetric errors and implement them as Gaussian. In the left plot, we assume four different production flavor composition hypotheses. We observe that all the allowed regions of astrophysical neutrino flavor content at Earth are close to (1:1:1), except when the initial flavor content is (1:0:0) [31]. In the right plot, we show the allowed region of the flavor content of the astrophysical neutrinos with all possible astrophysical production mechanisms, i.e., the production flavor

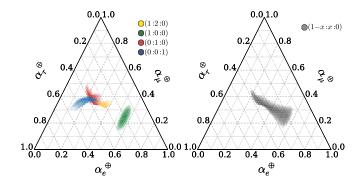


FIG. 1: Allowed regions of the flavor content at Earth using the priors on the mixing angles and errors given from the current neutrino oscillation measurements. In the left plot, the different colors correspond to different assumptions on flavor content at the production. The color intensity is proportional to the probability density. In the right plot, we further sample the initial flavor content as (x:1-x:0).

composition is sampled with (x:1-x:0) uniformly on x [32]. Therefore, this rather narrow band covers all possible scenarios of the standard neutrino oscillations with the standard astrophysical neutrino production mechanisms.

New Physics in Effective Hamiltonians — An effective way of introducing new physics in neutrino oscillations is by introducing new operators. The full Hamiltonian that incorporates the new physics operators, in the flavor basis, can be expressed as,

$$H = \frac{1}{2E} U M^2 U^{\dagger} + \sum_n \left(\frac{E}{\Lambda_n}\right)^n \tilde{U}_n O_n \tilde{U}_n^{\dagger} = V^{\dagger}(E) \Delta V(E) ,$$

where $O_n = \operatorname{diag}(O_{n,1}, O_{n,2}, O_{n,3})$ and $\Delta = \operatorname{diag}(\Delta_1, \Delta_2, \Delta_3)$. O_n and Λ_n set the scale of the new physics and \tilde{U}_n is the mixing matrix that describes the new physics flavor structure. In the effective theory approach, lower order operators are more relevant, thus in this work we will only study the first terms in the expansion, namely n = 0 and n = 1.

Although in this work we will study n=0 and n=1, results can be extended to higher orders. These new operators can be interpreted in different new physics contexts. Some examples for n=0 new physics are couplings between neutrinos and space time torsion [33], CPT-odd Lorenz violation [34–37], and non-standard neutrino interactions [38–41]. As for n=1 new physics operators, CPT-even Lorentz violation [42, 43] and equivalence principle violation [44, 45] are possible examples.

There are some constraints from neutrino oscillation experiments to these effective operators in the context of Lorentz and CPT violation [46]. The most stringent limits on certain parameters are obtained from Super-Kamiokande and IceCube atmospheric neutrino analyses [47, 48]. In this context the CPT-odd and CPT-even Lorentz violation coefficients are constrained to be

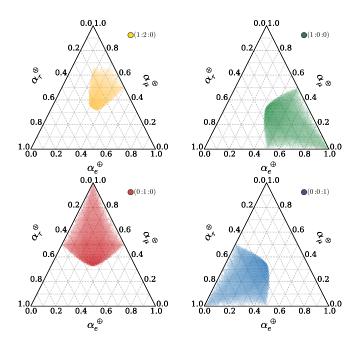


FIG. 2: Allowed region using anarchic sampling on the mixing angles for the new physics operator when the mass term in the Hamiltonian is neglected. The different plots correspond to different assumption on flavor content at production. The color intensity is proportional to the probability predicted by anarchic sampling.

 $\sim 10^{-23}$ GeV and $\sim 10^{-27}$ depending on the flavor structure \tilde{U}_n . These constraints can be used to set the scales of n=0 and n=1 operators introduced in this paper. For example, we set $O_0=1\times 10^{-23}$ GeV as a current limit of the n=0 operator, and $O_1=1\times 10^{-23}$ GeV with $\Lambda_1=1$ TeV as a current limit of n=1 operators, where $O_1=10^{-27}$. Through this paper we have assumed the scale of O_1 is of the order of O_0 without loss of generality.

Anarchic Sampling prediction and IceCube Results — In order to predict the flavor composition at Earth in the presence of new physics, the values of the mixing matrices \tilde{U}_n should be specified. In order to show a prediction with new physics operators, we have to account for all the free parameters in the mixing matrix; we use a random sampling scheme to construct the mixing matrix. A well established schema is the anarchic sampling [49–52], which samples a flat distribution given by the Haar measure,

$$d\tilde{U}_n = d\tilde{s}_{12}^2 \wedge d\tilde{c}_{13}^4 \wedge d\tilde{s}_{23}^2 \wedge d\tilde{\delta} , \qquad (5)$$

where, \tilde{s}_{ij} , \tilde{c}_{ij} , and $\tilde{\delta}$ correspond to sines, cosines, and phase for the new physics *n*-operator mixing angles. We omit the Majorana phases since they do not affect neutrino oscillations

In Fig.2 we show the allowed regions using an archic sampling in the case where $H=\left(\frac{E}{\Lambda_n}\right)^n \tilde{U}_n O_n \tilde{U}_n^{\dagger}$. In this

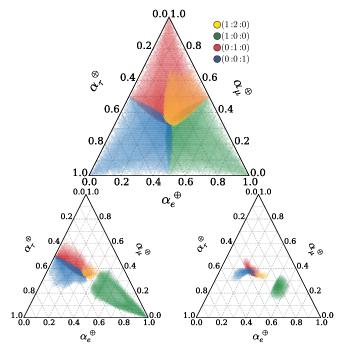


FIG. 3: Allowed region using an archic sampling on the mixing angles for the new physics n=0 operators. The top plot corresponds to the current limits on n=0 operator; the bottom left plot corresponds to $O_0=3.6\times 10^{-26}~{\rm GeV},$ while the bottom right plot corresponds to $O_0=6.3\times 10^{-28}~{\rm GeV}.$

case, we neglect the mass term and we are considering that the Hamiltonian has only one operator, i.e., $V = \tilde{U}_n$, and the result does not depend on n. Each plot in this figure correspond to a different production flavor composition. We show the pion decay production (1:2:0) [yellow], beta decay (1:0:0) [green], muon cooling (0:1:0) [red] and for completeness we show the exotic ν_{τ} dominant model (0:0:1) [blue]. The color density in these plots is a representation of the probability given by the anarchic sampling.

In Fig.3 we show the case where we have a mass term and the n=0 operators. In the top plot, we set $O_0=1.0\times 10^{-23}$ GeV, corresponding to the order of the current best limit on this operator. On the bottom left plot we set $O_0=3.6\times 10^{-26}$ GeV and the bottom right plot we set $O_0=6.3\times 10^{-28}$ GeV. These values are chosen because they have the same magnitude as the mass term with neutrino energy of $E_{\nu}=35$ TeV and $E_{\nu}=2$ PeV respectively. In this plot, the colors represent different assumptions in the production flavor content, and the color intensity is the probability given by the anarchic sampling as in Fig.2.

In Fig.4 we show the case for the n=1 operators. The color notations and their intensities have equivalent meaning as Fig.3. As before, the top plot we set the new physics operator to the current best limit $\frac{O_1}{\Lambda_1} \sim 10^{-27}$. This is achieved by choosing $O_1 = O_0 = 1.0 \times 10^{-23} \text{ GeV}$

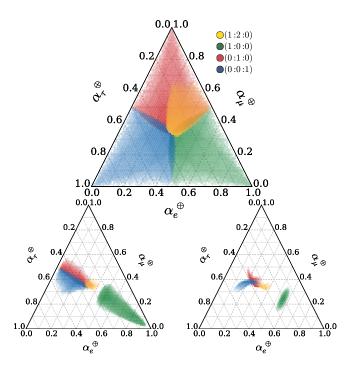


FIG. 4: Allowed region using anarchic sampling on the mixing angles for the new physics n=1 operators. The top plot corresponds to the current limits on n=1 operator; the bottom left plot corresponds to $O_1=3.6\times 10^{-26}$ GeV and $\Lambda_1=35$ TeV $(\frac{O_1}{\Lambda_1}=1.0\times 10^{-30})$, while the bottom right plot corresponds to $O_1=6.3\times 10^{-28}$ GeV and $\Lambda_1=2$ PeV $(\frac{O_1}{\Lambda_1}=3.2\times 10^{-34})$.

and $\Lambda_1=1$ TeV. In the bottom left plot, $O_1=3.6\times 10^{-26}$ GeV and $\Lambda_1=35$ TeV are used, and in the bottom right plot the parameters are $O_1=6.3\times 10^{-28}$ GeV and $\Lambda_1=2$ PeV. These choices make new physics to be the same magnitude as the mass term with neutrino energy of $E_{\nu}=35$ TeV and $E_{\nu}=2$ PeV, respectively. In other words, these choices explore new physics down to $\frac{O_1}{\Lambda_1}=1.0\times 10^{-30}$ and $\frac{O_1}{\Lambda_1}=3.2\times 10^{-34}$. This can be compared, for example, to the aforementioned best limits of Lorentz and CPT violation in neutrino sector [47, 48]. The potential limits from astrophysical neutrino flavor content can be well beyond what terrestrial neutrino experiments can achieve.

From Fig.3 and Fig.4 we observe that the allowed regions in the flavor triangle change in a similar way as a function of the energy scale. This is true for any higher operators, because what matters is the scale where they dominates over standard neutrino mass term, and these two operators are sufficient to predict behaviors of any higher order operators. Comparing Fig.3 and Fig.4 with respect to Fig.2 where the allowed regions are more symmetric, there is a preferred region along the vacuum oscillation triangle shown in Fig.1. It is interesting to notice that due to the unitary evolution and the fact that the oscillations are averaged, for a given production flavor

content, only a subset of the flavor triangle is accessible. The pion decay production mechanism (1:2:0) is one of the most natural astrophysical scenarios for high energy neutrino production. From Fig.3 and Fig.4 the allowed region for this case is the smallest, which means that if future measurements exclude this region, the pion production dominant mechanism is excluded regardless of the presence of new oscillation physics.

In the analyses of the IceCube high energy neutrino events, different results have been shown. The first result [53], using the IceCube result [2], showed a best fit at (1:0:0) disfavoring (1:1:1) at 92% C.L. Later, the same authors did an improved analysis [14] including energy dependence and extra systematic errors, finding that the best fit may move considerably depending on the features of the energy spectrum such as including an energy cutoff or not. The IceCube collaboration later published an analysis of the flavor ratio above 30 TeV [15] finding a best fit at $(0:\frac{1}{5}:\frac{4}{5})$, as well as excluding (1:0:0) and (0:1:0) at more than 90% C.L. This IceCube result shows a best fit dominated by the ν_{τ} component, which can be explained by the correlation between the energy cutoff and the Glashow resonance, as noted by [14]. In obtaining this best fit, the IceCube collaboration has assumed an equal amount of neutrinos and antineutrinos, which best corresponds to a proton-proton source. On the other hand, if the neutrino source is proton-photon dominated then the neutrino-antineutrino ratio weaken making the previous conclusion. It is interesting to notice that if this IceCube best fit does not change considerably after adding more data, the production mechanism has to include a ν_{τ} component. This is because the new physics in the propagation can not give the best fit value for any plausible astrophysical scenarios. This implies not only new physics in the neutrino oscillations, but also new physics in the production mechanism.

Conclusions — We performed the first new physics study on the astrophysical neutrino flavor content using effective operators in the standard three neutrino scenario. These operators can represent a variety of models such as Lorentz and CPT violation, violation of equivalent principle, cosmic torsion, non-standard interactions, etc, making this work to be the most general study of new physics in astrophysical neutrino flavor content to date.

We found that large effects in the flavor content at Earth are still allowed with given terrestrial bounds on new physics in the neutrino sector. This implies that an accurate measurement of the flavor content will provide stronger bounds on new physics. Furthermore, there are regions on the flavor triangle that cannot be accessed even in the presence of new physics in the neutrino oscillations for any of the plausible astrophysical mechanisms. Interestingly, that the most natural astrophysical mechanism, pion decay, has the smallest region in the flavor triangle even when new physics is considered. The

real astrophysical neutrino production mechanism in nature may be the combination of channels, but our results hold for such a case. Therefore, a higher statistics measurement by future neutrino telescopes, such as IceCube-Gen2 [54], could reveal not only the initial neutrino flavor ratios, but also the presence of new physics in neutrinos.

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- M. Aartsen et al. (IceCube), Science 342, 1242856 (2013), 1311.5238.
- [2] M. Aartsen et al. (IceCube Collaboration), Phys.Rev.Lett. 113, 101101 (2014), 1405.5303.
- [3] D. J. Chung, E. W. Kolb, and A. Riotto, Phys.Rev. D59, 023501 (1999), hep-ph/9802238.
- [4] B. Feldstein, A. Kusenko, S. Matsumoto, and T. T. Yanagida, Phys.Rev. D88, 015004 (2013), 1303.7320.
- [5] Y. Bai, R. Lu, and J. Salvado (2013), 1311.5864.
- [6] Y. Bai, A. Barger, V. Barger, R. Lu, A. Peterson, et al., Phys.Rev. **D90**, 063012 (2014), 1407.2243.
- [7] F. Krauß, M. Kadler, K. Mannheim, R. Schulz, J. Trtedt, et al., Astron. Astrophys. 566, L7 (2014), 1406.0645.
- [8] A. Esmaili and P. D. Serpico, JCAP 1311, 054 (2013), 1308.1105.
- [9] L. A. Anchordoqui, T. C. Paul, L. H. M. da Silva, D. F. Torres, and B. J. Vlcek, Phys.Rev. **D89**, 127304 (2014), 1405.7648.
- [10] W. Winter, Phys.Rev. **D88**, 083007 (2013), 1307.2793.
- [11] M. Aartsen et al. (IceCube Collaboration), Phys.Rev. D91, 022001 (2015), 1410.1749.
- [12] A. M. Taylor, S. Gabici, and F. Aharonian, Phys.Rev. D89, 103003 (2014), 1403.3206.
- [13] O. Mena, S. Palomares-Ruiz, and A. C. Vincent, Phys.Rev.Lett. 113, 091103 (2014), 1404.0017.
- [14] S. Palomares-Ruiz, A. C. Vincent, and O. Mena, Phys.Rev. **D91**, 103008 (2015), 1502.02649.
- [15] M. Aartsen et al. (IceCube) (2015), 1502.03376.
- [16] A. Palladino, G. Pagliaroli, F. Villante, and F. Vissani, Phys.Rev.Lett. 114, 171101 (2015), 1502.02923.
- [17] N. Kawanaka and K. Ioka (2015), 1504.03417.
- [18] V. A. Kostelecky and M. Mewes, Phys.Rev. D69, 016005 (2004), hep-ph/0309025.
- [19] J. Diaz, T. Katori, J. Spitz, and J. Conrad, Phys.Lett. B727, 412 (2013), 1307.5789.
- [20] G. Pagliaroli, A. Palladino, F. Vissani, and F. L. Villante (2015), 1506.02624.
- [21] J. Conrad, C. Ignarra, G. Karagiorgi, M. Shaevitz, and J. Spitz, Adv. High Energy Phys. 2013, 163897 (2013), 1207.4765.
- [22] A. Y. Smirnov and R. Zukanovich Funchal, Phys.Rev.

- **D74**, 013001 (2006), hep-ph/0603009.
- [23] J. Kopp, P. A. N. Machado, M. Maltoni, and T. Schwetz, JHEP 1305, 050 (2013), 1303.3011.
- [24] C. Ignarra, Nucl. Phys. Proc. Suppl. 237-238, 173 (2013).
- [25] M. Fukugita and T. Yanagida, Phys.Lett. **B174**, 45 (1986).
- [26] C. A. Arguelles Delgado, J. Salvado, and C. N. Weaver (2014), 1412.3832.
- [27] M. Gonzalez-Garcia, Phys.Dark Univ. 4, 1 (2014).
- [28] K. Abe et al. (T2K Collaboration), Phys.Rev.Lett. 112, 061802 (2014), 1311.4750.
- [29] P. Adamson et al. (MINOS), Phys.Rev.Lett. 110, 171801 (2013), 1301.4581.
- [30] A. Chatterjee, M. M. Devi, M. Ghosh, R. Moharana, and S. K. Raut, Phys. Rev. **D90**, 073003 (2014), 1312.6593.
- [31] A. Palladino and F. Vissani (2015), 1504.05238.
- [32] M. Bustamante, J. F. Beacom, and W. Winter (2015), 1506.02645.
- [33] V. De Sabbata and M. Gasperini, Nuovo Cim. A65, 479 (1981).
- [34] V. A. Kostelecky and M. Mewes, Phys.Rev. D69, 016005 (2004), hep-ph/0309025.
- [35] A. Kostelecky and M. Mewes, Phys.Rev. D85, 096005 (2012), 1112.6395.
- [36] V. D. Barger, S. Pakvasa, T. J. Weiler, and K. Whisnant, Phys.Rev.Lett. 85, 5055 (2000), hep-ph/0005197.
- [37] S. R. Coleman and S. L. Glashow, Phys.Rev. D59, 116008 (1999), hep-ph/9812418.
- [38] J. W. F. Valle, Phys. Lett. B199, 432 (1987).
- [39] M. M. Guzzo, A. Masiero, and S. T. Petcov, Phys. Lett. B260, 154 (1991).
- [40] Y. Grossman, Phys. Lett. B359, 141 (1995), hepph/9507344.
- [41] S. Bergmann, Y. Grossman, and E. Nardi, Phys. Rev. D60, 093008 (1999), hep-ph/9903517.
- [42] S. Glashow, A. Halprin, P. Krastev, C. N. Leung, and J. T. Pantaleone, Phys.Rev. **D56**, 2433 (1997), hepph/9703454.
- [43] J. S. Diaz, A. Kostelecky, and M. Mewes, Phys.Rev. D89, 043005 (2014), 1308.6344.
- [44] M. Gasperini, Phys.Rev. **D39**, 3606 (1989).
- [45] M. Butler, S. Nozawa, R. Malaney, and A. Boothroyd, Phys.Rev. **D47**, 2615 (1993).
- [46] V. A. Kostelecky and N. Russell, Rev.Mod.Phys. 83, 11 (2011), 0801.0287.
- [47] K. Abe et al. (Super-Kamiokande), Phys.Rev. D91, 052003 (2015), 1410.4267.
- [48] R. Abbasi et al. (IceCube), Phys.Rev. **D82**, 112003 (2010), 1010.4096.
- [49] L. J. Hall, H. Murayama, and N. Weiner, Phys.Rev.Lett. 84, 2572 (2000), hep-ph/9911341.
- [50] N. Haba and H. Murayama, Phys.Rev. **D63**, 053010 (2001), hep-ph/0009174.
- [51] A. de Gouvea and H. Murayama (2012), 1204.1249.
- [52] A. de Gouvea and H. Murayama, Phys.Lett. B573, 94 (2003), hep-ph/0301050.
- [53] S. Palomares-Ruiz, O. Mena, and A. C. Vincent (2014), 1411.2998.
- [54] M. Aartsen et al. (IceCube) (2014), 1412.5106.