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Theoretically palatable flavor combinations of astrophysical neutrinos

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The flavor composition of high-energy astrophysical neutrinos can reveal the physics governing their production, propagation, and interaction. The IceCube Collaboration has published the first experimental determination of the ratio of the flux in each flavor to the total. We present, as a theoretical counterpart, new results for the allowed ranges of flavor ratios at Earth for arbitrary flavor ratios in the sources. Our results will allow IceCube to more quickly identify when their data imply standard physics, a general class of new physics with arbitrary (incoherent) combinations of mass eigenstates, or new physics that goes beyond that, e.g., with terms that dominate the Hamiltonian at high energy.

Introduction.— The discovery of astrophysical neutrinos with energies up to a few PeV by the IceCube Collaboration [1–4] is tremendously important for multi-messenger astronomy as well as for new tests of neutrino properties. While the origin of these neutrinos is still unclear, there are important clues in the energy spectrum and sky distribution, and a component from cosmic distances (~ Gpc) is required [5–24]. These are the most extreme energies and distances for detected neutrinos.

The flavor composition is also expected to be important, because the ratio of flux in each flavor to the total cancels the unknown normalization. The ratios depend on the physical conditions at the source, the effects of standard flavor mixing, and on potential new physics [5, 25–36].

The first IceCube results on flavor composition have been published recently [35], and were followed by results obtained with a combined-likelihood analysis of several data sets with more statistics [37]. Accordingly, there has been intense interest in deducing flavor ratios from IceCube data [9, 31, 34, 38, 39].

In this Letter, we use ternary plots or “flavor triangles” to show the flavor composition at the Earth. We systematically explore which regions of this plot can be populated from theoretical perspectives —without or with new physics— including the uncertainties in source flavor composition and neutrino mixing parameters. We also note prospects for the proposed volume upgrade, IceCube-Gen2 [40].

We make no distinction between ν and ¯ν, because, except for yet-unobserved high-energy events, IceCube cannot distinguish between them. (In addition, their cross sections agree to better than ~5% in this energy range [41, 42].)

All plots shown in the main text are for the normal neutrino mass hierarchy (NH), in which ν1 is the lightest mass eigenstate. Corresponding plots for the inverted hierarchy (IH), in which ν3 is lightest, are given in the Supplemental Material; the differences are modest.

Flavor identification in IceCube.— IceCube can discriminate between muon tracks (from νµ, mostly) and cascades (from charged-current interactions of νe and ντ, mainly, and from neutral-current interactions of all flavors). If higher-energy events are observed, it will be possible to isolate νµ cascades via the Glashow resonance [43–45], and νµ and ¯νµ via double-bang and lozenged topology [46–48]. In their absence, there is an experimental degeneracy between the electron and tau neutrino flavor content at Earth [34, 35]. In contrast, theoretically predicted flavor ratios, even in models with new physics, have a µ-τ symmetry due to that mixing angle being near-maximal.

Flavor composition at the source.— The flavor composition at the source could be quite different depending on the physical conditions. For the pion decay chain, which is often used as standard (“pion beam”), one expects a composition (f e, S : f µ, S : f τ, S) = (1 : 1 : 1), with f e, S the ratio of νe + ¯νe to the total flux, where f e, S + f µ, S + f τ, S = 1. Synchrotron cooling of secondary muons in strong magnetic fields leads to a transition to (0 : 1 : 0)S (“muon damped”) at higher energies, which depends on the field strength; see, e.g., Refs. [5, 38, 49–51]. If these muons pile up at lower energies [51], or if there are contributions from charmed meson decays [29, 52, 53], then (1 : 1 : 1)S is expected. Neutron decays [5] lead to (1 : 0 : 0)S. Small deviations, < 5% in the νe/νµ ratio, are expected from effects such as the helicity dependence of muon decays [5, 54]. If several of the above processes in the source compete, arbitrary flavor compositions (f e, S : 1 − f e, S : 0) can be obtained [51]. If, in addition, ντ are produced, such as by oscillations in a matter envelope [55–57], even (f e, S : f µ, S : 1 − f e, S − f µ, S) (with 0 ≤ f µ, S ≤ 1 − f e, S) could be possible. Dark matter annihilation or decay could yield any mixture, but (1 : 1 : 1)S is the most natural.

Flavor composition at Earth.— Here we focus on a diffuse flux, which is composed of small contributions from many sources over a wide range of distances,
and detected with energy resolution $\gtrsim 10\%$ (and binned more coarsely). In this case, the neutrinos are, at least effectively, an incoherent mixture of mass eigenstates. Even for the solar $\Delta m^{2}_{23} \approx 8 \times 10^{-5}$ eV$^2$ and PeV energies, the vacuum oscillation length is only $\sim 10^{-13}$ Gpc, much smaller than the complete baseline. (Depending on the physics in the production region, there can be also wave packet decoherence in the source [58–60].) As a consequence, the flavor composition at Earth [58] is $f_{\beta,\alpha} = \sum_{\alpha} |U_{\alpha i}|^2 |f_{\alpha S}|$, with $U$ the PMNS matrix [61], implying $\sum_{\alpha} f_{\beta,\alpha} = 1$. For a pion beam, the flavor composition evolves roughly into flavor equipartition at the detector, $(\frac{1}{3} : \frac{1}{3} : \frac{1}{3})_{\beta}$.

New physics in neutrino propagation might modify the flavor composition. We categorize classes of new-physics models below.

**Flavor content of the mass eigenstates.**

Figure 1 shows the flavor content $|U_{\alpha i}|^2$ of the mass eigenstates, which is the fundamental input that determines flavor ratios at Earth without or with new physics. It also illustrates the underlying three-flavor unitarity of our analysis, i.e., $|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2 = 1$, which allows the flavor content to be displayed in a ternary plot [63]. This is appropriate because the mixing angles to sterile neutrinos must be quite small [64, 65].

The long axis of each region is set by the uncertainty in $\theta_{23}$ and $\delta_{CP}$, while the short axis is set by the uncertainty in $\theta_{13}$. The effect of the uncertainty in $\theta_{13}$ is tiny. Even if $\theta_{23}$ were to be precisely determined soon, it is less likely that $\delta_{CP}$ will be, and the uncertainty in the latter will still span a large range in $|U_{\tau 1}|^2$ and $|U_{\tau 2}|^2$.

**Standard flavor mixing.**

Figure 2 shows the allowed region for the flavor composition at Earth assuming arbitrary flavor composition at the source and standard neutrino mixing (including parameter uncertainties). The region is quite small: even at 3$\sigma$ it covers only about 10% of the available space. There is little difference between $f_{\tau S} = 0$ and $f_{\tau S} \neq 0$.

There is a theoretical symmetry along the line $(f_{e,\beta} : (1 - f_{e,\beta})/2 : (1 - f_{e,\beta})/2)$ coming from nearly-maximal mixing. On the other hand, the experimental degeneracy pulls towards $(f_{e,\beta} : 1 - 2f_{e,\beta} : f_{e,\beta})$, account of the difficulty of distinguishing between electromagnetic and hadronic cascades. Thus, theory and experiment are complementary, which enhances the discriminating power of flavor ratios.

The region shown includes the possibility of energy-dependent flavor composition at the source; see the Supplemental Material for an example. It also includes the possibility that the diffuse flux has contributions from sources with different flavor compositions, because of the linear mapping between those at the source and those at Earth.

Whereas the first IceCube flavor ratio analysis [35] used only three years of contained-vertex events, the updated analysis [37], whose exclusion curves are shown in Fig. 2, combines several different data sets collected over four years, including through-going muons. The exclusion curves of both analyses are compatible.

Figure 3 shows that if the flavor composition at the source could be restricted from astrophysical arguments, the allowed regions at Earth could become tiny (and will shrink when the mixing parameters are better known). A source composition of $(1 : 0 : 0)_{\beta}$ is already disfavored at $\gtrsim 2\sigma$. While the current IceCube fit is compatible with the standard $(\frac{1}{3} : \frac{1}{3} : \frac{1}{3})_{\beta}$ at 1$\sigma$, the best-fit point cannot be reached within the Standard Model.

An upgrade of IceCube would have excellent discrimination power, as indicated by the projected sensitivity curves we estimate for IceCube-Gen2 and show in Fig. 3. We reduced the IceCube uncertainties by a factor 5, corresponding to an exposure increased by a factor $\approx 25$ ($\sim 6$ times larger effective area [40] and twelve years instead of three). The true sensitivity might be worse (due to sparser instrumentation) or better (due to new techniques or to the discovery of flavor-identifying signals [43, 44, 46, 48, 51, 66–74]). To be conservative, we assumed the best fit will correspond to the most-frequently considered composition, $(\frac{1}{3} : \frac{1}{3} : \frac{1}{3})_{\beta}$, for which it will be most difficult to test for new physics.

**Flavor ratios with new physics.**

New physics can modify the flavor composition at production, during propagation, or in interaction. In the first two cases, it will affect the flavor composition that reaches the detector; this is our focus. In the last case—which includes, e.g., non-standard interactions [75] and renormalization group running of the mixing parameters [76]—we assume that new physics, possibly energy-dependent, can be separated by probing the interaction length in Earth via the angular dependence of the neutrino flux [77–80].

In extreme scenarios, there could be only one mass eigenstate present at detection, and the flavor composition would correspond to that of one eigenstate. This could happen if all but one mass eigenstate completely decays or if matter-affected mixing at the source singles out a specific one for emission.

Figure 4 shows the allowed region if we restrict ourselves to a general class of new-physics models—those in which arbitrary combinations of incoherent mass eigenstates are allowed (we give examples below of models that can access the area outside this region). The $\alpha$-flavor content of an allowed point is computed as $k_1 |U_{\alpha 1}|^2 + k_2 |U_{\alpha 2}|^2 + k_3 |U_{\alpha 3}|^2$, where the $k_i$ are varied under the constraint $k_1 + k_2 + k_3 = 1$ and the values of the mixing parameters are fixed. To generate the complete region, we repeat the procedure by varying the mixing parameters within their uncertainties.

For a particular new-physics model, the functional forms and values of the $k_i$ are determined by its parameters. The most dramatic examples include all variants of neutrino decay among mass eigenstates, both partial and complete [25, 81–84], and secret neutrino interactions [85–91]; the $k_i$ in these cases depend on neutrino lifetimes and new coupling constants, respectively. Other examples are pseudo-Dirac neutrinos [92–94] and decoherence on the Planck-scale structure of spacetime [95–99].
Even with this general class of new-physics models, only about 25% of the flavor triangle can be accessed. The current IceCube best fit cannot be reached even by invoking this class of physics models. IceCube-Gen2 will be needed to strongly constrain such new-physics models.

Interestingly, there is more than one way in which the standard \( \left[ \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right] \) composition can be generated, such as through the standard mixing of \( \left[ \frac{1}{3}, \frac{1}{3}, 0 \right] \), or through a fortuitous incoherent mix of mass eigenstates due to decay.

Already, complete decay in the most often used neutrino decay scenario (only \( \nu_1 \) stable) for the NH can be ruled out at \( \gtrsim 2\sigma \) (see Ref. [84] for a weaker exclusion at 1\( \sigma \) based on their own analysis of tracks and cascades), and bounds on the neutrino lifetimes can be set [102].

To access the white region in Fig. 4, a broader class of new-physics models is required. Possible examples are models with violation of CPT and/or Lorentz invariance (which alter the dispersion relations) [25, 100, 103–106], or the equivalence principle [107–109], and coupling to a torsion field [110].

All these have in common that they either invalidate the concept of decoherence in the astrophysical neutrino flavor composition or change the values of the mixing parameters. Ref. [111] adopted a generic effective theory approach in which the new-physics terms dominate the propagation Hamiltonian at high energies, and showed that such models are indeed able to populate almost the full triangle.

Another possibility is the existence of extra dimensions, which could lead to matter-like resonant mixing between active and sterile flavors [112]. Boosted dark matter [19, 113, 114] could generate neutrino-like events, even mimicking pure-flavor signatures.

Conclusions.— We have demonstrated that the allowed region of neutrino flavor composition at Earth under standard mixing is quite small, in spite of the uncertainties in the mixing parameters and flavor composition at the sources. The allowed region remains small even in the presence of a general class of new-physics models whose effect is to change the incoherent mix of mass eigenstates during propagation (e.g., neutrino decay and secret interactions). These results hardly depend on the mass hierarchy, and they hold for energy-dependent flavor compositions at the source or energy-dependent new physics, even when simultaneously present [115]; see the Supplemental Material.

In order to access the larger space of possible flavor combinations, a broader class of new physics during propagation —flavor-violating or capable of modifying the values of the mixing parameters— or at detection is required. Interestingly, the current IceCube best-fit composition lies in this region, though the standard \( \left[ \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right] \) case is not excluded.

The power of IceCube to determine the composition is enhanced by the complementarity between its experimental \( \nu_e - \nu_x \) degeneracy and the theoretical \( \nu_{\mu} - \nu_x \) symmetry coming from nearly-maximal mixing. The current bounds are not only compatible with most source compositions, but also with many potential new physics effects. However, the most favored neutrino decay scenario (only \( \nu_1 \) stable) can be already ruled out at \( \gtrsim 2\sigma \).

The smaller the allowed region with only standard mixing shown in Fig. 2 and Fig. 3, the more sensitive IceCube is to new physics. Likewise, the smaller the new-physics region shown in Fig. 4, the more sensitive IceCube is to the broader class of new physics. The recent successes in measuring neutrino mixing parameters have been essential to making these regions small. Our results provide new perspectives that will sharpen and accelerate tests of flavor ratios.

Ideally, flavor ratios would be determined using a single class of point sources at known distances. No high-energy astrophysical sources have been resolved yet, however. We have shown that, even using a diffuse flux, flavor ratios can reveal information about source conditions and neutrino properties.

Data from a volume upgrade of IceCube in combination with improved measurements of the mixing parameters, including \( \delta_{\text{CP}} \), have the potential to nail down the flavor composition at the source or to identify new physics in propagation. However, it is not possible to extract the value of \( \delta_{\text{CP}} \) from astrophysical data alone if the flavor composition at the source is not known; see the Supplemental Material.

To fully exploit the power of neutrino flavors, advances in four directions are needed:

1. A volume upgrade of IceCube (IceCube-Gen2 [40]) or a corresponding experiment in seawater (e.g., KM3NeT [116]).

2. Reduction of the uncertainties in the values of the mixing parameters (especially \( \theta_{23} \) and \( \delta_{\text{CP}} \)).

3. Improvements in experimental techniques to reconstruct neutrino flavor and energy.

4. More systematic model building to better understand, or constrain, the region of flavor ratios at Earth that could be accessed by new physics.

Given the wealth of information about neutrino production, propagation, and interaction that the flavor composition provides, its precise determination should become a high-priority goal of ongoing and near-future experimental analyses.

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FIGURES

FIG. 1. Flavor content of the three active mass eigenstates. The regions are given by the best-fit values of the mixing parameters (light yellow), and their 1σ (darker) and 3σ (darkest) uncertainty regions [62], assuming a normal mass hierarchy (NH). The tilt of the tick marks indicates the orientation with which to read the flavor content.

FIG. 2. Allowed flavor ratios at Earth with no new physics. The flavor ratios at the source are arbitrary (gray) or contain no tau flavor (red). The IceCube results are from Ref. [37].

FIG. 3. Allowed flavor ratios at Earth for different choices of source ratios, assuming standard mixing. Projected 1σ, 2σ, and 3σ exclusion curves from IceCube-Gen2 are included for comparison (gray, dotted); see main text.
FIG. 4. Allowed flavor ratios at Earth in a general class of new-physics models. These produce linear combinations of the flavor content of $\nu_3$, $\nu_2$, and $\nu_1$, shown as yellow (dashed) curves, from left to right. The standard mixing $3\sigma$ region from Fig. 2 is shown as a magenta (dotted) curve.
Flavor content
NH

Vary $\theta_{ij}, \delta_{CP}$

- Best Fit
- $1\sigma$
- $3\sigma$

$|U_{\nu i}|^2$ vs. $|U_{e i}|^2$
Std. mixing

$\theta_{ij}, \delta_{CP}: BF, 1\sigma, 3\sigma$

NH

$(f_e: f_\mu: f_\tau)_S$

- all free
- (1:2:0)
- (0:1:0)
- (1:0:0)
- (1:1:0)

IceCube 2015

IC-Gen2?
New physics
\(\theta_{ij}, \delta_{CP}: BF, 1\sigma, 3\sigma\)
NH

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IC-Gen2?