Non-Fermi-Liquid Crossovers in a Quasi-One-Dimensional Conductor in a Tilted Magnetic Field

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Non Fermi Liquid Crossovers in a Quasi-One-Dimensional Conductor in a Tilted Magnetic Field

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We consider a theoretical problem of electron-electron scattering time in a quasi-one-dimensional (Q1D) conductor in a magnetic field, perpendicular to its conducting axis. We show that inverse electron-electron scattering time becomes of the order of characteristic electron energy, $1/\tau \sim \epsilon \sim T$, in a high magnetic field, directed far from the main crystallographic axes, which indicates breakdown of the Fermi liquid theory. In a magnetic field, directed close to one of the main crystallographic axis, inverse electron-electron scattering time becomes much smaller than characteristic electron energy and, thus, applicability of Fermi liquid theory restores. We suggest that there exist crossovers (or phase transitions) between Fermi liquid and some non Fermi liquid states in a strong enough tilted magnetic field. Application of our results to the Q1D conductor $(\mathrm{Per})_2\mathrm{Au(mnt)}_2$ shows that it has to be possible to observe the above mentioned phenomenon in feasibly high magnetic fields of the order of $H \geq H^* \simeq 25\ T$.

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High magnetic field properties of quasi-one-dimensional (Q1D) and quasi-two-dimensional (Q2D) conductors have been intensively studied since the discovery of the so-called Field-Induced Spin-Density-Wave (FISDW) cascades of phase transitions in the Q1D materials $(\mathrm{TMTSF})_2\mathrm{X}$ (X=ClO_4, PF_6, etc.) [1-3]. It is important that successful theoretical explanations of the FISDW phases [3-8] were not done in the framework of the traditional theory of metals but required a novel notion - the so-called quasi-classical $3D \rightarrow 2D$ dimensional crossover. Later, different types of quasi-classical $3D \rightarrow 1D \rightarrow 2D$ dimensional crossovers were applied for explanations of such unusual properties of a metallic phase in Q1D conductors as Lebed's magic angles and Lee-Naughton-Lebed's oscillations [9]. Note, that general feature of the above mentioned dimensional crossovers is that electron spectrum changes its dimensionality in moderate magnetic fields, where the typical sizes of electron trajectories are bigger than the inter-plane distances in layered Q1D conductors. Meanwhile, it was also theoretically shown [10-13] that magnetic properties of Q1D and Q2D superconductors can become unique in very strong magnetic fields under conditions of the so-called quantum $3D \rightarrow 2D$ dimensional crossovers, where the typical sizes of electron trajectories are of the order or less than the inter-plane distances.

The goal of our Letter is to introduce quantum $3D \rightarrow 1D \rightarrow 2D$ dimensional crossover in a Q1D conductor and to show that it can be responsible for the Fermi liquid - non Fermi liquid crossovers (or phase transitions) in a tilted magnetic field. We calculate inverse electron-electron scattering time and demonstrate that it becomes almost 1D (i.e., of the order of the characteristic electron energy, $1/\tau \sim \epsilon \sim T$) in high magnetic fields, directed far from the main crystallographic axes. In this case, Landau quasi-particles in Fermi liquid are not well defined. Therefore, we can expect that Fermi liquid theory is broken and some novel electronic states, including the possible Luttinger liquid phase, appear. If magnetic field is directed close to one of the main crystallographic axes, then, as we show below, inverse electron-electron scattering time become 2D and, thus, much less than the characteristic electron energy, $1/\tau \ll \epsilon \sim T$. In this case, we have to expect Fermi liquid behavior of conducting electrons. It is important that in $(\mathrm{Per})_2\mathrm{Au(mnt)}_2$ layered Q1D conductor the above mentioned Fermi liquid - non Fermi liquid crossovers (or transitions) are expected to happen in feasibly high magnetic fields of the order of $25\ T$. We also discuss experimental results on investigation of Lebed's magic angles in $(\mathrm{Per})_2\mathrm{Au(mnt)}_2$ [14], where such crossovers (or transitions) may have been already observed at $H \simeq 30\ T$.

Let us first demonstrate the suggested phenomenon, using qualitative language. We consider a layered Q1D conductor with electron spectrum, corresponding to the following two slightly corrugated planes near $p_x = \pm p_F$:

$$\epsilon(\mathbf{p}) = \pm v_F(p_x \mp p_F) - 2t_y \cos(p_y a_y) - 2t_z \cos(p_z a_z), \quad (1)$$

where p_F and v_F are the Fermi momentum and Fermi velocity, respectively; $p_F v_F \gg t_y \gg t_z$; $\hbar \equiv 1$. Below, we study the case, where magnetic field is perpendicular to the conducting chains and makes angle α with the conducting planes,

$$\mathbf{H} = (0, \cos\alpha, \sin\alpha)H, \ \ \mathbf{A} = (0, -\sin\alpha, \cos\alpha)Hx. \quad (2)$$

To consider a quantum problem of the Q1D electrons (1) motion in the magnetic field (2), we make use of the so-called Peierls substitution method, formulated for a Q1D conductor in Ref.[4]. In our particular case, this method allows to introduce magnetic field by the following substitutions:

$$p_x \mp p_F \rightarrow \mp i(d/dx), \ \ p_y a_y \rightarrow p_y a_y - \omega_y(\alpha)/v_F, \\ p_z a_z \rightarrow p_z a_z + \omega_z(\alpha)/v_F, \quad (3)$$

where

$$\omega_y(\alpha) = ev_F a_y H \sin\alpha/c, \quad \omega_z(\alpha) = ev_F a_z H \cos\alpha/c \ . \tag{4}$$

After these substitutions electron energy (1) becomes the Hamiltonian operator and the corresponding Schrödinger-like equation for electron wave function in mixed $(x;p_y,p_z)$ representation can be written as

$$\biggl\{ \mp i v_F \frac{d}{dx} - 2t_y \cos\biggl[p_y a_y - \frac{\omega_y(\alpha)}{v_F}x\biggr] - 2t_z \cos\biggl[p_z a_z + \frac{\omega_z(\alpha)}{v_F}x\biggr]\biggr\} \psi^{\pm}_{\epsilon}(x;p_y,p_z) = \delta\epsilon \ \psi^{\pm}_{\epsilon}(x;p_y,p_z), \tag{5}$$

where electron energy is counted from the Fermi level, $\delta\epsilon = \epsilon - p_F v_F$. Note that Eq.(5) can be analytically solved. As a result, we obtain:

$$\psi^{\pm}_{\epsilon}(x;p_y,p_z) = \exp\biggl(\frac{\pm i\delta\epsilon x}{v_F}\biggr) \exp\biggl\{\mp i l_y(\alpha)\sin\biggl[p_y a_y - \frac{\omega_y(\alpha)}{v_F}x\biggr]\biggr\} \exp\biggl\{\pm i l_z(\alpha)\sin\biggl[p_z a_z + \frac{\omega_z(\alpha)}{v_F}x\biggr]\biggr\}, \tag{6}$$

where

$$l_y(\alpha) = \frac{2t_y}{\omega_y(\alpha)}, \ l_z(\alpha) = \frac{2t_z}{\omega_z(\alpha)}. \tag{7}$$

It is possible to show [3] that the parameters (7) are the "sizes" of quasi-classical electron trajectories along y and z axes, measured in terms of the corresponding lattice parameters, a_y and a_z.

For further qualitative analysis it is convenient to calculate the Fourier transformations of function (6) for integer values of variables $y = na_y$ and $z = ma_z$ (i.e., on the conducting chains):

$$\psi^{\pm}_{\epsilon}(x, y = na_y, z = ma_z) = \int_0^{2\pi} \frac{d(p_y a_y)}{2\pi} \exp(inp_y a_y) \times \int_0^{2\pi} \frac{d(p_z a_z)}{2\pi} \exp(imp_z a_z) \ \psi^{\pm}_{\epsilon}(x,p_y,p_z). \tag{8}$$

After substitution of wave function (6) in Eq.(8) and straightforward calculations it is possible to show that

$$\psi^{\pm}_{\epsilon}(x, y = na_y, z = ma_z) = \exp\biggl\{\frac{\pm i[\delta\epsilon \pm n\omega_y(\alpha) \mp m\omega_z(\alpha)]x}{v_F}\biggr\} \times J_n[\pm l_y(\alpha)] \ J_m[\mp l_z(\alpha)], \tag{9}$$

where we make use of the following property of the Bessel functions [15]:

$$J_n(z) = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} \ \exp(in\phi) \ \exp[-iz\sin(\phi)] \ . \tag{10}$$

We note that wave function in a real space (9) show the amplitudes for an electron to occupy the conducting chains with the coordinates $y = na_y$ and $z = ma_z$ in a Q1D conductor in case, where electron wave function is centered at $y = z = 0$. In particular, from Eq.(9), it follows that the total probability to occupy all possible chains at arbitrary magnetic field is

$$P = \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} J^2_n[\pm l_y(\alpha)] \ J^2_m[\mp l_z(\alpha)] = 1, \tag{11}$$

as it has to be, where we use that $\sum_{n=-\infty}^{+\infty} J^2_n(z) = 1$ for arbitrary value of the argument z [15].

Note that wave functions in a real space (9) are although one dimensional but in general occupy many conducting chains. Nevertheless, when the parameters (7) become smaller than 1 in high magnetic fields,

$$H \geq H^* = \max\biggl\{\frac{2t_y c}{ev_F a_y \sin\alpha}, \frac{2t_z c}{ev_F a_z \cos\alpha}\biggr\} \ , \tag{12}$$

electron wave functions (9) become localized on the conducting chain with $y = z = 0$. This fact is directly seen from the following properties of the Bessel functions [15]:

$$\lim_{z\to 0} \ J_0(z) \to 1; \ \lim_{z\to 0} \ J_n(z) \to 0, \ n \neq 0. \tag{13}$$

The above mentioned localization of electrons means that high enough magnetic fields fully "one-dimensionalize" Q1D electron spectrum (1). Therefore, we expect that, at high magnetic fields, Q1D electrons start to exhibit non Fermi liquid properties, since Fermi liquid is known not to exist in a pure $1D$ case. It is important that this can happen only in the case, where direction of a magnetic field is far from the crystallographic axes at $\alpha = 0^o$ and $\alpha = 90^o$. Indeed, if direction of a magnetic field is close to one of these axes, the "sizes" (7) of electron wave function (9) become large and, thus, Fermi liquid properties have to be restored. Therefore, we expect Fermi liquid - non Fermi liquid angular crossovers (or phase transitions) in a tilted high enough magnetic field. Let us estimate the value of the critical magnetic field (12) for the Q1D conductor $(\text{Per})_2\text{Au}(\text{mnt})_2$, using the following values for the parameters of its electron spectrum [14]: $v_F = 1.7 \times 10^7 cm/s$, $t_y = 20 \ K$, $t_z \leq t_y$, $a_y = 16.6 \ \mathring{A}$, and $a_z = 30 \ \mathring{A}$. In this case, from Eq.(12), we estimate that $H^* \simeq 25 \ T$ at $\alpha = 45^0$ - the value, which is available as a steady magnetic field.

Below, we calculate inverse electron-electron scattering time and directly demonstrate that the major Landau criterion [16,17] for Fermi liquid behavior is broken at high enough magnetic fields (12). We recall that this criterion says that Landau quasi-particles have to be well defined in Fermi liquid. In particular, this means that electron-electron scattering time has to be much less than the typical electron energy, $1/\tau \ll \epsilon \sim T$. For further calculations, it is important that, in a magnetic field (2), only electron momenta, perpendicular to the conducting chains, p_y and p_z, are conserved. This means that momentum conservation law can be written in the collision integral for Fermi particles [16,17] only for the above mentioned directions. On the other hand, the total electron energy is conserved in a magnetic field. In order, to calculate inverse electron-electron scattering time, averaged over electron energy, ϵ, and perpendicular components of momentum, p_y and p_z, we need to consider the following expression, extended electron-electron collision integral to the case of non-conservation of momentum along conducting axis $\mathbf{x}$:

$$\begin{aligned}\frac{1}{\tau} = \int d\epsilon_1 \int d\epsilon_2 \int d\epsilon_3 \int d\epsilon_4 \; \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4) \\ \times n(\epsilon_1) n(\epsilon_2)[1 - n(\epsilon_3)][1 - n(\epsilon_4)] \\ \times \int dp_y^1 \int dp_y^2 \int dq_y \int dp_z^1 \int dp_z^2 \int dq_z \\ W(\epsilon_1, p_y^1, p_z^1; \epsilon_2, p_y^2, p_z^2; \\ \epsilon_3, p_y^1 + q_y, p_z^1 + q_z; \epsilon_4, p_y^2 - q_y, p_z^2 - q_z). \end{aligned} \tag{14}$$

To find electron-electron scattering probability, $W(...)$ in Eq.(14), in a magnetic field (2), we make use of known electron wave functions (8). It is possible to show that the scattering probability, corresponding to electron-electron scattering amplitude, shown in Fig.1, is

$$\begin{aligned} W(\epsilon_1, p_y^1, p_z^1; \epsilon_2, p_y^2, p_z^2; \epsilon_3, p_y^1 + q_y, p_z^1 + q_z; \epsilon_4, p_y^2 - q_y, p_z^2 - q_z) \\ = U \int dx \exp[i(\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4)x/v_F] \\ \times \exp\Biggl\{ 8 i l_y(\alpha) \sin\Bigl[\tfrac{\omega_y(\alpha) x}{2 v_F}\Bigr] \sin\Bigl[\tfrac{p_y^1 + p_y^2}{2}\Bigr] \cos\Bigl[\tfrac{p_y^1 - p_y^2}{2}\Bigr] \sin\Bigl[\tfrac{q_y}{2}\Bigr] \Biggr\} \\ \times \exp\Biggl\{ 8 i l_z(\alpha) \sin\Bigl[\tfrac{\omega_z(\alpha) x}{2 v_F}\Bigr] \sin\Bigl[\tfrac{p_z^1 + p_z^2}{2}\Bigr] \cos\Bigl[\tfrac{p_z^1 - p_z^2}{2}\Bigr] \sin\Bigl[\tfrac{q_z}{2}\Bigr] \Biggr\} \end{aligned} \tag{15}$$

We point out that we use approximation, where electron-electron interaction, U, is independent on electron momenta in the absence of a magnetic field, which corresponds to electron-electron interaction term proportional to $\delta^3(\mathbf{r_1} - \mathbf{r_2})$ in a real space. In this case, all possible amplitudes of electron-electron scattering give the same probability (15).

After lengthly but straightforward calculations, we obtain from Eq.(15):

$$\begin{aligned} \frac{1}{\tau} = 2 g^2 T \int_0^\infty \left(\frac{2\pi T dx}{v_F} \right) \left[\frac{(\frac{2\pi T x}{v_F}) \cosh(\frac{2\pi T x}{v_F}) - \sinh(\frac{2\pi T x}{v_F})}{\sinh^3(\frac{2\pi T x}{v_F})} \right] \\ \times \left\langle J_0^2 \left\{ 4 l_y(\alpha) \sin\left[\frac{\omega_y(\alpha) x}{2 v_F} \right] \cos(\phi_1) \right\} \right\rangle_{\phi_1} \\ \times \left\langle J_0^2 \left\{ 4 l_z(\alpha) \sin\left[\frac{\omega_z(\alpha) x}{2 v_F} \right] \cos(\phi_2) \right\} \right\rangle_{\phi_2}, \end{aligned} \tag{16}$$

where $< ... >_\phi$ denotes averaging over variable ϕ, g stands for dimensionless electron-electron interaction constant. Note that the inverse electron-electron scattering time (16) is normalized in such a way that $1/\tau = g^2 T$ in a pure 1D case. We point out that in the derivation of

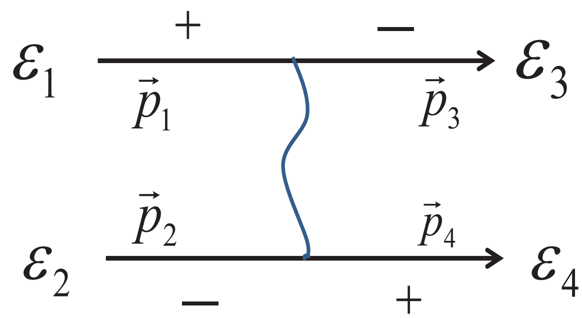

FIG. 1: One possible amplitude of electron-electron scattering, where the first electron is scattered from right sheet of the Q1D Fermi surface (1) to left sheet, whereas the second electron is scattered from left sheet to right sheet.

Eq.(16), we make use of Eq.(10) as well as the following equations [15]:

$$\begin{aligned} \int\int\int\int d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4 \exp[i(\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4)x/v_F] \\ \times \delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4) n(\epsilon_1) n(\epsilon_2)[1 - n(\epsilon_3)][1 - n(\epsilon_4)] \\ = 2\pi^2 T^3 \left[\frac{(\frac{2\pi T x}{v_F}) \cosh(\frac{2\pi T x}{v_x}) - \sinh(\frac{2\pi T x}{v_F})}{\sinh^3(\frac{2\pi T x}{v_F})} \right] \end{aligned} \tag{17}$$

and

$$\int_0^{2\pi} \frac{d\phi}{2\pi} J_0(2 z \sin\phi) = J_0^2(z). \tag{18}$$

Let us analyze Eq.(16) at high magnetic fields. First, we consider the case, where magnetic field is directed far from the crystallographic axes at $\alpha = 0^0$ and $\alpha = 90^0$. If magnetic field satisfies Eq.(12), as it directly follows from Eq.(16), both Bessel functions are of the order of 1 and inverse electron-electron scattering time is of the order of

$$\frac{1}{\tau} \sim g^2 T \sim T. \tag{19}$$

In other words, at high magnetic fields (12), inverse electron-electron scattering time is completely "one-dimensionalized" (i.e., becomes of the order of the characteristic electron energy, $\frac{1}{\tau} \sim \epsilon \sim T$). According to Landau [16,17], in this case the notion of quasi-particles in Fermi liquid loses its meaning. Therefore, under these conditions, we expect non Fermi liquid behavior of the Q1D electron gas (1). Now, let us consider inverse electron-electron scattering time (16) in the case, where magnetic field is applied along $\mathbf{y}$ axis, which corresponds to $\alpha = 0^0$. In this case, integral (16) can be estimated as

$$\begin{aligned} \frac{1}{\tau}(0^0) = 2 g^2 T \int_0^\infty \left(\frac{2\pi T dx}{v_F} \right) \left[\frac{\frac{2\pi T x}{v_F} \cosh(\frac{2\pi T x}{v_F}) - \sinh(\frac{2\pi T x}{v_F})}{\sinh^3(\frac{2\pi T x}{v_F})} \right] \\ \times \lim_{\alpha \to 0} \left\langle J_0^2 \left\{ 4 l_y(\alpha) \sin\left[\frac{\omega_y(\alpha) x}{2 v_F} \right] \cos(\phi_1) \right\} \right\rangle_{\phi_1}. \end{aligned} \tag{20}$$

Let us consider the case of low enough temperatures, where

$$T \ll t_y \simeq \omega_y(\alpha = 90^0). \qquad (21)$$

In this case for small enough angles,

$$\sin\alpha \ll T/\omega_y(\alpha = 90^0), \qquad (22)$$

integral (20) can be simplified as

$$\frac{1}{\tau}(0^0) = 2g^2 T \int_0^\infty dz \frac{z\cosh(z) - \sinh(z)}{\sinh^3(z)} \times \left\langle J_0^2\left(\frac{2t_y z}{\pi T}\cos\phi\right)\right\rangle_\phi . \qquad (23)$$

Note that the integral (23) can be analytically calculated with the so-called logarithmic accuracy:

$$\frac{1}{\tau}(0^0) \simeq \frac{g^2 T^2}{2\pi t_y} \ln^2\left(\frac{t_y}{T}\right) \ll T. \qquad (24)$$

As it follows from Eq.(24), for small enough angles (22) inverse electron-electron scattering time becomes smaller than the electron characteristic energy, $1/\tau \ll \epsilon \sim T$, and the concept of quasi-particles in Fermi liquid restores [16,17]. Therefore, we expect restoration of Fermi liquid behavior for $\alpha \simeq 0^0$. In Fig.2, the results of careful numerical calculations of Eq.(16) are presented, which confirm the above mentioned analytical analysis. To obtain inverse electron-electron relaxation time for magnetic field, directed close to $\mathbf{z}$ axis ($\alpha = 90^0$), we need to do the following substitutions $t_y \to t_z$ and $\omega_y(\alpha) \to \omega_z(\alpha)$ in Eqs.(22),(24). As a result, we obtain

$$\frac{1}{\tau}(90^0) \simeq \frac{g^2 T^2}{2\pi t_z} \ln^2\left(\frac{t_z}{T}\right) \ll T \qquad (25)$$

for

$$\cos\alpha \ll T/\omega_z(\alpha = 0^0), \qquad (26)$$

and, thus, Fermi liquid behavior is expected to restore also at angles close to 90^0. [We note that there are some mathematical similarities between microscopic problem, considered in this Letter, and semi-phenomenological calculations of conductivity of a Q1D metal in a magnetic field [18]. Nevertheless, the physical conclusions of our Letter and Ref.[18] are quiet different.]

In conclusion, we discuss possible experimental applications of the suggested above Fermi liquid - non Fermi liquid angular crossovers (or phase transitions) in a Q1D conductor in high magnetic fields. The most natural way is to perform the corresponding experiments in the Q1D conductor $(\mathrm{Per})_2\mathrm{Au(mnt)}_2$ under pressure, where charge-density-wave state is destroyed and metallic Fermi liquid phase is a ground state at $H = 0$ [14,19]. In addition to resistive experiments [14,19], we suggest also torque

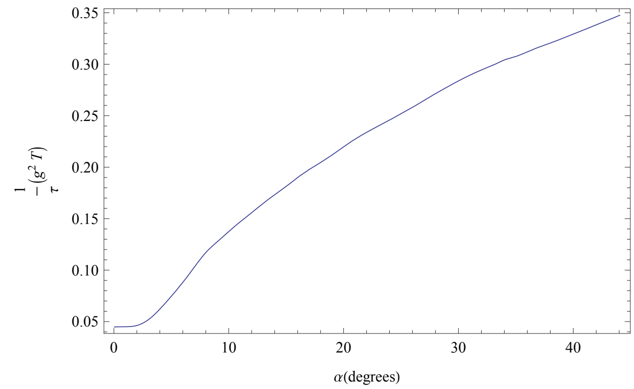

FIG. 2: Inverse electron-electron scattering time, calculated by means of Eq.(16) and expressed in term of g^2T, is shown as a function of angle α. The calculations are done for the parameters $l_y(0) = 1$, $t_z/t_y = 0.2$, $a_y = a_z/2$, $4\pi T/\omega_y(0) = 0.2$, which correspond to $H \simeq 25\ T$.

measurements in high magnetic fields, $H \simeq 25\ T$, perpendicular to the conducting chains, since the angular Fermi liquid - non Fermi liquid crossovers have to have also thermodynamic consequence [20]. In this context, we note that, as shown by Yakovenko [21], Lebed's Magic angle effects have to exist already in moderate magnetic fields in Fermi liquid phase of a Q1D conductor for different thermodynamic properties such as torque, specific heat, and magnetic moment. As to the resistive measurements of Lebed's Magic angle phenomenon [14], it seems that one feature of the above mentioned crossovers has been already observed in Ref.[14] - non-metallic temperature dependence of resistance for high magnetic fields, directed far from the main crystallographic axes. It is important that this non-metallic behavior cannot be a consequence of Fermi liquid magnetoresistance, since, for experimental current along the conducting axis, $\mathbf{I} \parallel \mathbf{b}$, Fermi liquid magnetoresistance is expected to be zero. On the other hand, it is known that it is not easy to measure conductivity in a Q1D conductor exactly along its conducting axis [3], therefore, more experimental works are needed.

We stress that the effects, suggested in the Letter, are rather general and have to be observed in other materials containing Q1D parts of the Fermi surfaces, such as $(\mathrm{TMTSF})_2\mathrm{X}$ salts and some BEDT-based materials. Nevertheless, the required magnetic field for Fermi liquid - non Fermi liquid crossovers in the above mentioned conductors are estimated as $H^* \simeq 250\ T$, which is order of magnitude higher than the value for the Q1D $(\mathrm{Per})_2\mathrm{Au(mnt)}_2$ conductor.

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[1] P.M. Chaikin, Mu-Yong Choi, J.F. Kwak, J.S. Brooks, K.P. Martin, M.J. Naughton, E.M. Engler, R.L. Green, Phys. Rev. Lett. **51**, 2333 (1983).

[2] M. Ribault, D. Jerome, J. Tuchendler, C. Weyl, K. Bechgaard, J. Phys. (Paris) Lett. **44**, L953 (1983).

[3] *The Physics of Organic Superconductors and Conductors*, edited by A.G. Lebed (Springer, Berlin, 2008).

[4] L.P. Gorkov and A.G. Lebed, J. Phys. (Paris) Lett. **45**, L433 (1984).

[5] M. Heritier, G. Montambaux, and P. Lederer, J. Phys. (Paris) Lett. **45**, L943 (1984).

[6] G. Montambaux, M. Heritier, and P. Lederer, Phys. Rev. Lett. **55**, 2078 (1985).

[7] P.M. Chaikin, Phys. Rev. B **31**, 4770 (1985).

[8] A.G. Lebed, Sov. Phys. JETP, **62**, 595 (1985) [Zh. Eksp. Teor. Fiz. **89**, 1034 (1985)].

[9] See, for example, Si Wu and A.G. Lebed, Phys. Rev. B **82**, 172504 (2010) and references therein.

[10] A.G. Lebed, JETP Lett. **44**, 114 (1986) [Pis'ma Zh. Eksp. Teor. Fiz. **44**, 89 (1986)].

[11] N. Dupuis, G. Montambaux, and C.A.R. Sa de Melo, Phys. Rev. Lett. **49**, 8993 (1994)].

[12] A.G. Lebed and K. Yamaji, Phys. Rev. Lett. **80**, 2697 (1998).

[13] A.G. Lebed, Phys. Rev. Lett. **107**, 087004 (2011).

[14] D. Graf J.S. Brooks, E.S. Choi, M. Almeida, R.T. Henriques, J.C. Dias, and S. Uji, Phys. Rev. B **80**, 155104 (2009).

[15] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, 5th edition (Academic Press, New York, 1994).

[16] See, for example, A.A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988).

[17] See, for example, L.P. Pitaevskii and E.M. Lifshitz, *Physical Kinetics* (Butterworth-Heinemann, Oxford, 1981).

[18] A.T. Zheleznyak and V.M. Yakovenko, Eur. Phys. J. B **11**, 385 (1999).

[19] D. Graf, J.S. Brooks, E.S. Choi, M. Almeida, R.T. Henriques, J.C. Dias, and S. Uji, Phys. Rev. B **75**, 245101 (2007).

[20] A.G. Lebed, in preparation.

[21] V.M. Yakovenko, Phys. Rev. Lett., **68**, 3607 (1992); Erratum **70**, 519 (1993).