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Phys. Rev. Lett. **115**, 121602 — Published 16 September 2015

DOI: [10.1103/PhysRevLett.115.121602](https://doi.org/10.1103/PhysRevLett.115.121602)

Entanglement entropy in two dimensional string theory

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Abstract

To understand an emergent spacetime is to understand the emergence of locality. Entanglement entropy is a powerful diagnostic of locality, because locality leads to a large amount of short distance entanglement. Two dimensional string theory is among the very simplest instances of an emergent spatial dimension. We compute the entanglement entropy in the large N matrix quantum mechanics dual to two dimensional string theory, in the semiclassical limit of weak string coupling. We isolate a logarithmically large, but finite, contribution that corresponds to the short distance entanglement of the tachyon field in the emergent spacetime. From the spacetime point of view, the entanglement is regulated by a nonperturbative ‘graininess’ of space.

INTRODUCTION

Locality is a key ingredient of conventional quantum field theories. To show that a given theory of quantum gravity exhibits an emergent semiclassical spacetime, an essential aspect to be understood is the emergence of local dynamics in the emergent spacetime. A robust and universal probe of locality is entanglement entropy. In a quantum system with degrees of freedom at all scales, such as a quantum field theory, local interactions imply a large amount of short distance entanglement in the ground state [1, 2]. This suggests the following strategy: Suppose we are given a quantum state that is a candidate to describe an emergent spacetime (say, the ground state of $\mathcal{N} = 4$ SYM theory [3] or the ground state of BFSS matrix quantum mechanics [4]). To ‘find’ the locality of the emergent spacetime within this state, that is, to identify the degrees of freedom that interact locally in the emergent spacetime, what we must do is identify strongly entangled degrees of freedom in the state.

In this paper we will implement the above strategy in the simplest possible example of emergent spacetime. This is the emergence of spacetime in two dimensional string theory from the matrix quantum mechanics of a single large N matrix. While relatively simple, this duality can be thought of as the baby cousin of the AdS/CFT correspondence. We will see that it illustrates how quantum gravity can provide nonperturbative cutoffs on UV divergences that appear due to short distance entanglement in quantum field theory.

The low energy (compared to the string scale) effective target space action for two dimensional bosonic string theory takes the form [5, 6]

$$S = \int dt dx \sqrt{-g} e^{-2\Phi} \left(R + 4(\nabla\Phi)^2 + 16 - (\nabla T)^2 + 4T^2 - 2\tilde{V}(T) \right). \quad (1)$$

The fields are the metric g , ‘tachyon’ T and dilaton Φ . This theory has only one propagating degree of freedom, which we can consider to be the tachyon. The background of interest is the linear dilaton solution together with a tachyon condensate

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi = 2x + \dots, \quad T = \mu \left(x + \frac{\log \mu}{2} \right) e^{2x} + \dots. \quad (2)$$

The dots indicate that the solution can only be trusted in the regime of weak string coupling

$$g_s \equiv e^\Phi \approx e^{2x}, \quad (3)$$

which becomes small as $x \rightarrow -\infty$. The relative coefficients of the two terms in the tachyon profile are fixed by nonlinearities in the tachyon action [6]. The single free parameter μ in the

background determines the string coupling at the ‘tachyon wall’ ($x \approx -\frac{1}{2} \log \mu$) to be $g_{\text{eff.}} \approx \mu^{-1}$. Therefore, $\mu \rightarrow \infty$ is a weakly coupled limit in which scattering off the tachyon wall can be described perturbatively.

Our objective in this paper is the following. We will evaluate the entanglement entropy of a spatial interval of length $\Delta x = x_2 - x_1$ that is to the left of the tachyon wall ($2x \lesssim -\log \mu$) and in the weakly interacting limit $\mu \rightarrow \infty$. In this regime the spacetime is semiclassical and so the entanglement entropy of a spatial region makes sense. About the background (2) the tachyon is a massless field. The nonlinearities of the tachyon action can still be important close to the tachyon wall, however, and couple linearized fluctuations of the tachyon to the non-translation-invariant tachyon background (2). These effects become small in the limit $x \rightarrow -\infty$. In this limit, with weak string interactions and weak nonlinearities, we expect to find the entanglement entropy of a two dimensional massless scalar [7]

$$S = \frac{1}{3} \log \frac{\Delta x}{\epsilon}, \quad (x \rightarrow -\infty, \Delta x \text{ fixed}). \quad (4)$$

Here ϵ is a UV cutoff. The semiclassical spacetime computation of this quantity is simply divergent and cannot see the cutoff. We will instead obtain the result (4), complete with an explicit cutoff, from the full underlying quantum state out of which the spacetime emerges.

Our result is given in equation (21) below. To compare with (4) we can again take the limit $x \rightarrow -\infty$. The result simplifies to

$$S = \frac{1}{3} \log \frac{\mu \Delta x}{\sqrt{g_s(x_1) g_s(x_2)}}, \quad (x \rightarrow -\infty, \Delta x \text{ fixed}). \quad (5)$$

A similar result (with $x_1 \approx x_2$) has been given some time ago in the prescient papers [8, 9]. Our result (21) is more precise and we believe our treatment is more transparent, although the essential physics is the same as that discussed in [8, 9]. The finite answer for the entanglement indicates a fundamental nonperturbative ‘graininess’ of spacetime at the scale set by the string coupling $g_s = e^\Phi \ll 1$. It is natural to think of this scale as the de Broglie wavelength of the D-particles in the theory, whose condensation has created the spacetime [10, 11]. Also, because the dominant short distance entanglement comes from the boundaries of the region $[x_1, x_2]$, it is natural that the cutoffs in (5) are set by the string coupling evaluated at the endpoints x_1 and x_2 .

The worldsheet string theory describing the background (2) is Liouville theory coupled to $c = 1$ matter. By discretizing the worldsheet, this theory is equivalent to a certain matrix quantum mechanics tuned to a critical point in a double scaling limit, as reviewed in [11–14]. The matrix

quantum mechanics has the action

$$S = \beta N \int dt \operatorname{tr} \left[\frac{1}{2} \dot{M}^2 + V(M) \right]. \quad (6)$$

Here M is an $N \times N$ Hermitian matrix and β is a coupling. We can restrict to singlet states, for instance by gauging the time derivative. The singlet states can be described by the eigenvalues of the matrix M . These eigenvalues experience the usual Vandermonde repulsion. For the quantum mechanics (6) of a single matrix, the Vandermonde repulsion is equivalent to Pauli exclusion of fermions. The dynamics of the eigenvalues can therefore be formulated as the dynamics of N non-interacting spinless fermions moving in the potential V . The emergent spacetime in which the string theory (1) lives is in fact the Fermi surface of this theory, and the tachyon field is the bosonization of the fermion dynamics, describing density fluctuations of the Fermi surface.

In order to obtain a continuum limit of the string theory worldsheet, the Fermi energy must be tuned to be close to a local maximum of the potential V , where there is a logarithmic divergence in the density of states. In this limit one obtains the following second-quantized Hamiltonian describing fermions in an inverted harmonic oscillator potential, see e.g. [14]:

$$H = \int d\lambda \left[\frac{1}{2} \frac{d\Psi^\dagger}{d\lambda} \frac{d\Psi}{d\lambda} - \frac{\lambda^2}{2} \Psi^\dagger \Psi + \mu \Psi^\dagger \Psi \right]. \quad (7)$$

The ground state of (7) is a Fermi sea in the inverted oscillator potential. The Fermi sea is populated up to a distance μ from the top of the potential. We will work with the original (and ultimately unstable) version of the theory in which the Fermi sea is only populated on one side of the local maximum. The results can surely be adapted to the stable theory in which both sides are populated [15, 16].

Extensive work on this duality [11–14] has mapped various quantities between the fermion description (7) and the spacetime description (1). The only result we will take from this past work is that the quantity μ appearing in (7) and in the background (2) is the same. In particular, this means that in order to access the weakly interacting semiclassical spacetime regime we must take $\mu \rightarrow \infty$. As we will see shortly, this corresponds to taking a WKB limit of the fermionic wavefunctions.

A further connection we will see with past work is that, in the weakly coupled regime as $\mu \rightarrow \infty$, and modulo the qualification below, the fermionic coordinate λ in (7) is related to the spacetime coordinate x in (1) by the ‘time of flight’ relation [17]

$$x = \tau(\lambda) \equiv -\frac{1}{\sqrt{2}} \int_{\lambda_*(\mu)}^{\lambda} \frac{d\lambda'}{\sqrt{-V(\lambda') - \mu}} = -\log \frac{\lambda + \sqrt{\lambda^2 - 2\mu}}{\sqrt{2\mu}}. \quad (8)$$

In the second relation we have used $V(\lambda) = -\lambda^2/2$ and hence the turning point $\lambda_*(\mu) = \sqrt{2\mu}$. In fact, one can also take our results to be an alternative *derivation* of the relation (8), based on the need to recover the short distance entanglement (4) of a one dimensional massless scalar field from the entanglement of the matrix eigenvalues. Note that in (8) we are sending positive λ , to the right of the maximum of the potential, to negative spacetime coordinate x .

It is known that the relation (8) is not precise [18]. The nonlocal nature of the full map between x and λ is due to the existence of additional massive string states at certain discrete momenta [18]. We use the local map (8) to reproduce the contribution of the spacetime tachyon field to the entanglement entropy of a spatial region in the weakly coupled limit $\mu \rightarrow \infty$. This map correctly describes the tachyon's massless dispersion [17]. The additional string states are both massive and non-dispersing [18]. Therefore, they do not give a singular contribution to the spatial entanglement entropy in the ground state.

Given the rather explicit map from the eigenvalue (= fermion) description to the emergent spacetime in this particular duality, there is a natural guess of which matrix quantum mechanics degrees of freedom will correspond to a given spatial region in spacetime. Namely, one expects that the eigenvalues taking some continuous range of values will map onto a region of spacetime according to the relation (8). With this in mind, we will proceed to compute the entanglement entropy of an interval in the theory of non-interacting fermions in an inverted harmonic oscillator potential (7). This is the entanglement in a state in a first quantized theory. While the total particle number is fixed, the number of particles in a given spatial region fluctuates. The reduced density matrix is therefore supported on states with varying particle number [19–23]. As discussed above, we expect this to give a complete and manifestly UV finite computation of the entanglement entropy that is not accessible directly from the spacetime perspective of the action (1).

ENTANGLEMENT AND DENSITY FLUCTUATIONS

The entanglement entropy of non-interacting fermions in a region A can be expressed as a sum over cumulants of the particle number distribution [19, 20]

$$S_A = \frac{\pi^2}{3} V_A^{(2)} + \frac{\pi^4}{45} V_A^{(4)} + \frac{2\pi^6}{945} V_A^{(6)} + \dots \quad (9)$$

Here

$$V_A^{(m)} = \left(-i \frac{d}{d\lambda} \right)^m \log \langle e^{i\lambda N_A} \rangle \Big|_{\lambda=0}, \quad (10)$$

with the integrated density operator in the region A given by $N_A = \int_A d\lambda n(\lambda)$. The expression for the entanglement entropy in terms of density fluctuations is ultimately derived from the explicitly

known reduced density matrix of a region for a system of non-interacting fermions. In particular, the reduced density matrix and entanglement entropy of a region can be expressed both in terms of the matrix of fermion two point correlation functions [21] and also in terms of the matrix of fermion wavefunction overlaps in the region [22]. We also found the further discussion in [23] helpful.

The expansion of the entanglement entropy in terms of the density cumulants in (9) will be especially useful for us for the following reason. It has been found that the leading singular behavior of the entanglement entropy in the limit of large fermion occupation number is determined by the second cumulant alone [24, 25]. We will see shortly below that the large μ limit of interest to us is a WKB limit for the fermions and hence indeed corresponds to large fermion occupation number. Therefore, we can expect that to leading order in the large μ limit

$$S_A = \frac{\pi^2}{3} V_A^{(2)} = \frac{\pi^2}{3} \int_A d\lambda d\lambda' \left(\langle n(\lambda) n(\lambda') \rangle - \langle n(\lambda) \rangle \langle n(\lambda') \rangle \right). \quad (11)$$

Actually, this result – that one can restrict to $V_A^{(2)}$ to leading order – has not been shown for noninteracting electrons in an arbitrary potential. In the additional limit in which the size of the entangling region becomes small relative to the scale of variation in the potential (this is the limit considered in [8, 9]), then the cancellations described in [24] for the short distance engagement will occur independently of the form of potential. We will see below that we can do better than this, by noting that in the WKB limit the singular contributions to all of the cumulants $V_A^{(m)}$ are functions of the time of flight variable (8).

The density operator is given by the second quantized fermionic field operators Ψ as $n(t, \lambda) = \Psi^\dagger(t, \lambda) \Psi(t, \lambda)$. The field operators can be expressed in terms of creation and annihilation operators weighted by energy eigenfunctions of the associated single particle Hamiltonian:

$$\Psi(t, \lambda) = \int_{-\infty}^{\infty} d\nu e^{i\nu t} a(\nu) \psi_\nu(\lambda). \quad (12)$$

That is

$$-\frac{1}{2} \frac{d^2 \psi_\nu}{d\lambda^2} + V(\lambda) \psi_\nu = -\nu \psi_\nu. \quad (13)$$

Note that ν is minus the energy. Similarly μ will be the chemical potential measured downwards towards negative energies, so that the state satisfies $a_\nu |\mu\rangle = 0$ if $\nu < \mu$, and $a_\nu^\dagger |\mu\rangle = 0$ if $\nu > \mu$. There is no sum over parities of the wavefunction in (12) because we are interested in a background in which the Fermi sea is filled only on one side of the local maximum. The density correlation in

(11) is at equal times. Standard manipulations then give [26]

$$\langle n(\lambda)n(\lambda') \rangle - \langle n(\lambda) \rangle \langle n(\lambda') \rangle = \int_{\mu}^{\infty} d\nu_1 \psi_{\nu_1}(\lambda) \psi_{\nu_1}(\lambda') \int_{-\infty}^{\mu} d\nu_2 \psi_{\nu_2}(\lambda) \psi_{\nu_2}(\lambda'). \quad (14)$$

We have used the fact that the wavefunctions will be real.

COMPUTATION OF THE PARTICLE VARIATION

To evaluate the entanglement entropy, following equations (11) and (14) above, we need to compute the integrals

$$3S_A = \pi^2 \int_{\mu}^{\infty} d\nu_1 \int_{-\infty}^{\mu} d\nu_2 \left(\int_{\lambda_1}^{\lambda_2} d\lambda \psi_{\nu_1}(\lambda) \psi_{\nu_2}(\lambda) \right)^2. \quad (15)$$

We have taken the region A to be the interval $[\lambda_1, \lambda_2]$. The λ integral is immediately performed by noting that

$$\int_{\lambda_1}^{\lambda_2} d\lambda \psi_{\nu_1}(\lambda) \psi_{\nu_2}(\lambda) = \frac{1}{2} \frac{1}{\nu_1 - \nu_2} \left[\frac{d\psi_{\nu_1}}{d\lambda} \psi_{\nu_2} - \psi_{\nu_1} \frac{d\psi_{\nu_2}}{d\lambda} \right]_{\lambda_1}^{\lambda_2}. \quad (16)$$

This result follows directly from the Schrödinger equation (13). In particular, (16) is exact and does not depend on any WKB limit. It also does not depend on the form of the Schrödinger potential.

We specialize now to the potential of interest $V(\lambda) = -\lambda^2/2$. The occupied states have $\nu > \mu$ in the Schrödinger equation (13). Therefore ν is large in the large μ limit, and the wavefunctions are correctly captured by a WKB limit. Performing the standard matching across the turning points, the WKB wavefunctions for $\lambda > \sqrt{2\nu}$ are, up to corrections that are exponentially small at large μ ,

$$\psi_{\nu}(\lambda) = \frac{\sqrt{2}}{\sqrt{\pi p}} \sin \left(\int_{\sqrt{2\nu}}^{\lambda} p d\lambda - \frac{\pi}{4} \right), \quad (17)$$

where $p = \sqrt{\lambda^2 - 2\nu}$, so that

$$P_{\nu}(\lambda) \equiv \int_{\sqrt{2\nu}}^{\lambda} p d\lambda = \frac{1}{2} \left(\lambda \sqrt{\lambda^2 - 2\nu} - 2\nu \log \frac{\lambda + \sqrt{\lambda^2 - 2\nu}}{\sqrt{2\nu}} \right). \quad (18)$$

Our wavefunctions have an extra $\sqrt{2}$ in their normalization relative to [26], as we are restricting to modes that are zero on one side of the local maximum.

Substituting the WKB wavefunctions (17) into (16), and squaring as required by (15), gives an integrand that can be written as exponentials of linear combinations of $P_{\nu_1}(\lambda_1)$, $P_{\nu_2}(\lambda_1)$, $P_{\nu_1}(\lambda_2)$, $P_{\nu_2}(\lambda_2)$.

Many of these terms are oscillatory. Upon integration, the oscillating terms experience significant cancellations. There are no stationery phase points in the region of integration and hence the oscillating terms vanish in the WKB limit $\mu \rightarrow \infty$. In order for the oscillations to be tamed, the exponents must cancel. This can happen in two ways. Firstly the exponent can cancel exactly, to leave a non-oscillating term. Secondly, the exponent can take forms such as $P_{\nu_1}(\lambda_1) - P_{\nu_2}(\lambda_1)$. These terms oscillate, but for $\nu_1 \sim \nu_2$ the two terms almost cancel and the oscillations are slowed down.

The contribution to the integral (15) from the oscillating terms in the $\nu_1 \sim \nu_2$ region and of the non-oscillating terms everywhere are computed in the Supplementary Material. The dominant contribution to the $\nu_1 \sim \nu_2$ region is found to come from states that are close to the Fermi surface. The answer is

$$I_{\text{osc.}} = \gamma_E + \log \epsilon + \log \frac{\tau(\lambda_2) - \tau(\lambda_1)}{\tau(\lambda_2) + \tau(\lambda_1)} + \frac{1}{2} \log \left(16\tau(\lambda_1)\tau(\lambda_2) \right). \quad (19)$$

Here $\tau(\lambda)$ is just the ‘time of flight’, as defined in equation (8) above. Also, ϵ is a cutoff on the difference $(\nu_1 - \nu_2)/2$. The divergence as $\epsilon \rightarrow 0$ will be cancelled by the non-oscillatory part of the integral. Indeed, in the Supplementary Material we show that the non-oscillating contribution to the integral (15) is

$$I_{\text{non-osc.}} = \frac{1}{2} \log [(\lambda_1^2 - 2\mu)(\lambda_2^2 - 2\mu)] - \log \epsilon. \quad (20)$$

The result (19) for the modes close to the Fermi surface in fact holds for electrons in any potential $V(\lambda)$. The result is always (19) in terms of the time of flight variable $\tau(\lambda)$ for the potential. Furthermore, by adapting the computation in the Supplementary Material, it is easily seen that the contribution of these modes close to the Fermi surface to the higher cumulants $V^{(m)}$ in (9) is also a function of the time of flight variable $\tau(\lambda)$.

Putting the above results together (i.e. $3S_A = I_{\text{osc.}} + I_{\text{non-osc.}}$), and using the inverse relation to (8) for the time of flight variable, i.e. $\lambda = \sqrt{2\mu} \cosh \tau(\lambda)$, we finally obtain the entanglement entropy as a function of $\tau(\lambda_1) = x_2$ and $\tau(\lambda_2) = x_1$ (we flipped the numbering $1 \leftrightarrow 2$ to preserve the ordering under the change of sign as $\lambda \leftrightarrow x$ in (8)). The answer can be written as

$$S_A = \frac{1}{3} \log \frac{x_2 - x_1}{\sqrt{\tilde{g}_s(x_1)\tilde{g}_s(x_2)}} + \frac{1}{6} \log \frac{16x_1x_2}{(x_1 + x_2)^2} + \frac{\gamma_E}{3} + \dots \quad (21)$$

In this expression we introduced the ‘string coupling’ [27]

$$\frac{1}{\tilde{g}_s(x)} \equiv 2\mu \sinh^2 \tau(\lambda). \quad (22)$$

Under the identification (8), in which $\tau(\lambda) = x$, and in the limit in which the expression (3) for the coupling can be trusted, $x \rightarrow -\infty$, then indeed we have

$$\tilde{g}_s(x) = \frac{g_s(x)}{2\mu}, \quad (x \rightarrow -\infty). \quad (23)$$

The dots in (21) denote the contribution from the higher cumulants in (9), that we discuss shortly. Equation (21) is our final expression for the entanglement entropy of the region $[x_1, x_2]$ in the target space theory. It is manifestly finite. We noted in (4) and (5) that the result reproduces the expected entanglement due to the tachyon field in the emergent spacetime, in the regime in which the comparison can be made reliably.

We have already mentioned that in some potentials the leading logarithmically singular term in the entanglement entropy at large particle number is known to be entirely captured by the second cumulant $V_A^{(2)}$ [24, 25]. For a region $[\lambda_1, \lambda_2]$ that is very small compared to the scale of variation of the potential, this result should hold for fermions in any potential. This singular term goes like $\log(\lambda_2 - \lambda_1)$ and arises due to modes close to the Fermi surface. We noted that the contribution of these modes to all the $V_A^{(m)}$ cumulants are functions of the time of flight variables $\tau(\lambda)$. Taken together, these observations suggest that the whole $\log[\tau(\lambda_2) - \tau(\lambda_1)]$ term found in (21) has been reliably captured by $V_A^{(2)}$ alone. We have checked this by verifying the absence of a $\log[\tau(\lambda_2) - \tau(\lambda_1)]$ term in the first higher cumulant correction, $V_A^{(4)}$. Therefore, we expect that the first term in the final result (21) correctly captures the leading non-analyticity in the entanglement entropy as $x_1 \rightarrow x_2$ at any x . In contrast, numerical investigation of the $V_A^{(4)}$ correction shows that the remaining terms in (21) do receive corrections from higher cumulants.

The cutoff in the short distance target space entanglement, as evidenced in the comparison of (4) and (5), arises technically from the fact that the inverse string coupling (22) is essentially the depth of the Fermi sea at the point λ . This finite depth provides a cutoff on the number of modes that are entangled, as was emphasized in [8, 9]. Therefore, the cutoff in the first term in (21) should also be robust, to leading order in the WKB limit, against corrections from the higher cumulants [24, 25]. We have explicitly checked that there is no logarithmically large contribution to the non-oscillating part of $V_A^{(4)}$.

DISCUSSION

We have shown that the emergent semiclassical locality of the tachyon field in two dimensional string theory can be seen as an accumulation of entanglement in the dual matrix quantum mechanics. The short distance entanglement of the tachyon field was identified in the entanglement between eigenvalues that were valued in some range with the remaining eigenvalues. We

found that this accumulation of entanglement is cut off, from the spacetime point of view, by a nonperturbative ‘graininess’ at a scale set by the (spatially varying) string coupling constant.

The emergent locality that we have diagnosed through entanglement goes all the way to the spacetime UV cutoff scale. Recent works in the AdS/CFT correspondence have noted that an emergent coarse-grained locality, at the larger AdS scale, can be represented by the entanglement structure of a tensor network [28–31]. This ‘skeleton’ of the emergent spacetime needs to be fleshed out with a large N number of degrees of freedom, that can then provide locality of the sort we have described: down to a microscopic scale. The single matrix quantum mechanics we have studied is, however, not powerful enough to produce spacetime physics with a hierarchy of scales.

In physically richer theories of quantum gravity, the entanglement of fields in the emergent spacetime (such as the tachyon mode whose entanglement we identified) will come hand in hand with the entanglement of microscopic ‘stringy’ degrees of freedom that act as the ‘architecture of spacetime’ [32–35]. Understanding emergent locality in fully fledged quantum gravity theories, using the perspective advocated here, will likely be closely tied to a deeper understanding of the Bekenstein-Hawking entropy of black holes [36, 37] as well as the Ryu-Takayanagi formula [38]. The singlet sector matrix quantum mechanics we have studied is not expected to capture black hole thermodynamics nor, likely, Ryu-Takayanagi type spacetime entanglement. See, for instance, the discussion in [39] and references therein.

We are happy to acknowledge useful discussions with Matt Headrick, Juan Maldacena and Lenny Susskind. This work was partially supported by a DOE Early Career Award, by the Templeton Foundation and an NSF Graduate Research Fellowship.

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