Balasubramanian et al. Reply:

Venkat Balasubramanian, Alex Buchel, Stephen R. Green, Luis Lehner, and Steven L. Liebling

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Reply to "Comment on two-mode stability islands around AdS"

Alex Buchel,[1, 2, *] Stephen R. Green,[2, †] Luis Lehner,[2, ‡] and Steven L. Liebling[3, §]

[1] *Department of Applied Mathematics, Department of Physics and Astronomy, University of Western Ontario, London, Ontario N6A 5B7, Canada*

[2] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*

[3] *Department of Physics, Long Island University, Brookville, NY 11548, U.S.A*

In a recent Comment [1], Bizoń and Rostworowski present a few criticisms of our recent Letter [2]. In particular, they present three arguments: (1) that their own evolutions of the two-mode initial data we had studied collapse to a black hole around a time $t \approx 1080$ whereas we found no collapse for times up to roughly $t = 1500$, (2) that our two-timescale framework (TTF) "*cannot be even used to infer stability*," and (3) that our "*claims about stability islands*" may not be correct.

We have studied evolutions of this initial data with resolutions higher than we had originally. As displayed in Fig. 1, we do find that higher resolutions display higher concentrations of energy, but we nevertheless have still not observed collapse to a black hole. We note that our code demonstrates convergence to a unique solution even at the late times in question (i.e., around $t \approx 1080$). We also emphasize that we have demonstrated that the solution to which our numerical results converge is in fact a solution of the scalar anti–de Sitter (AdS) system we seek to solve. This latter property (*consistency*) is demonstrated with tests that the constraint residuals and mass loss converge to zero (see our Supplementary Material). And so, while it is possible that the continuum solution does in fact collapse around $t \approx 1080$ (and indeed, the work of [3] does see collapse in a code for which both convergence and consistency are verified), we cannot confirm this (in time for the publication of [1]) and hence their claim (1).

However even if collapse does take place at a time $t \approx 1080$, the main claims of our paper still stand. We could not know if collapse occurred after the time which we ran our code, and so collapse for some time soon after $t = 1500$ was always a possibility. And so if instead collapse occurs at $t \approx 1080$, there is no change to our claims. Within that time one still finds both direct and inverse cascades which is now confirmed by the Comment.

Another important point is that, were we to decrease the initial amplitude ϵ, we would certainly observe any possible collapse pushed to times later than $t \approx 1080$.

Regarding their claims (2) and (3), there seems to be some misunderstanding of what we tried to communicate in our Letter. Determining whether some scalar perturbation of AdS with arbitrarily small amplitude ϵ collapses to a black hole is not possible with numerical evolutions which require some finite ϵ, finite resolution, and finite evolution time t.

More particularly regarding their claim (2), we agree that the TTF cannot be used to infer stability beyond times $\propto \epsilon^{-2}$. Indeed, we stressed that we have carried our TTF analysis to $O(\epsilon^3)$ so that one can only trust predictions within times scaling as ϵ^{-2}. However, for shorter times, we disagree with the claims of the Comment that our truncation to $j_{\max} = 47$ "*does not suffice to capture the dynamics of the turbulent cascade*" for 2-mode initial data. As is clear from Fig. 4 of [2], the vast majority of the energy remains in the lowest modes of the system during evolution, and the dynamics of these modes (in particular, the recurrence times) are very well-captured by the TTF evolution with $j_{\max} = 47$. We have clearly stated the limitations of our mode-truncation in the Supplementary Material; in particular, we explained that this leads to the discrepancy with the full numerical GR curve in Fig. 3 of [2] because the high-j modes are highly peaked about the origin (despite carrying very small amounts of energy). We note that the TTF has been verified in Ref. [4] and even the authors of the Comment have now used the method [5].

With regard to claim (3), we did not use the phrase "two-mode stability islands" in our Letter. In later work [6] we referred to "stability islands," but in that case we were referring to quasi-periodic solutions.

Their Comment provides independent confirmation of our main claims: (i) the presence of both direct and inverse energy cascades (a fact that was missed in the original perturbative analysis), and (ii) the validity of the TTF.

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* abuchel@perimeterinstitute.ca
† sgreen@perimeterinstitute.ca
‡ llehner@perimeterinstitute.ca
§ steve.liebling@liu.edu

[1] P. Bizoń and A. Rostworowski, (2014), arXiv:1410.2631 [gr-qc].

[2] V. Balasubramanian, A. Buchel, S. R. Green, L. Lehner, and S. L. Liebling, Phys. Rev. Lett. **113**, 071601 (2014), arXiv:1403.6471 [hep-th].

[3] N. Deppe, Personal communication (2015).

[4] B. Craps, O. Evnin, and J. Vanhoof, JHEP **1501**, 108 (2015), arXiv:1412.3249 [gr-qc].

[5] P. Bizoń, M. Maliborski, and A. Rostworowski, (2015), arXiv:1506.03519 [gr-qc].

[6] A. Buchel, S. R. Green, L. Lehner, and S. L. Liebling, Phys.Rev. **D91**, 064026 (2015), arXiv:1412.4761 [gr-qc].

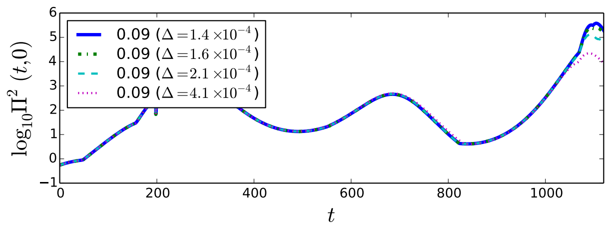

FIG. 1. The behavior of the scalar field at the origin for the evolution of equal-energy, two-mode initial data for $\epsilon = 0.09$ with increasing resolution.