Derivation of Dark Matter Parity from Lepton Parity

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Derivation of Dark Matter Parity from Lepton Parity

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Abstract

It is shown that in extensions of the standard model of quarks and leptons where additive lepton number L is broken by two units, so that Z_2 lepton parity, i.e. $(-1)^L$ which is either even or odd, remains exactly conserved, there is the possibility of stable dark matter without additional symmetry. This applies to many existing simple models of Majorana neutrino mass with dark matter, including some radiative models. Several well-known examples are discussed. This new insight leads to the construction of a radiative Type II seesaw model of neutrino mass with dark matter where the dominant decay of the doubly charged Higgs boson ξ^{++} is into W^+W^+ instead of the expected $l_i^+ l_j^+$ lepton pairs for the well-known tree-level model.

The origin of neutrino mass has been a fundamental theoretical issue for many years. It is not yet known experimentally whether it is Dirac so that an additive lepton number L is conserved, or Majorana so that L is broken to $(-1)^L$, i.e. lepton parity which is either even or odd, which remains conserved. Theoretically, it is usually assumed to be Majorana, i.e. self-conjugate, and comes from physics at an energy scale higher than that of electroweak symmetry breaking of order 100 GeV. As such, the starting point of any theoretical discussion of the underlying theory of neutrino mass is the effective dimension-five operator [1, 2]

$$\mathcal{L}_5 = -\frac{f_{ij}}{2\Lambda}(\nu_i\phi^0 - l_i\phi^+)(\nu_j\phi^0 - l_j\phi^+) + H.c., \quad (1)$$

where $(\nu_i, l_i), i = 1, 2, 3$ are the three left-handed lepton doublets of the standard model (SM) and (ϕ^+, ϕ^0) is the one Higgs scalar doublet. As ϕ^0 acquires a nonzero vacuum expectation value $\langle\phi^0\rangle = v$, the neutrino mass matrix is given by

$$\mathcal{M}^\nu_{ij} = \frac{f_{ij}v^2}{\Lambda}. \quad (2)$$

Note that $\mathcal{L}_5$ breaks lepton number L by two units.

Consider first the most well-known model where neutrino mass just comes from the canonical (Type I) seesaw mechanism with a massive Majorana ν_R. The new terms in the Lagrangian are $f\bar{\nu}_R\nu_L\phi^0$ and $(M/2)\nu_R\nu_R$. Hence lepton parity is conserved with ν_R odd. Now consider the simplest possible model of dark matter [3] with the addition of just one real singlet scalar particle s with odd Z_2 dark parity. How is this linked to lepton parity? The answer is very simple. Here lepton parity is odd for ν and l. If s is added, then to forbid the $s\nu_R\nu_R$ coupling, s must also be odd. Thus in this simplest model, dark parity is identical to lepton parity. The same holds true if s is replaced by a scalar doublet (η^+, η^0) [4], because its lepton parity must also be odd to forbid the $\bar{\nu}_R\nu_L\eta^0$ term. Suppose a neutral singlet fermion N_R is added to the SM and assumed to be dark matter, then to forbid the coupling $\bar{N}_R\nu_L\phi^0$, its lepton parity must now be even, but its dark parity should be odd.

In all cases, the formula for dark parity is then just $(-1)^{L+2j}$, where j is the intrinsic spin of the particle. At this point, it becomes obvious that this is completely analogous to the well-known R parity of supersymmetry, which also stabilizes dark matter.

In recent years, the notion that the underlying physics which generates $\mathcal{L}_5$ may be connected to dark matter has motivated a large number of studies. In most cases, an exactly conserved discrete Z_2 symmetry is imposed for the stability of dark matter, which appears to be unrelated to any existing symmetry of the standard model. As shown above, this is actually not the case. This dark Z_2 parity is in fact derivable from lepton parity, as discussed in the following three examples.

Consider first the simplest such model of radiative neutrino mass [5] through dark matter, called "scotogenic" from the Greek 'scotos' meaning darkness. The one-loop diagram is shown in Fig. 1. The new particles are a second scalar doublet (η^+, η^0) and three neutral

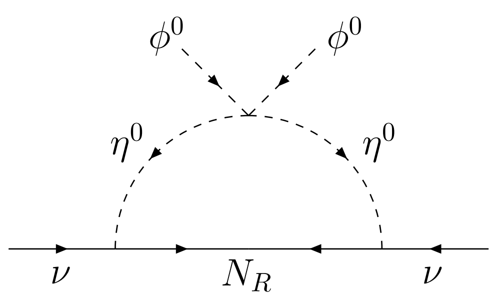

Figure 1: One-loop Z_2 scotogenic neutrino mass.

singlet fermions N_R. The imposed Z_2 is odd for (η^+, η^0) and N_R, whereas all SM particles are even. Under lepton parity with ν_L odd, the same Lagrangian is obtained with η odd and N_R even. The imposed dark parity is thus again $(-1)^{L+2j}$. If the conventional lepton parity assignment is made for N_R, i.e. odd, then it appears that the model has two Z_2 symmetries, but there is actually only one as shown above, because dark parity is derivable from the new assignment of lepton parity by virtue of the intrinsic spin of the new particles.

Consider next the three-loop model [6] with the diagram shown in Fig. 2. The new

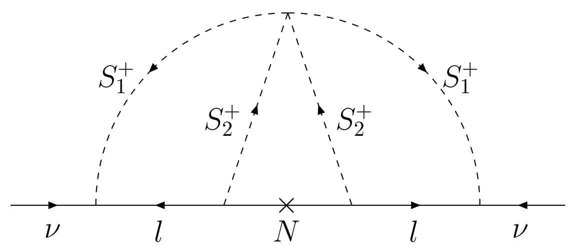

Figure 2: Three-loop neutrino mass.

particles are the S_1^+, S_2^+ scalar singlets, and the N_R singlet fermions, where S_2^+ and N_R are odd. Using lepton parity with ν, l, S_2^+ odd and N_R, S_1^+ even, the same Lagrangian is obtained. Again, dark parity is $(-1)^{L+2j}$.

Another three-loop model [7] has the diagram shown in Fig. 3. The new particles are a

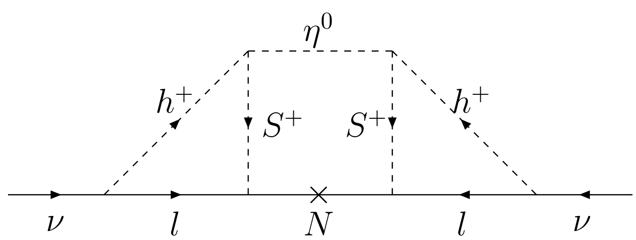

Figure 3: Another three-loop neutrino mass.

second Higgs doublet (h^+, h^0), a neutral scalar singlet η^0, and a charged scalar singlet S^+, together with N_R, where η^0, S^+, and N_R are odd. Using lepton parity with ν, l, S^+, η^0 odd, and h, N_R even, the same Lagrangian is again obtained with dark parity given by $(-1)^{L+2j}$.

There are also models of radiative neutrino mass with larger dark symmetries, such as Z_3 and $U(1)_D$. What role does lepton parity play in these cases? The obvious answer is that it cannot generate these symmetries, but the question is what happens to the lepton parity itself? To understand this, consider the Z_3 dark matter model [8] of neutrino mass as shown in Fig. 4. The new particles are three scalars χ transforming as ω under Z_3, one Dirac

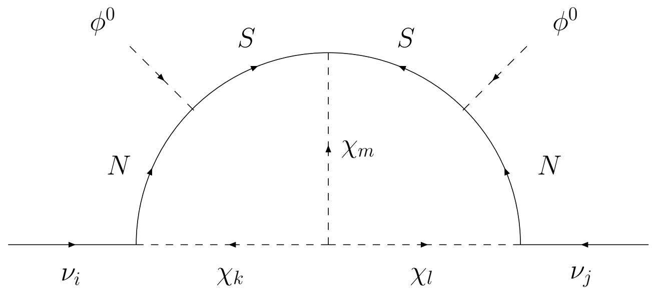

Figure 4: Two-loop neutrino mass.

fermion doublet (N, E) and one Dirac fermion singlet S transforming as ω^2, where $\omega^3 = 1$. To be consistent with the Lagrangian of this model, the lepton parity assignment has to be odd for ν, N and S, which means that the derived dark parity for all the new particles are even. In other words, there is no dark parity at all. The symmetry which stabilizes the new particles is the imposed Z_3.

Consider now the dark $U(1)_D$ case [9] as shown in Fig. 5. The new particles are two

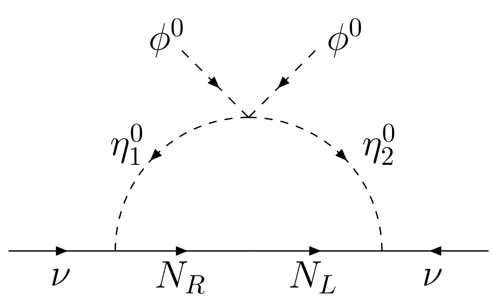

Figure 5: One-loop $U(1)_D$ scotogenic neutrino mass.

scalar doublets (η_1^+, η_1^0) and (η_2^+, η_2^0) transforming oppositely under $U(1)_D$, and three Dirac fermion singlets N transforming as η_1. Using lepton parity with ν, η_1, η_2 odd, a residual Z_2 symmetry of $U(1)_D$ is obtained. As expected, the full symmetry cannot be reproduced. For example, the $(\Phi^\dagger \eta_1)^2$ term is allowed by lepton parity but not $U(1)_D$.

Since $\mathcal{L}_5$ is a dimension-five operator, any loop realization is guaranteed to be finite. Suppose a Higgs triplet (ξ^{++}, ξ^+, ξ^0) is added to the SM, then a dimension-four coupling $\xi^0 \nu_i \nu_j - \xi^+(\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j$ is allowed. As ξ^0 obtains a small vacuum expectation value [10] from its interaction with the SM Higgs doublet, neutrinos acquire small Majorana masses, i.e. Type II seesaw. Is there an analogous radiative mechanism in this case? The answer is yes, using the notion of conserved lepton number violated only by two units with soft terms. This new model is discussed below.

Lepton number is imposed on all hard (dimension-four) terms of the Lagrangian, with ξ having $L = 0$. Its main purpose is to forbid the $\xi^0 \nu\nu$ term. The scalar trilinear $\bar{\xi}^0 \phi^0 \phi^0$ term is allowed and induces a small $\langle \xi^0 \rangle$, but ν remains massless. Note that this assignment is opposite to the well-known Gelmini-Roncadelli (GR) model [11] where ξ is assigned $L = -2$, so that $\xi^0 \nu\nu$ is allowed but $\bar{\xi}^0 \phi^0 \phi^0$ is forbidden. In the GR case, lepton number is spontaneously broken, so the neutrinos acquire mass together with the appearance of a massless Goldstone boson (majoron) which has long since been ruled out experimentally by Z decay. In the present model, neutrinos will acquire radiative masses with the explicit soft breaking of L to $(-1)^L$. This may be accomplished by adding a new Dirac fermion doublet (N, E) with $L = 0$, together with three complex neutral scalar singlets s with $L = 1$. The resulting one-loop diagram is shown in Fig. 6. Note that the hard terms $\xi^0 NN$ and $s \bar{\nu}_L N_R$

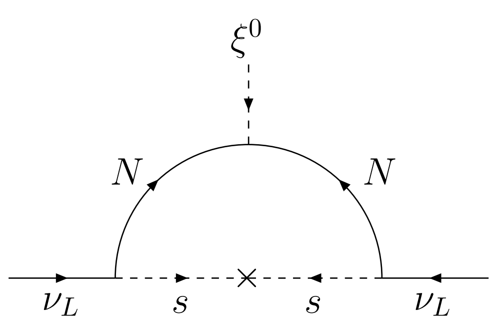

Figure 6: One-loop neutrino mass from $L = 0$ Higgs triplet.

are allowed by L conservation, whereas the ss terms break L softly by two units to $(-1)^L$.

Note again that the hard term $\xi^0\nu\nu$ is forbidden, or else the usual tree-level Type II seesaw mechanism would have prevailed. Again, (N,E) and s are odd under $(-1)^{L+2j}$. The three s scalars are the analogs of the three right-handed sneutrinos in supersymmetry, and $(N,E)_{L,R}$ are the analogs of the two higgsinos. However, their interactions are simpler here and less constrained. The lighest s is a possible dark-matter candidate [12], although it is highly constrained [13] from present data if it decouples from all other particles except the SM Higgs. Furthermore, from the allowed $(s^*s)(\Phi^\dagger\Phi)$ interactions, electroweak baryogensis [14] may be realized. Note that whereas one s cannot be both dark matter and induce a first-order phase transition in the Higgs potential, as shown in Ref. [15], there are three complex singlets here with mass splitting between the real and imaginary parts. The lightest one is dark matter, but the other five may have strong enough couplings to the Higgs boson with CP violation to allow successful baryogenesis. These large loop-induced deviations of the Higgs self couplings are presumably observable at a future e^+e^- accelerator for precision Higgs measurements.

The usual understanding of the Type II seesaw mechanism is that the scalar trilinear term $\mu\xi^\dagger\Phi\Phi$ breaks lepton number L by two units and a small vacuum expectation value $\langle\xi^0\rangle = u$ may be obtained if either μ is small or m_ξ is large or both. More precisely, consider the scalar potential of Φ and ξ.

$$\begin{aligned} V &= m^2\Phi^\dagger\Phi + M^2\xi^\dagger\xi + \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\xi^\dagger\xi)^2 + \lambda_3|2\xi^{++}\xi^0 - \xi^+\xi^+|^2 \\ &+ \lambda_4(\Phi^\dagger\Phi)(\xi^\dagger\xi) + \frac{1}{2}\lambda_5[|\sqrt{2}\xi^{++}\phi^- + \xi^+\bar{\phi}^0|^2 + |\xi^+\phi^- + \sqrt{2}\xi^0\bar{\phi}^0|^2] \\ &+ \mu(\bar{\xi}^0\phi^0\phi^0 + \sqrt{2}\xi^-\phi^0\phi^+ + \xi^{--}\phi^+\phi^+) + H.c. \end{aligned} \tag{3}$$

Let $\langle\phi^0\rangle = v$, then the conditions for the minimum of V are given by [10]

$$m^2 + \lambda_1 v^2 + (\lambda_4+\lambda_5)u^2 + 2\mu u = 0, \tag{4}$$

$$u[M^2 + \lambda_2 u^2 + (\lambda_4+\lambda_5)v^2] + \mu v^2 = 0. \tag{5}$$

For $\mu \neq 0$ but small, u is also naturally small because it is approximately given by

$$u \simeq \frac{-\mu v^2}{M^2 + (\lambda_4 + \lambda_5) v^2}, \tag{6}$$

where $v^2 \simeq -m^2/\lambda_1$. The physical masses of the $L = 0$ Higgs triplet are then given by

$$m^2(\xi^0) \simeq M^2 + (\lambda_4 + \lambda_5) v^2, \tag{7}$$

$$m^2(\xi^+) \simeq M^2 + (\lambda_4 + \frac{1}{2}\lambda_5) v^2, \tag{8}$$

$$m^2(\xi^{++}) \simeq M^2 + \lambda_4 v^2. \tag{9}$$

Since $m_\nu = f_\nu u$, where f_ν is a Yukawa coupling, there are two strategies for making m_ν small. (I) One is to keep f_ν not too small, say $f_\nu \sim 0.1$, but make $u \sim 1$ eV. This implies a very small μ unless M is very large. For M of order v so that the Higgs scalar triplet may be observable, $\mu \sim 1$ eV is required. To understand this small μ value, one approach is to ascribe it to the breaking of lepton number from extra dimensions [16]. Another approach is to forbid the μ term at tree level and generate it in one loop [17]. (II) The other strategy is to keep u not too small, say 0.1 GeV, but make f_ν very small. This is what happens here because the $L = 0$ assignment for ξ means $f_\nu = 0$ at tree level. It is then generated in one loop as shown in Fig. 6. Let the relevant Yukawa interactions be given by

$$\mathcal{L}_Y = f_s s \bar{\nu}_L N_R + \frac{1}{2} f_R \xi^0 N_R N_R + \frac{1}{2} f_L \xi^0 N_L N_L + H.c., \tag{10}$$

together with the allowed mass terms $m_E(\bar{N}N + \bar{E}E)$, $m_s^2 s^* s$, and the L breaking soft term $(1/2)(\Delta m_s^2) s^2 + H.c.$, then

$$m_\nu = \frac{f_s^2 u r x}{16\pi^2} [f_R F_R(x) + f_L F_L(x)], \tag{11}$$

where $r = \Delta m_s^2 / m_s^2$ and $x = m_s^2 / m_E^2$, with

$$F_R(x) = \frac{1+x}{(1-x)^2} + \frac{2x \ln x}{(1-x)^3}, \tag{12}$$

$$F_L(x) = \frac{2}{(1-x)^2} + \frac{(1+x)\ln x}{(1-x)^3}. \tag{13}$$

Since m_ν is now suppressed relative to u, the latter value may be as large as 0.1 GeV, using for example $x \sim f_R \sim f_L \sim 0.1$, $r \sim f_s \sim 0.01$. This implies that ξ may be as light as a few hundred GeV and be observable, with $\mu \sim 1$ GeV. Note that $u \sim 0.1$ GeV has negligible contribution (of order 10^{-6}) to the precisely measured ρ parameter $\rho_0 = 1.00040 \pm 0.00024$ [18]. For $f_s \sim 0.01$ and m_E a few hundred GeV, the new contributions to the anomalous muon magnetic moment and $\mu \to e\gamma$ are also negligible in this model.

As for the decay of ξ, its effective couplings to leptons are now very small, unlike the tree-level Type II seesaw model, where the decay of ξ^{++} to same-sign dileptons is expected to be dominant. Current experimental limits [19] on the mass of ξ^{++} into $e\mu$, $\mu\mu$, and ee final states are about 490 to 550 GeV, assuming for each a 100% branching fraction. These limits are not valid in the present model. Instead, $\xi^{++} \to W^+W^+$ should be considerd [20], for which the present limit on $m(\xi^{++})$ is only about 84 GeV [21]. A dedicated search of the W^+W^+ mode in the future is clearly called for.

If $m(\xi^{++}) > 2m_E$, then the decay channel $\xi^{++} \to E^+E^+$ opens up and will dominate. In that case, the subsequent decay $E^+ \to l^+s$, i.e. charged lepton plus missing energy, will be the signature. The present experimental limit [22] on m_E, assuming electroweak pair production, is about 260 GeV if $m_s < 100$ GeV for a 100% branching fraction to e or μ, and no limit if $m_s > 100$ GeV. There is also a lower threshold for ξ^{++} decay, i.e. $m(\xi^{++})$ sufficiently greater than $2m_s$, for which ξ^{++} decays through a virtual E^+E^+ pair to ssl^+l^+, resulting in same-sign dileptons plus missing energy.

The lepton number symmetry L may be promoted to the well-known $B-L$ gauge symmetry, but then three neutral singlet fermions ν_R transforming as -1 under $U(1)_{B-L}$ are usually added to satisfy the anomaly-free conditions. This means that neutrinos obtain tree-level Dirac masses from the allowed term $\bar{\nu}_L \nu_R \bar{\phi}^0$, and Type II seesaw would not be necessary. However, another possibility for anomaly cancellation is to have the three ν_R's tranform as

$(-4,-4,5)$ [23, 24]. In that case, $\bar{\nu}_L \nu_R \bar{\phi}^0$ is forbidden, and the spontaneous breaking of $U(1)_{B-L}$ leads to a radiative Type II seesaw model as described.

Finally, suppose the generation of neutrino mass is extended to (some) quarks and charged leptons through dark matter [25], then lepton parity may be promoted to matter parity, i.e. $(-1)^{3B+L}$ and dark parity becomes exactly R parity, i.e. $(-1)^{3B+L+2j}$, in complete analogy with what happens in the minimal supersymmetric standard model (MSSM). Of course, here the usual tree-level Yukawa couplings of the Higgs doublet to quarks and charged leptons must be forbidden by an imposed flavor symmetry [25].

In conclusion, it has been pointed out in this paper the very simple idea that for many well-studied models of neutrino mass with dark matter, lepton parity, if appropriately defined, leads automatically to dark parity. Several specific examples of existing models were discussed, together with a new radiative Type II seesaw model of neutrino mass with dark matter, which predicts a doubly charged Higgs boson with the interesting dominant decay mode of $\xi^{++} \to W^+ W^+$ [20, 21], below the threshold of the production of dark matter.

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