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Phys. Rev. Lett. **114**, 251103 — Published 26 June 2015

DOI: [10.1103/PhysRevLett.114.251103](https://doi.org/10.1103/PhysRevLett.114.251103)

# Ultra-high-energy debris from the collisional Penrose process

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Soon after the discovery of the Kerr metric, Penrose realized that superradiance can be exploited to extract energy from black holes. The original idea (involving the breakup of a single particle) yields only modest energy gains. A variant of the Penrose process consists of particle collisions in the ergoregion. The collisional Penrose process has been explored recently in the context of dark matter searches, with the conclusion that the ratio  $\eta$  between the energy of post-collision particles detected at infinity and the energy of the colliding particles should be modest ( $\eta \lesssim 1.5$ ). Schnittman [1] has shown that these studies underestimated the maximum efficiency by about one order of magnitude (i.e.,  $\eta \lesssim 15$ ). In this work we show that particle collisions in the vicinity of rapidly rotating black holes can produce high-energy ejecta and result in high efficiencies under much more generic conditions. The astrophysical likelihood of these events deserves further scrutiny, but our study hints at the tantalizing possibility that the collisional Penrose process may power gamma rays and ultra-high-energy cosmic rays.

PACS numbers: 04.70.-s, 04.70.Bw, 04.70.Dy

**I. Introduction.** It is tempting to say that modern relativistic astrophysics was born in 1963, when Kerr discovered the famous solution of Einstein’s equations describing rotating black holes [2] and Schmidt identified the first quasar [3]. The connection between these theoretical and observational breakthroughs became clear much later, but astronomers now agree that black holes are the engines behind many high-energy events in the Universe. Black holes are the most compact objects in Nature, and therefore particles in their vicinities can attain relativistic velocities, producing observable phenomena such as jets and gamma-ray bursts. The mechanisms converting the gravitational energy stored by black holes into high-energy fluxes of matter and radiation are a major area of investigation in relativistic astrophysics, and particle collisions near black holes could have important implications for dark matter searches (see e.g. [4, 5]).

In 1969, Penrose discovered a remarkable way to extract energy from Kerr black holes [6]. In the so-called “ergoregion” surrounding rotating black holes particles can have negative energies ( $\epsilon_i < 0$ ) as measured by an observer at infinity. Therefore an object of energy  $\epsilon_0$  can fragment into two pieces, one of which escapes to infinity with energy  $\epsilon_3$ , while the other is absorbed at the horizon with energy  $\epsilon_4 < 0$  (the reason for the odd choice of subscripts will be clear soon). This results in  $\epsilon_3 = \epsilon_0 - \epsilon_4 > \epsilon_0$ , a net energy gain at the expense of the rotational energy of the hole. Unfortunately this process yields a modest efficiency  $\epsilon_3/\epsilon_0 \leq (1 + \sqrt{2})/2 \simeq 1.207$ , where the maximum is achieved for disintegration into two massless particles [7–10].

The original Penrose process relies on the somewhat unrealistic disintegration of a single particle. An astrophysically more promising variant of the idea is the *collisional Penrose process* [9]: two bodies with energy

( $\epsilon_1, \epsilon_2$ ) – which could be elementary particles – collide resulting in two bodies with energy ( $\epsilon_3, \epsilon_4$ ). This process has recently attracted interest in connection with dark matter searches as a result of work by Banados, Silk and West [11], who found that particle collisions near rapidly rotating black holes could yield (in principle) unbound center-of-mass energies, perhaps producing observable exotic ejecta (see also [10], [12, 13] for caveats, and [14] for a review). A crucial issue is that the highest center-of-mass energies are achieved close to the horizon, and therefore the collision products experience large gravitational redshifts, resulting in modest efficiencies

$$\eta \equiv \frac{\epsilon_3}{\epsilon_1 + \epsilon_2} \quad (1)$$

for the escaping particle (that we will assume, without loss of generality, to be particle 3):  $\eta \lesssim 1.5$ , where the precise upper bound depends on the nature of the colliding particles [10, 15, 16].

Surprisingly, Schnittman [1] recently reported an order-of-magnitude increase in the efficiencies ( $\eta \lesssim 15$ ) by allowing for a small kinematic change in the collision, illustrated schematically in Fig. 1. One of the colliding particles (say, particle 1) falls in the effective potential for radial motion around an extremal black hole, defined in Eq. (2) below, and depicted by a dash-dotted (blue online) curve in Fig. 1. Particle 1 rebounds at the turning point  $r = r_t$ , so it has *outgoing* radial momentum ( $p_1^r > 0$ ) when it collides (at some radius  $r_0$  such that  $r_t < r_0 < r_{\text{ergo}}$ ) with the incoming particle 2. This outgoing momentum favors ejection of a high-energy particle 3 after the collision.

In this *Letter* we confirm the results of [1], and we reach the striking conclusion that *arbitrarily large* efficiencies can be achieved when we do not require both of the col-

liding particles to fall into the black hole from infinity. In our “super-Penrose” processes the outgoing particle 1 has angular momentum below the critical value for escape (corresponding to the solid black line in Fig. 1) when it collides with the ingoing particle 2. The radial motion of particle 1 is determined by the dashed (red online) effective potential: particle 1 is confined to the vicinity of the black hole, and it must have been created in the ergoregion via previous scattering events (see [17] for a similar proposal). The likelihood of such multiple scatterings is a delicate matter to be resolved by detailed cross-section calculations for specific processes, but we demonstrate that there is nothing preventing the creation of such particles in the ergosphere: our “super-Penrose” collisions are *at least kinematically* allowed. From now on we will use geometrical units ( $G = c = 1$ ).

**II. Setup.** Our setup is similar to that of Ref. [1]. We consider two bodies (or particles) 1 and 2 colliding with four-momenta  $p_1^\mu$  and  $p_2^\mu$  at Boyer-Lindquist coordinate position  $r = r_0$  in the equatorial plane ( $\theta = \pi/2$ ) of a Kerr black hole. The final state consists of two bodies 3 and 4 with four-momenta  $p_3^\mu$  and  $p_4^\mu$ , also moving in the equatorial plane. In Boyer-Lindquist coordinates, the geodesic equations for equatorial particles with rest mass  $m$ , energy  $\epsilon \equiv -g_{t\mu}p^\mu$  and angular momentum  $\ell \equiv g_{\varphi\mu}p^\mu$  (as seen by an observer at infinity) are given by

$$\begin{aligned} \dot{r}^2 &= \frac{r^3 + a^2(r + 2M)}{r^3} (E - V^+) (E - V^-), \\ V^\pm &= \frac{2aLM \pm \sqrt{r\Delta [L^2r + (r^3 + a^2(r + 2M))\delta]}}{r^3 + a^2(r + 2M)}, \\ \dot{\theta} &= 0, \\ \dot{\varphi} &= \frac{1}{\Delta} \left[ \left(1 - \frac{2M}{r}\right) L + \frac{2aM}{r} E \right], \end{aligned} \quad (2)$$

where the dot denotes differentiation with respect to the geodesic affine parameter  $\lambda$ , and  $\Delta = r^2 - 2Mr + a^2$ . Here  $E \equiv \epsilon/m$  and  $L \equiv \ell/m$  denote the energy and angular momentum per unit mass for massive particles, while for massless particles this distinction does not apply ( $E = \epsilon$  and  $L = \ell$ ). The constant  $\delta = 1$  (0) for massive (massless) particles, respectively. For massless particles the four-momentum is  $p^\mu = \dot{x}^\mu$ ; for massive particles we can choose  $\lambda = \tau/m$  ( $\tau$  being proper time), so that  $p_\mu p^\mu = -m^2$ . To enforce causality and guarantee that the locally measured energy is always positive, physical geodesics must satisfy  $\dot{t} > 0$ .

Local conservation of four-momentum implies

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu. \quad (3)$$

Using Eqs. (2), this condition can be written as a set of three equations:

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4, \quad (4)$$

$$\ell_1 + \ell_2 = \ell_3 + \ell_4, \quad (5)$$

$$p_1^r + p_2^r = p_3^r + p_4^r. \quad (6)$$

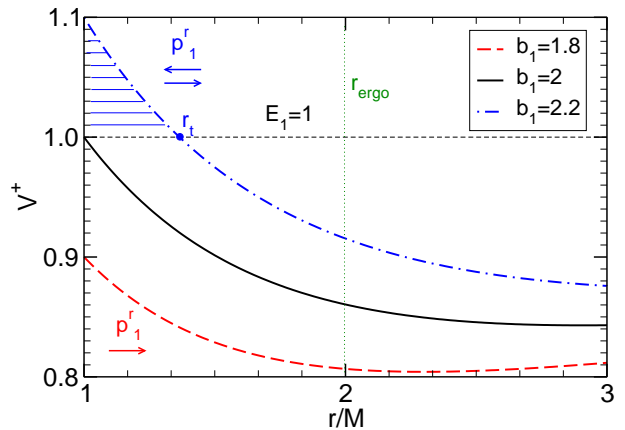


FIG. 1. Effective radial potential  $V^+$  [cf. Eq. (2)] for an extremal black hole, a particle with energy  $E_1 = 1$  and angular momentum  $L_1 = b_1 E_1$ . The horizon is located at  $r = 1$ , and the ergoregion corresponds to  $r < r_{\text{ergo}} = 2$ . The dash-dotted (online: blue) curve shows the potential felt by particle 1 for the process discussed in [1]; particle 1 cannot access the dashed (blue) region with  $r < r_t$ . The dashed (online: red) curve corresponds to the process considered in this paper. The solid (online: black) curve depicts the potential for the critical value of  $b_1$  separating these two processes.

The initial state is completely specified by fixing  $E_{1,2}$ ,  $L_{1,2}$  (and  $m_{1,2}$  for massive particles). For massive particles, we can specify the final state by providing a relation between  $m_3$  and  $m_4$ : for example, for two identical collision products we can set  $m_4 = m_3$ . Finally, we provide the Lorentz boost  $E_4$  and specific angular momentum  $L_4$  and we are left with three unknowns ( $m_3$ ,  $E_3$ ,  $L_3$ ) that can be determined by solving Eqs. (4)–(6). To be observationally interesting, the solutions of the system above must satisfy the requirement that particle 3 can escape and reach an observer at infinity (i.e., there cannot be any turning points  $r_3^{\ddagger}$  for particle 3 with  $r_3^{\ddagger} > r_0$ ). To find the maximum efficiency  $\eta_{\text{max}}$ , we simply repeat the previous procedure for a range of values of  $L_4$ . We find that the efficiency only depends on the impact parameters of the initial particles  $b_{1,2} \equiv L_{1,2}/E_{1,2}$  and on the ratio of their energies  $R \equiv \epsilon_1/\epsilon_2$ . Therefore all of our results will be shown as functions of these quantities. For simplicity, we will also set the black-hole mass  $M = 1$ .

**III. Super-amplification collisions.** We focus on collisions for which particle 1 is outgoing (i.e.,  $p_1^r > 0$ ) while particle 2 is ingoing. Our results are summarized in Figs. 2–5. We start by using the same initial conditions as in [1]: an extremal black hole and  $E_1 = E_2$ . For  $b_1 = 2$  we confirm that in this case the peak efficiency is  $\eta_{\text{max}} \sim 6.4$  (a value sensibly larger than the predictions of previous studies [14–16]) for collisions that occur close enough to the horizon. In Fig. 2 we show the maximum efficiency, varying  $b_1$  close to the critical value  $b_1 = 2$  and fixing the impact parameter of particle 2 to the value

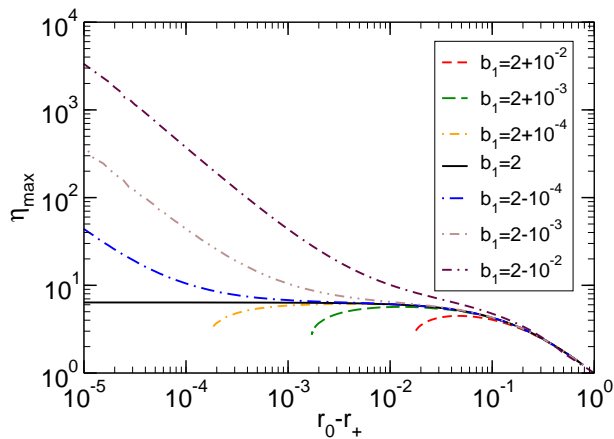


FIG. 2. Maximum efficiency  $\eta_{\max}$  for the collision of equal-energy particles ( $R = 1$ ) as a function of the radius at which the reaction occurs. We consider  $p_1^r > 0$ ,  $p_2^r < 0$ ,  $b_2 = -2(1 + \sqrt{2})$ , and an extremal black hole ( $a = 1$ ). The curves for  $b_1 > 2$  terminate at the turning point of particle 1.

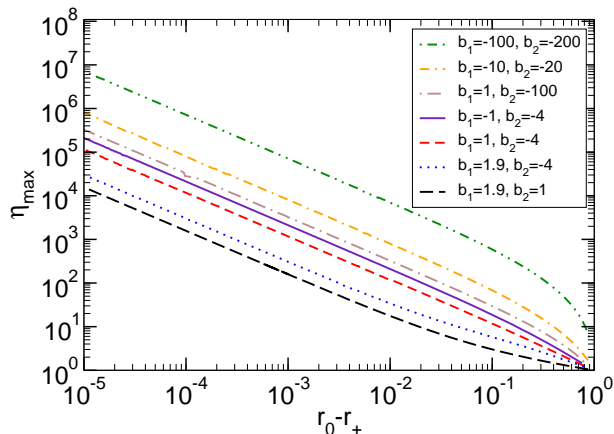


FIG. 3. Maximum efficiency  $\eta_{\max}$  for the same process considered in Fig. 2 as a function of the radius at which the reaction occurs, for  $a = 1$ . When  $b_2 < -2(1 + \sqrt{2})$ , particle 2 must also be produced close to the black hole.

$b_2 = -2(1 + \sqrt{2})$ . We find that the maximum efficiency quickly decreases with increasing  $b_1$ , because particle 1 is now allowed to move in a smaller region inside the ergosphere (see Fig.1). For sub-critical particles ( $b_1 < 2$ ) the scenario is completely different, and the efficiency becomes arbitrarily large as we approach the horizon. In our searches we found that these “super-Penrose collisions” occur for  $b_2 < b_1 < 2$ .

Fig. 3 shows that the efficiencies of super-Penrose collisions can be orders of magnitude larger than those found in [1] in a large region of the parameter space of initial conditions. For very large ( $|b_1|, |b_2|$ ) and collisions close to  $r = r_+$ , the peak efficiency scales as  $\eta_{\max} \sim 0.5\sqrt{(2 - b_1)(2 - b_2)}/(r_0 - r_+)$ . Efficiencies  $\eta \gtrsim 10^3$  are

easily achieved for maximally spinning black holes.

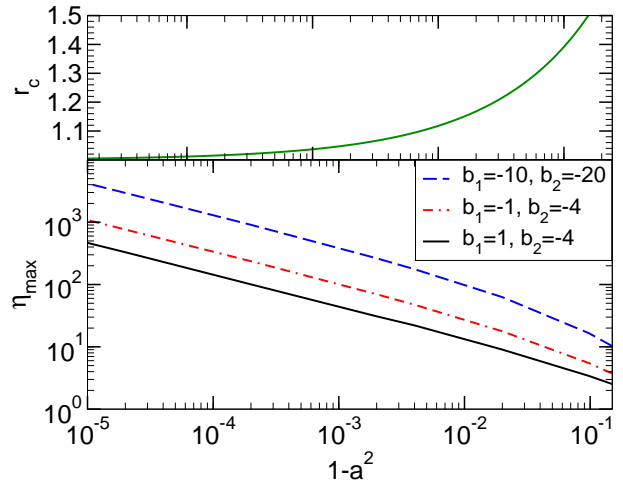


FIG. 4. Maximum efficiency as a function of the black hole spin  $a$  for the same process as in Fig. 2. The maximum efficiency is attained at the photon sphere,  $r_c$ , below which the infalling particle 3 can not escape. For  $a \simeq 1$ , the maximum efficiency scales as  $1/\sqrt{1 - a^2}$ .

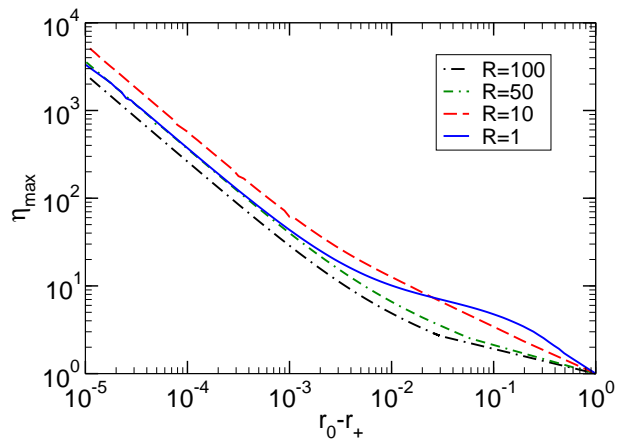


FIG. 5. Maximum gain for a Compton-like scattering between a massless particle with  $p_1^r > 0$ ,  $b_1 = 1.99$  and a massive particle with  $p_2^r < 0$ ,  $b_2 = -2(1 + \sqrt{2})$ . We plot the maximum gain for different values of  $R \equiv \epsilon_1/\epsilon_2$ .

Fig. 4 shows the dependence of the super-amplification efficiency on the black-hole spin  $a$ . For  $a < 1$  the maximum gain quickly drops (bottom panel), and particle 3 can only escape for collisions occurring at  $r_0 > r_c$ , where  $r_c$  (top panel) is the Boyer-Lindquist radius of the photon sphere in the equatorial plane. The maximum efficiency is attained when the collision occurs precisely at the photon sphere, where we find the scaling  $\eta_{\max} \sim (1 - a^2)^{-1/2}$ . Nonetheless, very high efficiencies are still allowed for astrophysically relevant black holes, as long as particles with large negative impact parameters

$b_1$  and  $b_2$  are created inside the ergosphere and collide. For collisions where particle 2 falls from spatial infinity and black holes spinning at the Thorne limit  $a \sim 0.998$  [18] we still find efficiencies as large as  $\eta \sim 10^2$ .

Finally, in Fig. 5 we consider the Compton-like scattering of a massless particle with  $p_1^r > 0$  and a massive particle with  $p_2^r < 0$  and we show the dependence of the maximum efficiency on the energy ratio  $R \equiv \epsilon_1/\epsilon_2$  of the initial particles. For reactions occurring near the edge of the ergoregion the efficiency is largest when  $R = 1$ , while near the horizon ( $r/r_+ - 1 \ll 1$ ) the maximum efficiency is achieved for  $R \approx 10$ . By comparing the solid (blue online) and dash-dash-dotted (green online) curves, we see that the maximum efficiency at the horizon occurs for  $1 < R \lesssim 50$ , and for  $R \gtrsim 50$  the efficiency decreases again. In conclusion, reactions such as the inverse Compton scattering between photons and massive particles could (at least in principle) produce highly energetic photons by multiple scattering processes.

**IV. The origin of the colliding particles.** Super-Penrose amplification cannot result from the collision of two particles coming from spatial infinity: as shown schematically in Fig. 1, particle 1 (with  $p_1^r > 0$  and  $b_1 < 2$  for  $a = 1$ ) must be created inside the ergosphere by previous scattering events. Furthermore, for  $b_{1,2} < -2(1+\sqrt{2})$  there are turning points of the motion outside the ergoregion, and particles would be deflected back before reaching the ergoregion (cf. Fig. 1 of [1]).

The colliding particles giving rise to super-amplification are physically relevant initial states, because they can be created (for example) by the collision of particles coming from infinity. In fact there is a wide range of realistic initial conditions that can result in “super-Penrose initial conditions”. For example, a particle with  $E_1 = 1$ ,  $m_1 = 1$ ,  $L_1 = 1.9$  (dotted blue line in Fig. 3) in the extremal Kerr background can be generated by two particles with rest masses  $(1, m_*)$  falling from rest at infinity with angular momenta ( $L_1 = -4, L_2 = 2$ ) and colliding at  $r = 1.01$ , as long as  $m_* > 6$ . The threshold mass ratio  $m_*$  depends on the angular momentum of the colliding particles and on the spin of the black hole: it is proportional to  $|L_1|$ , and it scales like  $(r - 1)^{-2}$  for extremal black holes. Note in particular that we can have super-Penrose collisions for arbitrarily large negative angular momenta of particle 2. Due to frame dragging, the cross section for counterrotating incoming particles is much larger. Counterrotation may be a common feature in astrophysics, e.g. because of disk fragmentation, as in the “chaotic accretion” scenario [19, 20]. Finally, we could (very conservatively) define a combined efficiency  $\eta'$  so that  $\epsilon_{1,2}$  in Eq. (1) refers to the “parent” particles falling from infinity. With this redefinition, the maximum efficiency typically seems to lower to the levels predicted in [1] (cf. Sec. V below).

In summary, the initial conditions giving rise to super-

amplification are kinematically allowed as the result of collisions of particles falling into the hole from large distances.

**V. Multiple scattering.** A consequence of the argument presented above is that multiple scattering events can increase the energetic gain achievable with (and the astrophysical relevance of) the “traditional” collisional Penrose process [9–11] and of Schnittman’s variant of the process [1]. This is because the energy of particles that cannot escape to infinity may be substantially larger than the energy of those that can. Even if “trapped” and unable to escape themselves, these particles may collide with other particles and give rise to high-energy collision products that *may* escape and be detectable.

Multiple energy-extracting collisions may lead to very large efficiencies [1]. Each Penrose scattering decreases the black-hole spin, but efficiencies can still be moderately high, even away from  $a = 1$ . These events may well be rare, but it is tempting to propose that they could play a role in the production of observable gamma rays or ultra-high-energy cosmic rays. A detailed assessment of these possibilities is beyond the scope of this *Letter*.

**VI. Discussion.** Previous studies [1, 10, 15, 16] (which we reproduced) focused on a region of the parameter space that excludes by construction the amplification mechanism studied in our work. The astrophysical likelihood of the super-Penrose amplification process proposed here is obviously a critical issue that requires further work. For very large efficiencies, the energy of the escaping particle can be as large as the black hole’s, and our geodesic approximation clearly breaks down. In this regime back-reaction effects must be included. Furthermore, numerical simulations and detailed calculations of production rates (along the lines of [10, 21, 22]) are needed to make conclusive statements about the astrophysical relevance of these results.

We made the simplifying assumption that the reaction occurs in the equatorial plane. Very few off-equatorial calculations of the Penrose process have been performed in the past, mainly due to computational difficulties. The results reported here, together with recent studies of the off-equatorial collisional Penrose process [23, 24], suggest that surprises may be in store, and a generalization of our calculations to the off-equatorial case is urgently needed.

Another important extension of our work would be the inclusion of external magnetic fields, a common feature of astrophysical black holes. The Penrose process for charged particles in the presence of electromagnetic fields is known to be more efficient than the original process [25, 26], with efficiencies as large as  $\eta \sim 10$  [27]. The possibility that electromagnetic fields could trigger and enhance super-Penrose collisions makes this an important line of research for the future.

We hope that this *Letter* will stimulate further work in these directions and improve our understanding of some of the most energetic events in the Universe.



After the completion of this paper we learned about work by Zaslavskii [28] that confirms our main findings.

**Acknowledgments.** We thank Jorge Rocha and Jeremy Schnittman for useful comments. E.B. was supported by NSF CAREER Grant No. PHY-1055103. R.B. acknowledges financial support from the FCT-IDPASC program through the Grant No. SFRH/BD/52047/2012, and from the Fundação Calouste Gulbenkian through the Programa Gulbenkian de Estímulo à Investigação Científica. V.C. acknowledges financial support provided under the European Union’s FP7 ERC Starting Grant “The dynamics of black holes: testing the limits of Einstein’s theory” grant agreement no. DyBHo–256667. This research was supported in part by the Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Economic Development & Innovation. This work was supported by the NRHEP 295189 FP7-PEOPLE-2011-IRSES Grant.

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