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Single to multi quasiparticle excitations in the itinerant helical magnet CeRhIn₅

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CeRhIn₅ is an itinerant magnet where the Ce³⁺ spins order in a simple helical phase. We investigate the spin excitations and observe sharp spin-waves parameterized by a nearest neighbor exchange $J_{RKKY}=0.88 \pm 0.05$ meV. At higher energies, the spin fluctuations are heavily damped where single quasiparticle excitations are replaced by a momentum and energy broadened continuum constrained by kinematics of energy and momentum conservation. The delicate energy balance between localized and itinerant characters results in the breakdown of the single quasiparticle picture in CeRhIn₅.

The noninteracting quasiparticle description of excitations is fundamental to condensed matter physics and the understanding of low energy fluctuations. However, interacting quasiparticle states have recently been recognized as important for the understanding of anomalous phases. For example, composite states including resonating valence bond states [1], Zhang-Rice singlets [2] or spinon-holons in the pseudogap, [3] have been suggested to be fundamental to superconductivity, frustrated magnetism, and even quantum criticality. [4–6] We use neutron scattering to measure the breakdown of the single quasiparticle description of the spin excitations in a helical itinerant heavy fermion magnet.

CeRhIn₅ is a heavy fermion metal, part of the CeTIn₅ (T=Rh, Ir, and Co) series displaying an interplay between localized antiferromagnetism and superconductivity. [7–10] The presence of two-dimensional layers of Ce³⁺ ions connects the physics of these systems with other unconventional superconductors as in the cuprates [11– 15] or iron based pnictide/chalcogenide superconductors. [16–18] CeRhIn₅ magnetically orders at T_N=3.8 K [19–21] and enters an unconventional superconducting phase that can be accessed under hydrostatic pressures or temperatures below ~ 75 mK. [22–26]

CeRhIn₅ is isostructural with CeCoIn₅, which is superconducting at ambient pressures with a $T_c=2.3$ K. [14] The order parameter of the superconducting phase has a *d*-wave symmetry with nodes in the *ab* plane. [27, 28] Magnetism and superconductivity are strongly coupled as evidenced by neutron scattering measurements reporting a doublet spin-resonance peak connected with superconductivity and indicating an order parameter that changes sign, consistent with *d*-wave symmetry. [29–31] At high magnetic fields near H_{c2} , an unusual magnetic "Q-phase" has been reported to exist in a narrow field region further confirming the interplay between super-



FIG. 1. The magnetic structure of CeRhIn₅ investigated using spherical polarimetry. (a) An illustration of the magnetic structure. (b) is a plot of $P_{measured}$ vs $P_{calculated}$ based upon the isotropic helical model shown in (a).

conductivity and the localized magnetism. [32, 33]

Neutron measurements were performed at NIST (Gaithersburg, USA) using MACS [34] and at the ILL (Grenoble, France) using the IN12 spectrometer and the D23 and D3 diffractometers. The HHL aligned sample was prepared using self-flux method. [14] To correct for the large neutron absorption, [35, 36] a finite element analysis has been done. Further details are provided in the supplementary information.



FIG. 2. Constant energy scans taken on IN12 and MACS in the antiferromagnetic phase. (a)-(b) illustrate fluctuations polarized along c with the horizontal bar being the spectrometer resolution. (c)-(e) show constant energy slices showing the energy dependence of the spin fluctuations. Fluctuations at large L characteristic of predominately a-b plane polarized fluctuations are present to high energy transfers.

We first review the low temperature magnetic structure using spherical polarimetry. [37–39] As found in the pioneering work by Bao et al. [19], the magnetic structure (Fig. 1 (a)) is characterized by an incommensurate Bragg peak Q = (0.5, 0.5, 0.297). Fig. 1 (b) plots the results of our polarized diffraction experiment confirming this with measured polarization matrix elements $(P_{measured})$ against calculated $(P_{calculated})$ assuming a perfect a - b helix with the moment defined by $\mathbf{M} = \mathbf{M}_a + i\mathbf{M}_b$ (with $|\mathbf{M}_{\mathbf{a}}| = |\mathbf{M}_{\mathbf{b}}|$) and propagation vector along c. Expressions for the matrix elements are given in the supplementary information. Confirming the helical magnetism, a volume imbalance between the two chiral domains η =0.68 \pm 0.05 was needed to account for off-diagonal matrix elements. Unpolarized diffraction measures the ordered magnetic moment to be $0.34 \pm 0.05 \ \mu_B$ per cerium ion, consistent with expectations from crystal field theory. [40] The derived magnetic structure and symmetry analysis is also consistent with predictions from Landau theory for the phase transition as outlined in the supplemental information. [41, 42]

We now discuss the inelastic scattering probing the dynamics. Figure 2 illustrates a summary of con-



FIG. 3. Constant energy (a-c) and Q(d-f) cuts integrating over L=[-4, -1.25] sensitive to predominately in-plane fluctuations. The solid lines in (a)-(c) are to gaussians displaced from the commensurate $(\frac{1}{2}, \frac{1}{2})$ position. Solid lines in (d)-(f) are fits to damped harmonic oscillators. The shaded region is the broad heavily damped component. (d-f) are integrated over $\pm (0.025, 0.025)$. The solid bars represent the experimental resolution.

stant energy scans. Figure 2 (a) shows a momentum scan along [110] finding the scattering to be peaked at (0.5, 0.5) indicating antiferromagnetic correlations within the a - b plane. Figure 2 (b) shows a scan along the [001] direction (corrected for absorption) finding momentum broadened correlations which decay rapidly with L. The solid line is a fit to $I(\mathbf{Q}) \propto f(Q)^2 \times [1 - 1]^2$ $(\hat{\mathbf{Q}}\cdot\hat{\mathbf{c}})^2 \sinh(c/\xi_c) / [\cosh(c/\xi_c) + \cos(\mathbf{Q}\cdot\mathbf{c})]$ which represents short-range antiferromagnetically correlated Ce^{3+} moments polarized along c with a dynamic correlation length ξ_c . $f(Q)^2$ is the magnetic form factor. [43] The dynamic correlation length was derived to be $\xi_c = 3.1$ ± 0.7 Å indicating little coupling between the Ce³⁺ layers. The strong decrease in intensity with momentum transfer along L illustrates that these fluctuations are predominately out of the a - b plane (c-axis polarized) and hence referred to to as out-of plane fluctuations here (see supplementary information).



FIG. 4. Constant-Q scans taken on IN12 and MACS. (a) illustrate the energy dependence of the c axis polarized spin fluctuations. (c) shows a constant-Q slice taken on MACS (integrating L=[-4,-1.25]) with the solid points fits to constant-Q scans and the open circles fits to constant energy. A continuum of scattering is present above the top of the "1-magnon" band. (d) shows a calculation considering the parameterization in single ("1") and multiparticle ("2") states with the \vec{Q} integrated intensities plotted in (b).

Figures 2 (c)-(e) illustrate full constant energy maps taken on MACS at energy transfers of 1.2-3 meV (c)-(e). Panel (c) illustrates that, as well as the magnetic scattering near L=0 from the out of plane fluctuations, strong scattering is also present at large L indicative fluctuations predominately polarized within the a - b plane, referred to as in-plane fluctuations here. We note that the energy transfer is significantly less than the first crystal field excitation at ~ 7-9 meV, indicating that the transition results from excitations within the lowest energy Ce³⁺ doublet. [44, 45] The correlated scattering is present at higher energies as evidenced by similar scans in (d) and (e).

We now discuss the energy dependence. Constant energy and momentum cuts are shown in Figs. 3 (a)-(c) and (d)-(f) respectively. As seen in both types of cuts, at low-energies the magnetic dynamics are described by two components - one which is sharp and resolution limited in energy and momentum and second higher energy component which is broadened in both momentum and

energy.

Figure 4 (c) displays a constant Q slice (integrating over L=[-1.5,-4]) sensitive to the predominately in-plane scattering. When all of the scattering is integrated over the magnetic Brillouin zone, the total spectral weight (accounting for absorption) is estimated at $2.0 \pm 0.5 \mu_B^2$ agreeing with expectations from single ion crystal field analysis (see supplementary information). Both components need to to be considered to satisfy sum rules and obtain all of the required dynamic spectral weight.

Neutron scattering is constrained by strict selection rules with the scattering process having $\Delta S_z = \pm 1$ or 0. Transverse spin excitations derive from harmonic theory and can be written as single quasiparticle or magnon excitations which are long lived in a magnetically ordered structure with resolution limited inelastic peaks. Other anharmonic processes can occur including scattering from two magnons with opposite sign (ie. $\Delta S_z=0$ process) provided there is an interaction term between the single magnon quasiparticles in the Hamiltonian. For collinear magnets, such terms are predicted to be weak from symmetry considerations, however for a noncollinear magnet, such as magnetic spiral or helix, such constraints are relaxed. [46, 47] This additional cross section in the neutron response is constrained by momentum and energy conserving processes, and is possible over a wide range in energy and momentum which is determined by the single magnon dispersion. Analogous classic examples of this cross section are found in model insulating low spin chains. [48–54] We now investigate whether the two component lineshape found here can be understood in terms of a single and multiparticle parameterization.

We first consider the low-energy component of the cross section that is also resolution limited in energy. Magnetic excitations for a planer helical magnet with a characteristic wavevector $\vec{q_c}$ are described by three modes with $\vec{Q} = \pm \vec{q_c}$ being in-plane modes and a commensurate mode describing out of plane fluctuations. [55–58]

Fig. 4 (a) shows a constant \vec{Q} =(0.5,0.5,0.3) scan which is derived to have a strong *c*-axis polarized character. An antisymmetric lorentzian fit gives a peak energy position of $\hbar\Omega$ =1.21 ± 0.06 meV and line width (half-width) of $\hbar\Gamma$ =0.22 ± 0.14 meV. The out of plane fluctuations are therefore gapped as well as weakly dispersing.

To extract a dispersion and hence an estimate for the in-plane exchange interaction, we have fit constant energy scans (examples shown in Fig. 3 (a)-(c)) to gaussians symmetrically displaced from the $\vec{Q} = (\frac{1}{2}, \frac{1}{2})$ and illustrated by the open circles in Fig. 4 (c). The constant energy fits show dispersing excitations at wave vectors close to $(\frac{1}{2}, \frac{1}{2})$, but at the zone boundary near $(\frac{1}{4}, \frac{1}{4})$ the "dispersion" becomes nearly vertical.

Constant momentum scans in Fig. 3 show that this vertical dispersion at the zone boundaries is due to the second short-lived and damped-in-energy component to

the cross section. To fully separate these two components, we have fit energy scans to two harmonic oscillators with one being resolution limited and the second damped in energy. The sharp component is denoted by the filled circles in Fig. 4 (c). To extract an estimate for the localized J_{RKKY} exchange, we have fit the peak locations of the sharp component to the dispersion for a $j_{eff} = \frac{1}{2}$ "spins" (capturing the doublet nature of the ground state) on a square lattice. We have followed the classic model previously applied to Rb₂MnF₄ where a lattice periodic dispersion of $E(\vec{q}) = 2J_{RKKY}\sqrt{\alpha^2 - \gamma(\vec{q})^2}$, with $\gamma(\vec{q}) = \cos[\pi(H+K)] \cos[\pi(-K+H)]$ was used. This provides a simple means of parametrizing the data and estimate of the nearest neighbor in-plane exchange. We note that this model does not capture the out of plane mode which is found to show little dispersion and originate from weak coupling between the Ce^{3+} layers. Based on the fit in Fig. 4 to this parametrization, we extract $J_{RKKY}=0.88\pm0.05$ meV and an anisotropy $\alpha=1.06\pm$ 0.02 meV.

Having described the sharp component sensitive to the antiferromagnetic exchange, we now discuss the broad continuum of scattering at higher energies. We interpret and describe this component in terms of a multi magnon model termed the "1+2" model. The heavily damped features originates from unstable particles where energy and momentum conservation result in a decay process. As noted in Ref. 59, the presence of the three modes imposed by the helical structure imply that excitations can decay into lower energy quasiparticles assuming there is a binding interaction. For a given momentum transfer k, the two particle excitations form a continuum of states and the energy and momentum positions where the cross section is finite is defined by conservation of momentum and energy. Following the classical theory outlined in Ref. 60 and 61 and using our parametrization of the single magnon scattering, we have calculated the energy and momentum dependence of the allowed multimagnon scattering. Fig. 4 (d) shows a plot of the scaled calculation with the one-magnon term superimposed to give the sharp component. The momentum integrated intensity from the calculations is over plotted in Fig. 4 (b). Deviations from calculations at low energies are likely due to experimental limitations owing to resolution, incoherent nuclear scattering, and absorption.

Several features are reproduced in the multiparticle calculation: first, the broad continuum of scattering which extends up to nearly $2 \times J_{RKKY}$; and second, the nearly vertical columns of scattering which extend up in energy near the zone boundary. Near the magnetic zone boundary, as illustrated in Fig. 3, the two components can be separated with both accounting for roughly equal amounts in terms of the integrated intensity. The multiparticle model therefore provides an account of the neutron cross section once the single magnon component is parameterized with it giving the correct energy band-

width and momentum dependence. The multiparticle continuum is also predicted to have a longitudinal polarization, [46] consistent with the persistence to large L shown in Fig. 2.

One thing that is not explicit in this analysis is how the coupling between single quasiparticles originates and what determines the relative spectral weight between the single and multiparticle components. In insulating magnets, the spectral weight in the continuum comes from the Bragg peak, yet in CeRhIn₅, our analysis shows that the spectral weight draws from the inelastic component. The symmetry of the helical magnetic structure simply implies that such multiparticle scattering is allowed in the neutron scattering cross section. Such processes maybe determined by cubic terms in the Hamiltonian or possibly coupling resulting from the itinerant electronic nature of CeRhIn₅ as discussed elsewhere. [62–65] However, we note that in classical and insulating magnets the multiparticle continuum is weak comprising $\sim 1-2$ % of the total spectral weight in Rb_2MnF_4 . [60] The relatively large size of the multiparticle continuum in CeRhIn₅ suggests that localized effects are not the cause and that the itinerant properties are important. Our experiment suggests a low energy scale in CeRhIn₅ where the single quasiparticle description breaks down and interactions become important.

The physics here might be more general and in particular, enhanced broadening in the neutron cross section has been observed near the zone boundary in metallic $Fe_{1+x}Te$ [66], and the cuprates $YBa_2Cu_3O_{6.35}$ [67, 68], La_2CuO_4 [69], and $Sr_2CuO_2Cl_2$. [70] These might indicate an interaction similar that discussed here yet much weaker due to symmetry constraints determined by the collinear structures. An alternate view is that the continuum in CeRhIn₅ results from the single magnon branch at low energies interacting with a continuum of electronic excitations as suggested in itinerant ferromagnets magnets such as MnSi [71] and Fe [72]. However, this scenario results in the disappearance or strong dampening of the single magnon branch and not the presence of two distinct components observed here in CeRhIn₅. This high energy continuum may represent a direct measure of the hybridization gap which characterizes the energy scale where the quasiparticles cross over from localized to itinerant and such energy scales are expected to be on the order of $\sim \text{meV}$ in CeRhIn₅. [10]

In summary, we have studied the excitations in helical CeRhIn₅ and found the presence of a strong continuum along with sharp single magnon excitations. Given both components are required to satisfy neutron scattering sum rules, we understand the cross section in terms of a "1+2" particle model where the broad component originates from multiparticle states with the energy and momentum dependence fixed by energy and momentum conservation laws determined by the single magnon cross section. We propose the multiparticle component originates from coupled magnonss, observable given the relaxed symmetry constraints from a helical magnet. Our measurements directly observe the breakdown of a the single quasiparticle, or magnon, picture for an itinerant magnet.

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