Constraint of the Astrophysical $^{26g}\text{Al}(p,\gamma)^{27}\text{Si}$ Destruction Rate at Stellar Temperatures


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The galactic 1.809-MeV $\gamma$-ray signature from the $\beta$ decay of $^{26g}$Al is a dominant target of $\gamma$-ray astronomy, of which a significant component is understood to originate from massive stars. The $^{26g}$Al($p,\gamma$)$^{27}$Si reaction is a major destruction pathway for $^{26}$Al at stellar temperatures, but the reaction rate is poorly constrained due to uncertainties in the strengths of low-lying resonances in $^{27}$Si. The $^{26g}$Al($d,p$)$^{27}$Al reaction has been employed in inverse kinematics to determine the spectroscopic factors, and hence resonance strengths, of proton resonances in $^{27}$Si via mirror symmetry. The strength of the 127-keV resonance is found to be a factor of four higher than the previously adopted upper limit, and the upper limit for the 68-keV resonance has been reduced by an order of magnitude, considerably constraining the $^{26g}$Al destruction rate at stellar temperatures.

Astronomical observables that can be related directly and unambiguously to individual nuclear isotopes provide unparalleled constraints on nucleosynthetic processes in astrophysical environments. However, such signatures are still only sparsely available, coming largely from isotopic ratios in meteoritic grains and astronomical observation of a handful of $\gamma$-ray lines, the latter providing a relatively direct probe of the source distribution. Arguably the most studied $\gamma$-ray signature is the 1.809-MeV line from the $\beta$ decay ($t_{1/2} = 7.2 \times 10^5$ y) of the $J^\pi = 5^+$ ground state of $^{26}$Al ($^{26g}$Al), a direct indication of the ongoing formation of $^{26}$Al within our Galaxy. This signature is insensitive to $^{26}$Al synthesized in its $0^+$ metastable state at 228 keV ($^{26m}$Al), which $\beta$ decays to the $^{26}$Mg ground state ($t_{1/2} = 6.34$ s [1]). Following inferences of $^{26}$Al from meteoric isotopic ratios [2] and the subsequent landmark astronomical detection of the $^{26g}$Al $\gamma$-ray signature [3], $^{26g}$Al has been the focus of increasingly sensitive measurements, aided by and driving the development of satellite-based $\gamma$-ray telescopes [4, 5]. The first Galactic intensity map of an individual $\gamma$-ray line [6], culminating in a nine-year exposure, and a subsequent 1.5-year measurement by the INTEGRAL $\gamma$-ray observatory which measured the Doppler-shift of this line with respect to the Galactic center [7], demonstrate that $^{26g}$Al has Galaxy-wide origins, suggesting a commonly-occurring progenitor. Additionally, in localized sources such as Cygnus and Orion, detailed spatial studies have used $^{26g}$Al as a tracer for the dispersal of massive-star ejecta within surrounding molecular clouds [8].

Directional comparison between $^{26g}$Al and other astronomical observables constrains the dominant sources of $^{26g}$Al [9]. A strong correlation between $^{26g}$Al and a Cosmic Background Explorer survey of 53-GHz microwave free-free emission, an indicator of ionized gas clouds and hence HII regions of massive ($M > 20M_\odot$) star formation [10], suggests a significant massive-star component to galactic $^{26g}$Al production. However, the relative contributions to the $^{26g}$Al flux remains uncertain, with contributors including classical novae [11], asymptotic giant branch (AGB) stars [12], and massive stars. For the latter, it is unclear whether $^{26g}$Al is predominantly distributed by stellar winds during the star’s lifetime, or synthesized during the explosive demise as a type II supernova (SNII) [13, 14]. An observational constraint is the ratio of $^{26g}$Al to $^{60}$Fe ($t_{1/2} = 2 \times 10^6$ y), another radioisotope detected astronomically. As SNII are understood to produce both species, but stellar winds are not a significant source of $^{60}$Fe, considerable effort has been expended on measuring and interpreting the astronomical $^{60}$Fe/$^{26g}$Al ratio. Uncertainties in stellar metallicity and rotations, and the thermoneutron reaction rates for massive stars, impact this interpretation.

A quantitative understanding of the $^{26g}$Al flux requires...
detailed knowledge of the thermonuclear rates of formation and destruction of $^{26}$Al. In thermal environments above $\sim 200$ MK, dynamic coupling between $^{26}$Al and $^{26}$Al via levels at 0.417 MeV ($3^+$) and 1.058 MeV ($1^+$) [23] decreases the effective $\beta$-decay lifetime for $^{26}$Al. Below $\sim 150$ MK, the ground and metastable states are isolated due to their vastly different structure (an M5 transition is required), resulting in a destruction rate governed by capture reactions. The $^{26}$Al($p,\gamma$)$^{27}$Si reaction contributes to the destruction rate in novae and AGB stars, and is understood to be the dominant reaction destroying $^{26}$Al during convective H burning in massive stars [24]. At such temperatures, the $^{26}$Al($p,\gamma$)$^{27}$Si reaction rate is determined by the properties of resonances in $^{27}$Si below $\sim 300$ keV. These resonances are listed in Table I, in which energies, $J^\pi$ and mirror assignments in $^{27}$Al are from a recent $\gamma$-ray spectroscopy measurement [17] (unless noted). The resonance strengths ($\omega\gamma$) from the 189-keV resonance and higher are constrained by direct measurements of the $^{26}$Al($p,\gamma$)$^{27}$Si reaction [20–22]. At lower energies, as $^{26}$Al beam intensities have been insufficient to date for direct measurements, upper limits only have been placed on $\omega\gamma$ via indirect techniques. In most cases, the single constraint is a coarse upper limit from the minimum possible orbital angular momentum $\ell_p$ for each resonance, under the assumption of maximum reduced proton width, $\theta^2 = 1$ [18]; in reality $\omega\gamma$ could be orders of magnitude lower than these limits, depending on the spectroscopic structure of these states.

The 127-keV resonance ($E_x = 7590$ keV) is the only state that can be populated via $\ell_p = 0$ proton capture on the $5^+ 26$Al for which $\omega\gamma$ is not measured directly. Lying at an energy relevant for AGB [12] and Wolf-Rayet (WR) [24] stars, the properties of this resonance are particularly important. However, only an upper limit has been placed on $\omega\gamma$, stemming from spectroscopic factors from a measurement of $^{26}$Al($^3$He,$d$)$^{27}$Si [19]. This measurement, performed on a carbon-backed Al$_2$O$_3$ target (enriched to 6.3% $^{26}$Al+$^{27}$Al), was hampered by background and the weak population of this state, the upper limit being constrained by data at a single angle. Nevertheless, by necessity this limit on $\omega\gamma$ has been widely adopted for rates including the 127-keV resonance [16, 24, 25].

TABLE I. Resonances in $^{27}$Si above the $^{26}$Al + $p$ threshold at 7463.0(2) keV [15], previous experimental constraints on $^{26}$Al + $p$ resonance strengths, and mirror assignments in $^{27}$Al (from Ref. [16, 17]).

<table>
<thead>
<tr>
<th>$E_x$ (keV)</th>
<th>$E_{res}$ (keV)</th>
<th>$J^\pi$</th>
<th>$\omega\gamma$ (meV)</th>
<th>$^{27}$Al $E_x$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7469</td>
<td>6</td>
<td>(1/2, 5/2)$^+$</td>
<td>$&lt; 1.8 \times 10^{-60}$ [18]$^*$</td>
<td>7676</td>
</tr>
<tr>
<td>(7493)$^b$</td>
<td>(30)</td>
<td>(3/2)$^+$</td>
<td>-</td>
<td>7799</td>
</tr>
<tr>
<td>7532</td>
<td>68</td>
<td>5/2$^+$</td>
<td>$&lt; 2.3 \times 10^{-10}$ [18]$^a$</td>
<td>7790</td>
</tr>
<tr>
<td>(7557)$^c$</td>
<td>(94)</td>
<td>(3/2)$^+$</td>
<td>$&lt; 3.4 \times 10^{-12}$</td>
<td>7858</td>
</tr>
<tr>
<td>7590</td>
<td>127</td>
<td>9/2$^+$</td>
<td>$&lt; 5.9 \times 10^{-6}$ [19]$^d$</td>
<td>7807</td>
</tr>
<tr>
<td>7652</td>
<td>189</td>
<td>11/2$^+$</td>
<td>0.055(9) [20], 0.035(7) [21]</td>
<td>7950</td>
</tr>
<tr>
<td>7694</td>
<td>231</td>
<td>5/2$^+$</td>
<td>$\leq 0.010$ [20]</td>
<td>7722</td>
</tr>
<tr>
<td>7704</td>
<td>241</td>
<td>7/2$^-$</td>
<td>0.010(5) [20]</td>
<td>7900</td>
</tr>
<tr>
<td>7739</td>
<td>276</td>
<td>9/2$^+$</td>
<td>3.8(10) [22], 2.9(3) [20]</td>
<td>7998</td>
</tr>
</tbody>
</table>

$^a$ Derived assuming a reduced proton width of 1, and $\ell_p \geq 2$.

$^b$ This level in $^{27}$Si, tentatively placed at 7493 keV, was cautiously associated with the only nearby unpaired mirror in $^{27}$Al, thereby receiving a (3/2)$^+$ assignment [17], requiring $\ell_p \geq 4$.

$^c$ This tentatively-observed level is surmised to be the mirror to a low-spin $T = 3/2$ state, with $\omega\gamma$ well below the previous upper limit [18], and is expected to contribute negligibly to the $^{26}$Al($p,\gamma$)$^{27}$Si rate [16] (see text).

$^d$ Derived from the $^{26}$Al + $p$ spectroscopic factor constrained in Ref. [19].

Of the resonances below 127 keV, the 68-keV resonance is likely to have the most astrophysical significance. A possible resonance at 94 keV has only been observed tentatively [18, 27, 28], and was not populated in the recent fusion-evaporation measurement [17]. If it exists, it has been surmised to be mirror to a low-spin $T = 3/2$ state with a 3/2$^+$ assignment [16]; it is probable that $\omega\gamma$ is well below the theoretical upper limit listed in Table I, thereby contributing negligibly to the $^{26}$Al($p,\gamma$)$^{27}$Si rate. The remaining two levels (one of which is tentative) are
too low in energy to have significant astrophysical impact. Consequently, \( \omega \gamma \) for the 68-keV resonance is the other dominant remaining uncertainty. The upper limit reported in Table I was estimated by Lotay et al. [16] assuming (without experimental constraint) a proton spectroscopic factor of \( C^2 S \leq 0.3 \).

The \(^{26}\text{Al}(d,p)\(^{27}\text{Al} \) reaction reported herein has been employed to study the single-neutron spectroscopic structure of \(^{27}\text{Al} \) states, in order to constrain via mirror symmetry the single-proton spectroscopic structure, and hence resonance strengths, of the astrophysically important \( 5/2^+ \) and \( 9/2^+ \) resonances in \(^{27}\text{Si} \). These states are located in \(^{27}\text{Si} \) (\(^{27}\text{Al} \)) at 7532 keV (7790 keV) and 7590 keV (7807 keV) respectively (see Table I); the mirror assignments stem from their population and decay in fusion-evaporation reactions [17]. A beam of \(^{26}\text{Al} \) (99% pure) was produced from a sputter ion source and accelerated to 117 MeV using the 25-MV tandem electrostatic accelerator at the Holifield Radioactive Ion Beam Facility (HRIBF) at Oak Ridge National Laboratory. The beam impinged upon a deuterated polyethylene (CD\(_2\)) target of areal density 150(14) \( \mu \text{g/cm}^2 \), at a typical intensity of \( \sim 5 \times 10^6 \) ions per second, for a period of \( \sim 5 \) days. The measurement employed the ORRUBA [29] and SIDAR [30] arrays of silicon detectors to detect ejectiles between 95° and 165° in the laboratory (\( \sim 6^\circ \) to \( \sim 55^\circ \) in the center-of-mass system). In order to determine the total beam exposure, elastically-scattered deuterons were monitored in ORRUBA detectors close to \( \theta_{lab} = 90^\circ \). The rate of this scattering was calibrated directly at a lower beam intensity (\( \sim 2 \times 10^5 \) ions per second) at which the incident beam particles could be counted reliably using a self-efficiency-calibrating dual microchannel plate (MCP) detector system [31, 32]. The target thickness was determined by measuring the energy loss of alpha particles from a \(^{24}\text{Cm} \) source traversing the target.

In detectors at backward angles in the laboratory, peaks from the \((d,p) \) reaction were observed along with a smooth background from reactions induced by the carbon content of the target. The form of this background was measured by running with a carbon target, the yield from which was scaled to a region containing no peaks from the \((d,p) \) reaction and subtracted from the \( \text{CD}_2 \)-target data. As the \(^{26}\text{Al} \) beam contained a small fraction (~1%) of stable \(^{26}\text{Mg} \) contaminant, a short measurement with a pure \(^{26}\text{Mg} \) beam was undertaken to quantify the contribution from \(^{26}\text{Mg} \)-induced reactions, which was subsequently subtracted from the data taken with the \(^{26}\text{Al} \) beam. Only two significantly-populated peaks from \(^{26}\text{Mg}(d,p)\(^{27}\text{Mg} \) were observed, neither of which overlap the states of interest from \(^{26}\text{Al}(d,p)\(^{27}\text{Al} \) [33]. The (carbon and \(^{26}\text{Mg} \)) background-subtracted excitation energy spectrum, see Fig. 1, exhibits a number of strong peaks from \(^{26}\text{Al}(d,p)\(^{27}\text{Al} \), predominantly populated through \( \ell = 0 \) and \( \ell = 2 \) neutron transfer, as evidenced by their angular distributions [33]. A peak located at 7805(7) keV potentially contains three unresolved states in \(^{27}\text{Al} \), including both the 7790 keV (5/2\(^+\)) and 7807 keV (9/2\(^+\)) states (mirrors to the 68-keV and 127-keV resonances in \(^{27}\text{Si} \), respectively), along with a 3/2\(^+\) state at 7790 keV.

The angular distribution associated with the peak at 7805(7) keV is displayed in Fig. 2. Error bars represent the statistical uncertainty in proton yields only, stemming from Gaussian fits to the excitation energy spectrum at each angle. The uncertainty in the overall normalization is \( \sim 12\% \), with dominant contributions from the target thickness (9%) and beam normalization (5%). Angular distributions were analyzed using Finite-Range Adiabatic Distorted Wave Approximation (FR-ADWA) calculations, performed using the code FRESCO [34], to which a 15% uncertainty in normalization is attributed. Optical potentials were generated with the global parameterization of Chapel-Hill (CH89) [35] using the code T WOFNR [36]. Standard radius and diffuseness parameters \( (r = 1.25, a = 0.65) \) were used for the Woods-Saxon potential used to generate the single-particle wave functions. A description of the experimental angular distribution requires a combination of \( \ell_n = 0 \) and \( \ell_n = 2 \) contributions; a least-squares fit to the angular distribution is overlaid in Fig. 2, along with the individual components to the fit. As the 9/2\(^+\) state at 7807 keV is the only known state within the width of the peak (see inset of Fig. 1) that can be populated via \( \ell_n = 0 \) transfer from the 5\(^+\) ground state of \(^{26}\text{Al} \), we associate the \( \ell_n = 0 \) strength in this peak entirely with the 7807-keV state, yielding a spectroscopic factor of 0.0102(27). As all three states can be populated directly via \( \ell_n = 2 \) transfer, it is unclear how much of the \( \ell_n = 2 \) component is asso-

![FIG. 1. (Color online) Excitation energy spectrum from \(^{26}\text{Al}(d,p)\(^{27}\text{Al} \) at 4.5 A-MeV and 6.5\(^\circ\) in the center-of-mass, with a resolution of 72 keV (FWHM). The peak (see inset) identified as containing the 7807 keV 9/2\(^+\) level is indicated by the solid (red) vertical line; the nearest neighbouring states (7664 keV and 7950 keV) which can be populated via \( \ell = 0 \) neutron-transfer are indicated with the dashed (blue) lines.](image-url)
associated with each state. Under the assumption that the totality of the $\ell_n = 2$ strength is associated with the $5/2^+$ state at 7790 keV, a spectroscopic factor of $0.029(16)$ is obtained from the fit; we adopt a $2\sigma$ upper limit for this spectroscopic factor when calculating the excitation energy spectrum at each angle.

In order to determine the proton spectroscopic factors for the mirror states in $^{27}$Si, calculations of spectroscopic factors were performed for states in $^{27}$Al and $^{27}$Si within the Shell Model Embedded in the Continuum (SMEC) formalism [37], using the USD-b effective interaction and a continuum coupling constant of $650$ MeV fm$^{-3}$ (found in previous studies to be typical for this mass region [38]). The ratio of the theoretical spectroscopic factors ($C^2S^\text{th}_\pi$ for $^{27}$Si to $C^2S^\text{th}_\nu$ for $^{27}$Al, for the same shell model state) is used to scale the experimentally-determined neutron spectroscopic factor $C^2S^\exp_\nu$ for $^{27}$Al to calculate the proton spectroscopic factor $C^2S_\pi$ for the mirror state in $^{27}$Si. This procedure accounts for the different continuum coupling due to the bindings of the mirror states in the two nuclides. For both the 68-keV and 127-keV resonances, the only shell-model state with the correct $J^\pi$, and similar excitation energy and $C^2S$, was adopted in each case for this procedure. The selection of any neighboring state for this analysis affects the final result by a negligible fraction of the uncertainty we ascribe to this part of our procedure, which stems from making large variations to the extent of continuum coupling ($0, -650$ and $-1300$ MeV fm$^3$) in order to generate a conservative estimate of the uncertainty in the ratio of spectroscopic factors. These spectroscopic factors are reported in Table II, along with resonance strengths derived under the assumption of $\Gamma_\pi \ll \Gamma_\gamma$, in the manner of Ref. [19]. The systematics of our entire procedure was validated by analyzing a decomposition of the differential cross sections associated with partially-resolved states located near 8 MeV [33], which contain the mirror states to the resonances at 189, 231, 241, 276 and 369 keV. The spectroscopic factors associated with $\ell = 1,2,3$ transfers in this region are consistent with the known strengths of these resonances, all of which have been constrained by direct measurements.

For the 127-keV resonance, $C^2S_\pi = 0.0085^{+0.21}_{-0.11}$ is extracted, over a factor of four higher than the upper limit of 0.002 set by the $^{26}$Al($^3$He,$d$)$^{27}$Si measurement of Vogelaar [19]. The resultant strength of the 127-keV resonance ($\omega\gamma = 2.6_{-0.9}^{+0.7} \times 10^{-5}$ meV) is correspondingly four times higher than the previously adopted upper limit. For the 68-keV resonance, the first experimental constraint on its spectroscopic factor yields $C^2S_\pi \leq 0.054$; correspondingly an upper limit of $3.0 \times 10^{-12}$ meV is assigned to the resonance strength. This reduces the upper limit by almost an order of magnitude from that adopted by Lotay et al [16]. The contributions from these resonances to the $^{26}$Al($p,\gamma$)$^{27}$Si rate as a function of temperature are plotted in Fig. 3, with curves comparing the current "ORNL" rates to those based upon the previ-
ous spectroscopic information [16]. The new strengths indicate that the 127-keV resonance is dominant over a larger temperature range than previously estimated, and the potential contribution of the 68-keV resonance is relatively diminished. These considerable changes in the reaction rate and reduction in the rate uncertainty occur over the temperature range relevant to nucleosynthesis in AGB stars and in H core burning in \( \sim 80 M_\odot \) WR stars, as highlighted by recent sensitivity studies [14, 24, 26]. The increase in the 127-keV resonance strength should impact the possibility of a direct measurement of this reaction which, though technically difficult, should be considered the highest priority in further constraining the \( ^{26}Al(p,\gamma)^{27}Si \) rate at stellar temperatures.

In summary, the \( ^{26}Al(p,\gamma)^{27}Si \) reaction rate has been considerably constrained at stellar temperatures via a spectroscopic measurement of single-neutron states in the mirror nucleus \(^{27}Al\). The 127-keV resonance is found to be a factor of four stronger than the previous upper limit, dominating over the temperature range important for H core burning in massive stars, which are known to be directionally correlated with the highly-studied \( ^{26}Al \) \( \gamma \)-ray signature.

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**TABLE II.** Spectroscopic factors ($C^2S_{\text{exp}}$) for single-neutron overlaps for states within the 7805(7)-keV peak in $^{27}\text{Al}$, and derived proton spectroscopic factors $C^2S_\pi$ for their mirror states in $^{27}\text{Si}$, yielding $^{26}\text{Al}+p$ resonance strengths (see text). Single-particle widths $\Gamma_{\text{s.p.}}$ are calculated using a radius parameter of $r = 1.25$. Uncertainties reflect combined experimental and theoretical uncertainties (see text).

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$^{27}\text{Al}$</th>
<th>$^{27}\text{Al}$</th>
<th>$^{27}\text{Al}^a$</th>
<th>$^{27}\text{Si}^a$</th>
<th>$^{27}\text{Si}$</th>
<th>$\Gamma_{\text{s.p.}}$</th>
<th>$\Gamma_p$</th>
<th>$\omega_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/2$^+$</td>
<td>7807</td>
<td>0.0102 ± 0.0021</td>
<td>0.0112$^{+0.0007}_{-0.0002}$</td>
<td>0.0094$^{+0.0016}_{-0.0024}$</td>
<td>0.0085$^{+0.0024}_{-0.0031}$</td>
<td>6.70 $\times 10^{-3}$</td>
<td>5.7$^{+1.6}_{-1.8}$ $\times 10^{-5}$</td>
<td>2.6$^{+0.7}_{-0.9}$ $\times 10^{-5}$</td>
</tr>
<tr>
<td>5/2$^+$</td>
<td>7790</td>
<td>$\leq 0.061$</td>
<td>0.0100$^{+0.0006}_{-0.0002}$</td>
<td>0.0088$^{+0.0010}_{-0.0022}$</td>
<td>$\leq 0.054$</td>
<td>2.06 $\times 10^{-10}$</td>
<td>$\leq 1.1 \times 10^{-11}$</td>
<td>$\leq 3.0 \times 10^{-12}$</td>
</tr>
</tbody>
</table>

$^a$ From SMEC calculations using the USD-b effective interaction, using a continuum coupling constant of $-650$ MeV fm$^3$. 


[35] M. T. J. Tostevin, M. Igarashi and N. Kishida, University of Surrey modified version of the code TWOFNR, private communication.