Ferrie and Combes Reply:

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Our Letter [1] generalizes the notion of weak value to an operational quantity so that it is applicable to a generic class of operationally meaningful probabilistic theories. Specifically we define the generalized weak value, a_w, to be a conditional expectation (given pre selection on ψ and post selection on ϕ) of the intermediate measurement outcome (a random variable denoted s) divided by a measurement strength or coupling parameter (denoted by λ)

$$a_w \equiv \mathbb{E}_{s|\phi,\psi}\left[\frac{s}{\lambda}\right] \quad \text{(generalized weak value)} \qquad (1)$$

We proved that this agrees with the quantum definition, i.e. $a_w = \Re[\langle\phi|A|\psi\rangle/\langle\phi|\psi\rangle]$, for our coarsed-grained quantum mechanical experiment and Garretson *et al* showed it was true without coarse-graining [2]. This allows us to explore whether anomalous weak values can exist in classical theory—without such a generalization the question is not meaningful. We [1], and others [3], showed that classical anomalous weak values do exist, which has two important consequences: (1) any experiment claiming to observe anomalous generalized weak values has a set of data which could have been generated by our model; and (2) generalized weak values can be seen as a consequence of non-standard reasoning based on classical statistics alone, and thus are not necessarily evidence of quantumness.

We thank Brodutch for highlighting the lack of "quantum" features in our model. However, citing the differences between our model and quantum theory does not invalidate the analogy for the purposes it was made—in fact, it strengthens our argument. Importantly, the only thing we aimed to capture in our model was a classical analog to the expectations considered anomalous and *no other features*. In this way, we have isolated the claimed "quantum" aspect of weak values and have not hidden "quantumness" elsewhere in our model. For a interesting and nonpartisan discussion of the quantum features our model necessarily omits, we suggest [3–6].

The actual content of Brodutch's Comment [7] is a failed attempt at a straw man argument. Recall: a straw man, when performed properly, is a valid logical argument against a *misrepresentation* of an opponent's position, which is clearly an invalid argument against the *actual* position of the opponent. Although Brodutch does indeed attack a misrepresentation of the argument in our Letter [1], the argument against the misrepresentation itself is fallacious. Since this technique of compounding fallacies has no name, we coin the term *anomalous straw man* for it—anomalous since it goes outside the range of usual logical fallacies.

Specifically Brodutch gives his own generalized definition of a weak value—different to our definition!—then tries to argue that this implies our result is wrong. He then compares his modified model with quantum weak and strong measurements. While this is irrelevant for the purpose of arguing against our Letter, there is a further problem that the argument against the straw man itself is fallacious.

First, Brodutch claims "the Gaussian was shifted by $x \cdot A_w$. As in the strong limit only x sets the scale or normalization, if x is kept constant while σ is changed no rescaling is necessary." However, it has been known since the early days of weak values [8] that $xA_w\sigma \ll 1$. Thus, xA_w *cannot* be an anomalous shift alone and *must* be re-scaled. Brodutch's circular argument collapses upon itself.

Second, Brodutch states, "Under the right conditions, eq 17 of [1] can be used to calculate the weak value from the expectation value of $\frac{s}{\lambda}$. However, contrary to the claim of the authors eq. 17 is not 'an equivalent definition of the weak value'." Brodutch admits our definition is numerically identical to the original definition, yet disagrees they are equivalent. Unless there is another meaning to the word equivalent, his statement is a non sequitar.

Third, Brodutch's "counterexample" in footnote [15] is not a weak measurement of A, it is a weak measurement of A followed by a conditional rotation. In the original system-and-meter picture of AAV, after the conditional rotation suggested by Brodutch, the shift of the meter would also not correspond to the weak value. In other words, the definition we give in Eq. (17) is still equivalent to the re-scaled shift in the meter of the original AAV model, in the scenario envisioned by Brodutch.

Finally, Brodutch states "In the FC model the strong limit $\lambda \to 1$ implies $\Pr(\phi = 1|\psi = +1) = 1$ and deterministic $s = 1$ so we expect $a_w = 1$." However, in Eq. (29) of our letter, we have $\Pr(\phi = -1|\psi = +1) = 1 - \delta$, which is clearly independent of λ. Moreover, $\Pr(\phi|\psi)$ is marginalized over s and thus it makes no sense to consider "deterministic $s = 1$."

We stress, even if Brodutch were able to correct the errors in his anomalous straw man argument, it would still be a straw man argument and hence invalid.

Note added: With regards to Brodutch's added note, we refer the reader to Eq. (2.45) of [9].

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[1] C. Ferrie and J. Combes, Phys. Rev. Lett. **113**, 120404 (2014).
[2] J. L. Garretson, H. M. Wiseman, D. T. Pope and D. T. Pegg, J. Opt. B: Quantum Semiclass. Opt. **6** S506 (2004).

[3] J. Dressel and A. N. Jordan, Phys. Rev. A **85**, 022123 (2012).
[4] M. F. Pusey, Phys. Rev. Lett. **113**, 200401 (2014).
[5] A. C. Ipsen, arXiv:1409.3538.
[6] J. Dressel, arXiv:1410.0943.
[7] Aharon Brodutch, arXiv:1410.8510.
[8] I. M. Duck, P. M. Stevenson, and E. C. G. Sudarshan, Phys. Rev. D **40**, 2112 (1989).
[9] A. G. Kofman, S. Ashhab and F. Nori, Physics Reports **520**, 43 (2012).