Jet Quenching Phenomenology from Soft-Collinear Effective Theory with Glauber Gluons

Zhong-Bo Kang, Robin Lashof-Regas, Grigory Ovanesyan, Philip Saad, and Ivan Vitev

Phys. Rev. Lett. **114**, 092002 — Published 3 March 2015

DOI: 10.1103/PhysRevLett.114.092002

Jet quenching phenomenology from soft-collinear effective theory with Glauber gluons

Zhong-Bo Kang,[1] Robin Lashof-Regas,[1,2] Grigory Ovanesyan,[3] Philip Saad,[1,2] and Ivan Vitev[1]

[1] *Theoretical Division, Los Alamos National Laboratory MS B283, Los Alamos, NM 87545 USA*

[2] *Department of Physics, University of California, Santa Barbara, CA 93106 USA*

[3] *Physics Department, University of Massachusetts Amherst, Amherst, MA 01003, USA*

(Dated: May 11, 2014)

We present the first application of a recently-developed effective theory of jet propagation in matter $\mathrm{SCET_G}$ to inclusive hadron suppression in nucleus-nucleus collisions at RHIC and the LHC. $\mathrm{SCET_G}$-based splitting kernels allow us to go beyond the traditional energy loss approximation and unify the treatment of vacuum and medium-induced parton showers. In the soft gluon emission limit, we establish a simple analytic relation between the QCD evolution and energy loss approaches to jet quenching. We quantify the uncertainties associated with the implementation of the in-medim modification of hadron production cross sections and show that the coupling between the jet and the medium can be constrained with better than 10% accuracy.

PACS numbers:

Suppression of the production cross section for high transverse momentum particles and jets in ultra-relativistic collisions of heavy nuclei, commonly referred to as jet quenching [1], is one of the most-important signatures of quark-gluon plasma (QGP) formation in such reactions and a quantitative probe of its properties. This phenomenon has been established experimentally at the Relativistic Heavy Ion Collider (RHIC) [2, 3] and the Large Hadron Collider (LHC) [4–6]. It was understood theoretically in a framework based on perturbative QCD calculations of parton propagation and energy loss in the QGP [7].

More recently, progress has been made on formulating and applying effective theories of QCD, suitable for calculations of jet properties in hot and dense strongly-interacting matter. The well-established soft-collinear effective theory (SCET) [8, 9] has been extended to include the interactions with the medium quasiparticles via a transverse t-channel momentum exchange. The resulting soft-collinear effective theory with Glauber gluons ($\mathrm{SCET_G}$) [10, 11] has been used to calculate all $\mathcal{O}(\alpha_s)$ $1 \to 2$ medium-induced splitting kernels [12] and study $\mathcal{O}(\alpha_s)$ effects on the in-medium parton shower [13]. The power counting of $\mathrm{SCET_G}$ correctly captures the behavior of the in-medium branchings when the lightcone momentum fraction $x = Q^+/p^+$ of the *emitted* parton becomes large ($x \to 1$). It is important to emphasize that these large-x corrections are absent in traditional energy loss calculations.

A critical step in improving the jet quenching phenomenology is to understand the implication of the finite-x corrections. Their implementation requires new theoretical methods, since in the large momentum fraction limit the leading parton can change flavor and the splitting process cannot be interpreted as energy loss. A natural language to capture this physics is that of the well-known DGLAP evolution equations [14]. As a first application of the $\mathrm{SCET_G}$ medium-induced splitting kernels, we revisit the evaluation of the nuclear modification factor R_{AA} for inclusive hadron production at high transverse momentum p_T (and rapidity y), defined as:

$$R_{AA}(p_T) = \frac{d\sigma^h_{AA}/dyd^2p_T}{\langle N_{\mathrm{coll}}\rangle d\sigma^h_{pp}/dyd^2p_T}\,, \tag{1}$$

which continues to attract strong theoretical interest [15, 16]. We consider central lead-lead (Pb+Pb) reactions at $\sqrt{s_{NN}} = 2.76$ TeV at the LHC as an example. In Eq. (1) $\langle N_{\mathrm{coll}}\rangle$ is the average number of binary nucleon-nucleon collisions. DGLAP evolution equations have been used to address hadron production in semi-inclusive deep inelastic scattering with initial conditions obtained using an energy loss approach [17, 18].

In the presence of a QGP, all parton splitting kernels are a direct sum of the universal vacuum part and a medium-dependent component $\mathrm{d}N_i/\mathrm{d}x\,\mathrm{d}^2\boldsymbol{Q}_\perp$, which has been calculated in Ref. [12]. Those are real emission graphs in the DGLAP language. The splitting functions are related to the medium-induced splitting kernels as follows:

$$\begin{aligned}P_i^{\mathrm{real}}(x,\boldsymbol{Q}_\perp;\beta) &= \frac{2\pi^2}{\alpha_s}\,\boldsymbol{Q}_\perp^2\,\frac{\mathrm{d}N_i(x,\boldsymbol{Q}_\perp;\beta)}{\mathrm{d}x\,\mathrm{d}^2\boldsymbol{Q}_\perp}\\ &\equiv P_i^{\mathrm{vac}}(x)\,h_i(x,\boldsymbol{Q}_\perp;\beta)\,.\end{aligned} \tag{2}$$

The equation above explicitly indicates that, unlike the vacuum case where the splitting function only depends on x, the medium-induced splitting function also depends on $\boldsymbol{Q}_\perp$ and the characteristics of the QGP collectively denoted by β. It also defines the reduced kernels h_i, where i denotes $q \to qg$, $g \to gg$, $g \to q\bar{q}$ and $q \to gq$. We relate the temperature and density of the gluon-dominated plasma to the measured charged particle rapidity density [7]. The position and time dependence of the Debye screening scale m_D and the quark and gluon scattering lengths, necessary to evaluate $P_i^{\mathrm{real}}(x,\boldsymbol{Q}_\perp;\beta)$, are obtained using an optical Glauber model for the collision geometry and a Bjorken expansion ansatz. The coupling g between the jet and the medium is a free parameter in the calculation and the phase space for the $\mathcal{O}(\alpha_s)$ splittings is discussed in [12].

Special attention has to be paid to the gluon splitting function because it diverges for both $x \to 0$ and $x \to 1$. The first divergence is regulated with a plus function prescription, while the second divergence need not be regulated owing to the form of the evolution equations. The splitting functions equal

$$P_{q\to qg}(x) = \left[P^{\rm real}_{q\to qg}(x)\right]_+ + A\,\delta(x)\,, \tag{3}$$

$$P_{g\to gg}(x) = 2C_A\left\{ \left[\left(\frac{1-2x}{x} + x(1-x)\right) h_{g\to gg}\,(x)\right]_+ + \frac{h_{g\to gg}\,(x)}{1-x}\right\} + B\,\delta(x)\,, \tag{4}$$

$$P_{g\to q\bar{q}}(x) = P^{\rm real}_{g\to q\bar{q}}(x)\,, \quad P_{q\to gq}(x) = P^{\rm real}_{q\to gq}(x)\,. \tag{5}$$

In the equations above we have suppressed the explicit $\boldsymbol{Q}_\perp$ and β dependence for simplicity. The virtual pieces of the splitting functions can be extracted from flavor and momentum sum rules in complete analogy to the vacuum case:

$$A = 0\,, \tag{6}$$

$$B = \int_0^1 {\rm d}x' \left\{ -2n_f(1-x')P_{g\to q\bar{q}}(x') + 2C_A\left[x'\left(\frac{1-2x'}{x'} + x'(1-x')\right) - 1\right] h_{g\to gg}\,(x')\right\}. \tag{7}$$

The DGLAP evolution equations for the fragmentation functions (FFs) read:

$$\frac{{\rm d}D_q(z,Q)}{{\rm d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{{\rm d}z'}{z'}\Big[P_{q\to qg}(z')D_q\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z')D_g\left(\frac{z}{z'},Q\right)\Big]\,, \tag{8}$$

$$\frac{{\rm d}D_g(z,Q)}{{\rm d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{{\rm d}z'}{z'}\left[P_{g\to gg}(z')D_g\left(\frac{z}{z'},Q\right) + P_{g\to q\bar{q}}(z')\sum_q\left(D_q\left(\frac{z}{z'},Q\right) + D_{\bar{q}}\left(\frac{z}{z'},Q\right)\right)\right]\,, \tag{9}$$

where $z \equiv 1-x$ in the splitting functions and $Q \equiv |\boldsymbol{Q}_\perp|$. The equation for the evolution of the anti-quark FF can be found from quark equation by substituting everywhere $D_q \to D_{\bar{q}}$.

QCD evolution and the energy loss approach represent two very different implementations of jet quenching. It is critical to establish this connection between them in light of the fact that energy loss phenomenology has been very successful [7, 15, 16]. This can be achieved *only* in the soft gluon bremsstrahlung limit, where the two diagonal splitting functions $P_{q\to qg}$ and $P_{g\to gg}$ survive. Up to $(2\pi^2/\alpha_s)\,\boldsymbol{Q}_\perp^2$, these are the Gyulassy-Levai-Vitev (GLV) double differential medium-induced gluon number distributions to first order in opacity [19]. There is no flavor mixing, and the entire branching is given by a plus function. The DGLAP evolution equations decouple and reduce to:

$$\frac{{\rm d}D(z,Q)}{{\rm d}\ln Q} = \frac{\alpha_s}{\pi}\int_z^1 \frac{{\rm d}z'}{z'}\,\left[P(z',Q)\right]_+ D\left(\frac{z}{z'},Q\right)\,. \tag{10}$$

Because the fragmentation functions $D(z)$ are typically steeply falling with increasing $z = p_T^{\rm hadron}/p_T^{\rm parton}$, the main contribution in Eq. (10) comes predominantly from $z' \approx 1$. We expand the integrand in this limit, keeping the first derivative terms, and approximate the steepness of the fragmentation function with its unperturbed vacuum value:

$$n(z) = -{\rm d}\ln D^{\rm vac}(z)/{\rm d}\ln z\,. \tag{11}$$

The analytical solution to the Eq. (10) reads:

$$D^{\rm med}(z,Q) \approx {\rm e}^{-(n(z)-1)\left\langle\frac{\Delta E}{E}\right\rangle_z - \langle N_g\rangle_z} D^{\rm vac}(z,Q)\,, \tag{12}$$

and shows explicitly that the vacuum evolution and the medium-induced evolution factorize. We have used the following definitions in the above formula:

$$\left\langle\frac{\Delta E}{E}\right\rangle_z = \int_0^{1-z}{\rm d}x\,x\,\frac{{\rm d}N}{{\rm d}x}(x) \xrightarrow{z\to 0} \left\langle\frac{\Delta E}{E}\right\rangle\,, \tag{13}$$

$$\langle N_g\rangle_z = \int_{1-z}^1 {\rm d}x\,\frac{{\rm d}N}{{\rm d}x}(x) \xrightarrow{z\to 1} \langle N_g\rangle\,, \tag{14}$$

where xdN/dx is the medium-induced gluon intensity distribution [19]. Note, that we have made the choice to put all the in-medium effects into the DGLAP evolution and the scale Q in Eq. (12) is the hard scale $\sim E$. This scale is also used as an upper limit of integration on $\boldsymbol{Q}_\perp$ in obtaining xdN/dx. The analytic formula in Eq. (12) gives us for the first time an insight into the deep connections between the evolution and energy loss approaches to jet quenching. Over most of the z range the suppression of the FFs is dominated by the the fractional energy loss, amplified by the steepness of $D(z)$. Near threshold ($z = 1$) the modification is determined by the probability *not* to emit a gluon, $\exp(-\langle N_g\rangle)$. Conversely, solving Eqs. (8) and (9) numerically allows us to unify the treatment of the vacuum and medium-induced parton showers.

We now turn to the numerical comparison between the medium-modified evolution approach to jet quenching and the traditional energy loss formalism. We elect to

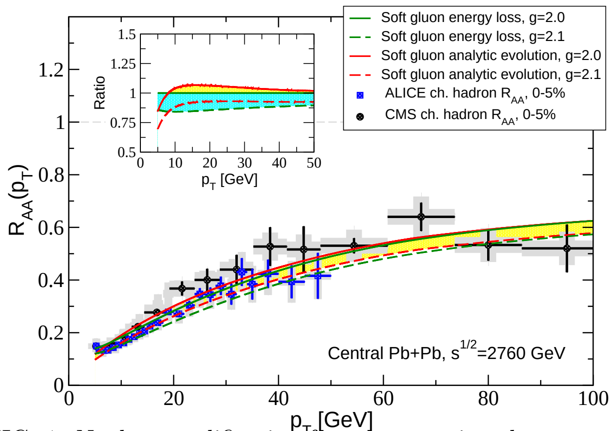

FIG. 1: Nuclear modification factor comparison between the traditional energy loss approach (cyan band) and the analytic solution to QCD evolution in the soft gluon limit (yellow band). The upper and lower edges of the bands correspond to couplings between the jet and the medium $g = 2.0$ and $g = 2.1$, respectively. The insets show the ratios of different R_{AA} curves. Data is form ALICE an CMS.

include all QGP effects in the fragmentation functions, such that the invariant inclusive hadron production cross section reads:

$$\frac{1}{\langle N_{\rm coll}\rangle}\frac{d\sigma^h_{AA}}{dyd^2p_T}=\sum_c\int_{z_{\rm min}}^1 dz\,\frac{d\sigma^c(p_c=p_T/z)}{dyd^2p_{T_c}} \times\frac{1}{z^2}D_c^{\rm med/quench}(z)\,. \qquad (15)$$

Here, $c = \{q, \bar{q}, g\}$ and we choose the factorization, fragmentation and renormalization scales $Q = p_{T_c}$, and $d\sigma^c/dyd^2p_{T_c}$ is the unmodified hard parton production cross section.

Should an energy loss approach be adopted, it is important to realize that the soft gluon emission limit must be consistently implemented. If the fractional energy loss becomes significant, it is carried away through multiple gluon bremsstrahlung. In the independent Poisson gluon emission limit, we can construct the probability density $P_c(\epsilon)$ of this fractional energy loss $\epsilon = \sum_i \omega_i/E \approx \sum_i Q_i^+/p^+$, such that:

$$\int_0^1 d\epsilon\; P(\epsilon) = 1\,, \quad \int_0^1 d\epsilon\;\epsilon\, P(\epsilon) = \left\langle \frac{\Delta E}{E}\right\rangle\,. \qquad (16)$$

A more detailed discussion is given in [7]. If a parton loses this energy fraction ϵ during its propagation in the QGP to escape with momentum $p_{T_c}^{\rm quench}$, immediately after the hard collision $p_{T_c} = p_{T_c}^{\rm quench}/(1-\epsilon)$. Noting the additional Jacobian $|dp_{T_c}^{\rm quench}/dp_{T_c}| = (1-\epsilon)$, the kinematic modification to the FFs due to energy loss is:

$$D_c^{\rm quench}(z) = \int_0^{1-z} d\epsilon\,\frac{P_c(\epsilon)}{(1-\epsilon)}D_c\left(\frac{z}{1-\epsilon}\right)\,, \qquad (17)$$

and can be directly implemented in Eq. (15).

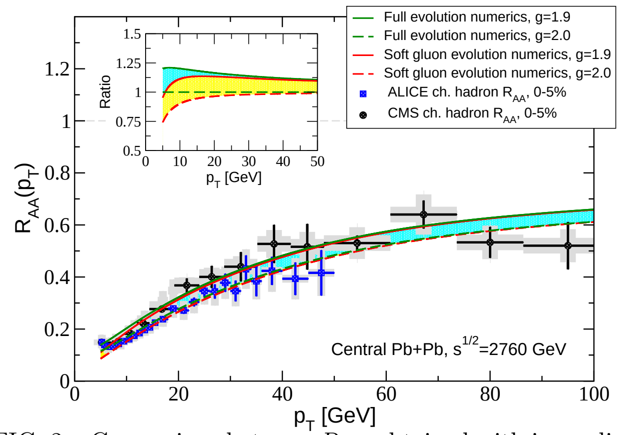

FIG. 2: Comparison between R_{AA} obtained with in-medium numerically evolved fragmentation functions using the full splitting kernels (cyan band) and their soft gluon limit (yellow band) to ALICE and CMS data. The upper and lower edges of the bands correspond to $g = 1.9$ and $g = 2.0$, respectively.

In Figure 1 we present our calculations of the nuclear modification factor R_{AA} in the limit of soft gluon bremsstrahlung. Results are obtained from the parton energy loss approach (cyan band) and by using the analytic solution to the in-medium evolution given in Eq. (12) (yellow band). The upper edge of the uncertainty bands (solid lines) corresponds to a coupling between the jet and the medium $g = 2.0$ and the lower edge (dashed lines) corresponds to $g = 2.1$. The results of the two calculations are remarkably similar and both reproduce well the suppression of inclusive charged hadron production in 0-10% central Pb+Pb collisions at the LHC measured by ALICE [4] and CMS [5] at $\sqrt{s} = 2.76$ TeV. In both approaches the coupling g between the jet and the medium can be constrained with an accuracy of 5% and the transport properties of the medium, which scale as g^4, can be extracted with 20% uncertainty. The inset shows the ratio for the different R_{AA} curves relative to the $g = 2.0$ energy loss result. We observe from this inset that the only difference between the two approaches is a small variation in the shape of the nuclear modification ratio as a function of p_T. At any fixed transverse momentum the difference in the predicted magnitude of jet quenching can be absorbed in less than 2% change of the coupling g between the jet and the medium.

In Figure 2 we show R_{AA}s obtained with medium-modified FFs that are numerical solutions to the DGLAP evolution equations, Eqs. (8), (9), with full medium-induced splitting kernels [12] (cyan band) and their small-x energy loss limit [20] (yellow band). In this figure, the uncertainty bands correspond to $g = 1.9 - 2.0$. The difference between the small-x and full evolution is only noticeable below $p_T = 20$ GeV, as can be seen from the inset. At small and intermediate transverse momenta the solution to the DGLAP equations beyond the soft gluon limit yields a slightly better agreement between theory and experiment.

Comparision of the four different evaluations of R_{AA},

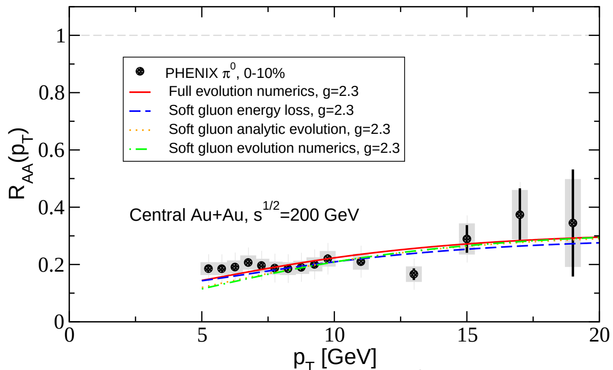

FIG. 3: Four different calculations of π^0 R_{AA} at RHIC with $g = 2.3$ are compared to the PHENIX suppression measurements in central Au+Au collisions.

this time for RHIC $\sqrt{s} = 200$ GeV in central Au+Au reactions, is shown in Fig. 3. They proide adequate description of the attenuation of the inclusive π^0 cross section measured by the PHENIX experiment [21]. The difference between the full solution to the DGLAP evolution equation (solid red curve) and the traditional energy loss approach (dashed blue curve) is again very small. We find that the coupling $g = 2.3$ between the jet and the medium at RHIC is $\sim 15\%$ larger than that at the LHC.

To understand the numerical results, we further scrutinize the in-medium modification of FFs in Figure 4 for 40 GeV quarks and gluons, respectively. As a function of the hadron-to-parton transverse momentum fraction z, the differences between the various methods of computing this modification can be much more pronounced than in R_{AA}. This is especially true for gluon fragmentation at large z. The observed hadron production cross section, however, samples a wide range of momentum fractions and in the presence of a QGP is biased toward lower values of z. Furthermore, the quark contribution is enhanced since $D_q^{\rm med}(z)$ is much less suppressed than $D_g^{\rm med}(z)$.

To summarize, we presented results for the suppression of inclusive hadron production in Pb+Pb reactions at the LHC based upon QCD factorization and DGLAP evolution with $\mathrm{SCET_G}$-based medium-induced splitting kernels. This method allowed us to unify the treatment of vacuum and medium-induced parton showers. In the soft gluon bremsstrahlung limit, we demonstrated the connection between this new approach and the traditional energy loss-based jet quenching phenomenology. Numerically, the agreement between the two methods is quite remarkable and they give a good description of the experimentally measured R_{AA} by ALICE and CMS at the LHC, and by PHENIX at RHIC. We find that the coupling between the jet and the medium can be constrained with better than 10% accuracy when the uncertainties that arise from the choice of method and the fit to the data are combined. In the future, it will be interesting to investigate whether better differentiation between the QCD evolution and energy loss approaches can be achieved using parton flavor separation techniques [22, 23].

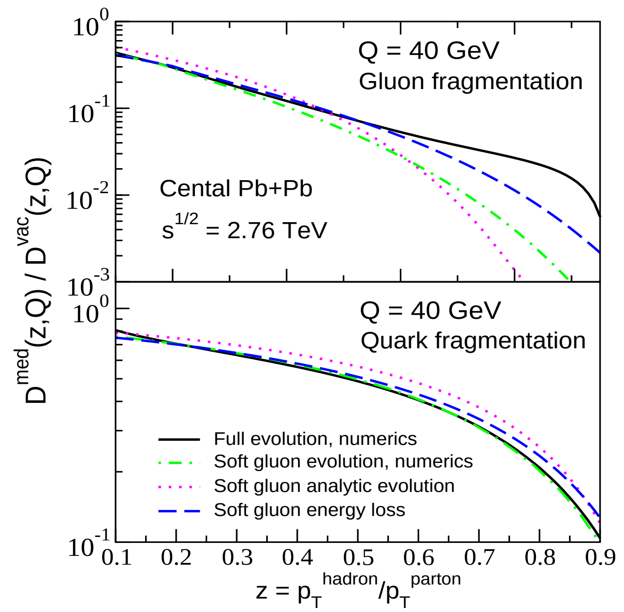

FIG. 4: The modification of the fragmentation functions for gluons (top panel) and quarks (bottom panel) are shown for $Q = 40$ GeV and central Pb+Pb collisions at the LHC, using four different methods to compute the in-medium modification with $g = 2.0$.

Acknowledgments: This work is supported by DOE Office of Science and in part by the LDRD program at LANL.

[1] X.-N. Wang and M. Gyulassy, Phys.Rev.Lett. **68**, 1480 (1992).
[2] PHENIX Collaboration, S. Adler *et al.*, Phys.Rev.Lett. **91**, 072301 (2003), nucl-ex/0304022.
[3] STAR Collaboration, J. Adams *et al.*, Phys.Rev.Lett. **91**, 172302 (2003), nucl-ex/0305015.
[4] ALICE Collaboration, K. Aamodt *et al.*, Phys.Lett. **B696**, 30 (2011), 1012.1004.
[5] CMS Collaboration, S. Chatrchyan *et al.*, Eur.Phys.J. **C72**, 1945 (2012), 1202.2554.
[6] ATLAS Collaboration, G. Aad *et al.*, Phys.Lett. **B719**, 220 (2013), 1208.1967.
[7] M. Gyulassy, I. Vitev, X.-N. Wang, and B.-W. Zhang, (2003), nucl-th/0302077.
[8] C. W. Bauer, S. Fleming, and M. E. Luke, Phys.Rev. **D63**, 014006 (2000), hep-ph/0005275.
[9] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys.Rev. **D63**, 114020 (2001), hep-ph/0011336.
[10] A. Idilbi and A. Majumder, Phys.Rev. **D80**, 054022 (2009), 0808.1087.
[11] G. Ovanesyan and I. Vitev, JHEP **1106**, 080 (2011), 1103.1074.
[12] G. Ovanesyan and I. Vitev, Phys.Lett. **B706**, 371 (2012), 1109.5619.

[13] M. Fickinger, G. Ovanesyan, and I. Vitev, JHEP **1307**, 059 (2013), 1304.3497.
[14] G. Altarelli and G. Parisi, Nucl.Phys. **B126**, 298 (1977).
[15] B. Betz and M. Gyulassy, (2014), 1404.6378.
[16] M. Djordjevic and M. Djordjevic, (2013), 1307.4098.
[17] W.-t. Deng and X.-N. Wang, Phys.Rev. **C81**, 024902 (2010), 0910.3403.
[18] N.-B. Chang, W.-T. Deng, and X.-N. Wang, Phys.Rev. **C89**, 034911 (2014), 1401.5109.
[19] M. Gyulassy, P. Levai, and I. Vitev, Phys.Rev.Lett. **85**, 5535 (2000), nucl-th/0005032.
[20] M. Gyulassy, P. Levai, and I. Vitev, Nucl.Phys. **B594**, 371 (2001), nucl-th/0006010.
[21] PHENIX Collaboration, A. Adare *et al.*, Phys.Rev.Lett. **101**, 232301 (2008), 0801.4020.
[22] S. D. Ellis, C. K. Vermilion, J. R. Walsh, A. Hornig, and C. Lee, JHEP **1011**, 101 (2010), 1001.0014.
[23] J. Gallicchio and M. D. Schwartz, Phys.Rev.Lett. **107**, 172001 (2011), 1106.3076.