Gap Reversal at Filling Factors 3+1/3 and 3+1/5: Towards Novel Topological Order in the Fractional Quantum Hall Regime

Ethan Kleinbaum, Ashwani Kumar, L. N. Pfeiffer, K. W. West, and G. A. Csáthy

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Gap Reversal at $\nu = 3 + 1/3$ and $3 + 1/5$: Towards Novel Topological Order in the Fractional Quantum Hall Regime

Ethan Kleinbaum,[1] Ashwani Kumar,[2] L.N. Pfeiffer,[3] K.W. West,[3] and G.A. Csáthy[1, 4]

[1]*Department of Physics and Astronomy, Purdue University, West Lafayette, IN 47907*
[2]*Department of Physics, Monmouth College, Monmouth, IL 61462*
[3]*Department of Physics, Princeton University, Princeton, NJ 08544*
[4]*Birck Nanotechnology Center, Purdue University, West Lafayette, IN 47907*

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In the region of the second Landau level several theories predict fractional quantum Hall states with novel topological order. We report the opening of an energy gap at the filling factor $\nu = 3 + 1/3$, firmly establishing the ground state as a fractional quantum Hall state. This and other odd-denominator states unexpectedly break particle-hole symmetry. Specifically, we find that the relative magnitudes of the energy gaps of the $\nu = 3 + 1/3$ and $3 + 1/5$ states from the upper spin branch are reversed when compared to the $\nu = 2 + 1/3$ and $2 + 1/5$ counterpart states in the lower spin branch. Our findings raise the possibility that at least one of the former states is of an unusual topological order.

The need to understand ordered states of strongly correlated quantum systems gave rise to the concept of topological order [1]. Fractional quantum Hall states (FQHS), such as the one at $\nu = 1/3$ [2], possess such order [3]. Other systems with topological order include topological insulators [4] and superconductors [5] as well as certain spin liquids [6, 7]. We witnessed a rapid development of the theory of topological order evident in efforts to classify topological phases, to identify topological invariants, as well as to extend the theory beyond the known topological phases.

Certain FQHSs may have more intricate topological order than the ones described by Laughlin's wavefunction [8] and Jain's theory of free composite fermions [9]. Of the novel FQHSs the ones supporting non-Abelian quasiparticles have generated the most excitement [10–12]. The $\nu = 5/2$ FQHS forming in the region $2 < \nu < 4$, commonly called the second Landau level (SLL), is believed to be such a non-Abelian state [13].

The nature of other FQHSs forming in the SLL, such as that of the $\nu = 2 + 1/3$ and $2 + 1/5$ FQHSs, remains unknown despite sustained efforts in theory [14–27]. The FQHS at $\nu = 2 + 1/3$ [28–34] admits both a conventional Laughlin-Jain description [8, 9] as well as non-Abelian candidate states [14–16]. The relatively poor overlap between the exact and numerically obtained wavefunctions [18–27] and the unusual excitations [17] do not provide firm evidence for Laughlin correlations in the $\nu = 2 + 1/3$ FQHS. A number of recent experiments on the $\nu = 2 + 1/3$ FQHS, however, found its bulk [33] and edge [35–37] properties consistent with the Laughlin description. The other prominent FQHS at $\nu = 2 + 1/5$ [31, 32] is generally believed to be of the conventional Laughlin type [18, 19, 24–27], although there is a non-Abelian construction for it as well [15]. It is therefore currently not clear whether or not the prominent odd-denominator FQHSs in the SLL, such as the ones at $\nu = 2 + 1/3$ and $2 + 1/5$, require a description beyond the conventional Laughlin-Jain theory.

Experiments on the odd-denominator FQHS in the SLL have been restricted almost exclusively to the $2 < \nu < 3$ range, called the lower spin branch of the SLL (LSB SLL). Motivated by their poor understanding, we have performed transport studies of these FQHSs in the little explored upper spin branch of the SLL (USB SLL), i.e. in the $3 < \nu < 4$ region. We establish a new FQHS at $\nu = 3+1/3$ by detecting the opening of an energy gap. A quantitative comparison of the gap at this and other filling factors reveals two surprising findings: 1) the ground state at $\nu = 3 + 2/3$, a symmetry-related filling factor to $\nu = 3 + 1/3$, is not a FQHS, despite the existence of a strong depression in the longitudinal magnetoresistance and 2) most intriguingly, the activation energy gaps Δ of the prominent odd-denominator FQHSs are reversed across different spin branches of the SLL. Indeed, in stark contrast to the well established relation $\Delta_{2+1/3} > \Delta_{2+1/5}$ between the gaps of FQHSs of the the LSB SLL, in the USB SLL we find $\Delta_{3+1/3} < \Delta_{3+1/5}$. Within the conventional Laughlin-Jain picture we are unable to account for this anomalous gap reversal. We think that the observed gap reversal is due to modified electron-electron interactions within the USB SLL. Our result raises the possibility that at least one of the FQHSs in the upper spin branch has a non-conventional origin and suggests that controlling electron-electron interactions is of fundamental importance in tuning topological order.

In order to thermalize electrons to ultra-low temperatures of a few mK we use a He-3 immersion cell [28, 38]. Cooling is ensured by eight sintered silver heat exchangers which are immersed in the liquid He-3 bath. Thermometry is performed using a quartz tuning fork viscometer which monitors the temperature dependent viscosity of the He-3 bath [38].

We measured a high quality sample, in which we have already studied transport in the LSB SLL [33]. Figure 1 shows this region of the LSB SLL at magnetic fields

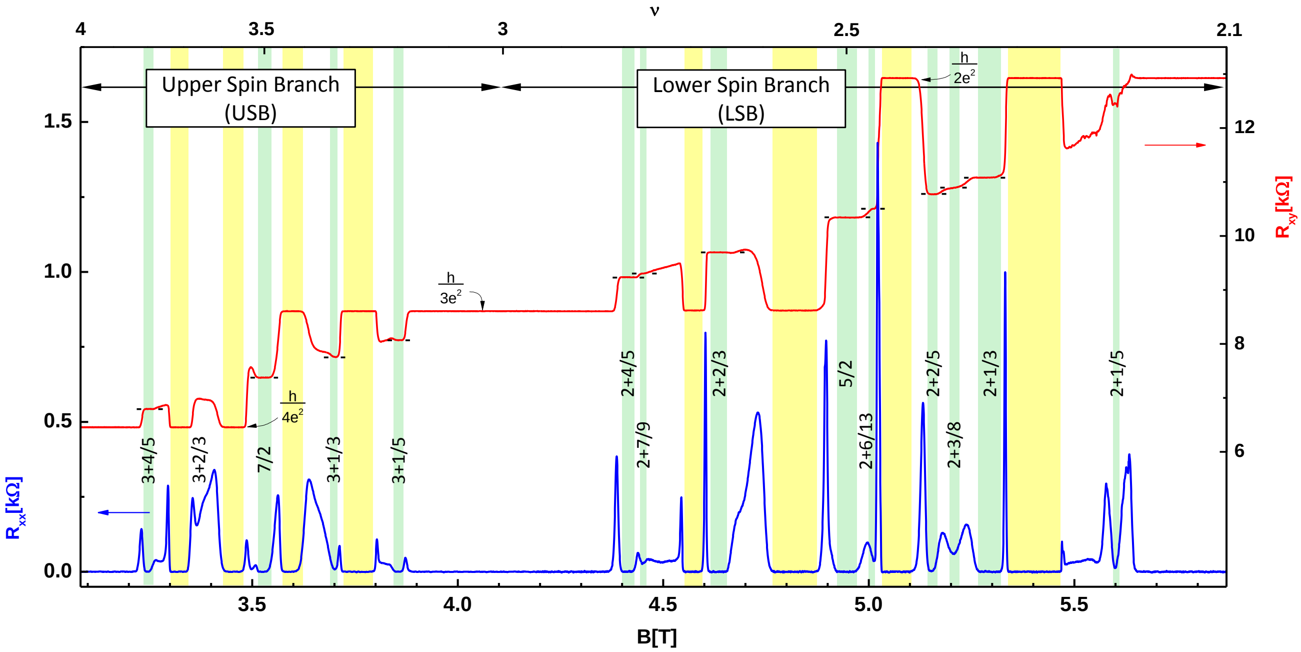

FIG. 1. Magnetoresistance traces in the second Landau level, i.e. in the filling factor range $2 < \nu < 4$, measured at $T = 6.9$ mK. The region of the lower spin branch (LSB) and upper spin branch (USB) are clearly marked. Fractional quantum Hall states are shaded in green, while the reentrant integer quantum Hall states in yellow. Data in the LSB is from Ref. [33].

$B > 4.1$ T. In this region we observe a large number of FQHSs as identified by their vanishing longitudinal magnetoresistance R_{xx} and Hall resistance R_{xy} quantized to h/fe^2 [2] at filling factors $\nu = f$, where f is the ratio of simple integers. We also observe four reentrant integer quatum Hall states (RIQHSs) signaled by quantization of R_{xy} to an integer, either $h/2e^2$ or $h/3e^2$ [39, 40]. These RIQHSs are believed to be exotic electronic solids [41].

Extending measurements to lower B-fields, we access the USB SLL. As seen in Fig.1, in this region we observe known FQHSs at filling factors $\nu = 7/2$, $3 + 1/5$, $3 + 4/5$ [39] and four RIQHSs [39, 40]. These FQHSs and RIQHSs form in the USB at the same partial filling factors, defined as the decimal part of the filling factor ν, as similar states in the LSB. The various ground states in the two spin branches are connected by particle-hole symmetry [42], therefore the ground states at ν, $5 - \nu$, $1 + \nu$, and $6 - \nu$ are said to be symmetry-related or conjugated states. For example, the FQHSs shown in Fig.1 at $\nu = 2 + 1/5$, $2 + 4/5$, $3 + 1/5$, and $3 + 4/5$ belonging to the different spin branches are symmetry-related.

Our data in the USB SLL exhibits a novel feature at $B = 3.50$ T, which does not have a symmetry related counterpart in the LSB SLL. As seen in Fig.1 and marked by the star symbol in Fig.2, at $B = 3.50$ T R_{xx} is nearly vanishing and R_{xy} exceeds the classical Hall value. Such a behavior is inconsistent with a FQHS; we think it is a signature of a new type of ground state. The data at $B = 3.50$ T is consistent with an incipient RIQHS. However, this incipient RIQHS is different from the known RIQHSs [39, 40]. Indeed, the two known RIQHSs at $\nu > 7/2$, which develop at $B = 3.32$ T and 3.45 T have R_{xy} quantized to $h/4e^2$. In contrast, R_{xy} of the incipient RIQHS at $B = 3.50$ T appears to develop towards $h/3e^2$ in the limit of $T = 0$.

As seen in Fig.1, strong local minima in R_{xx} also develop in the USB SLL at $\nu = 3 + 1/3$ and $\nu = 3 + 2/3$. However, the presence of these minima does not guarantee the formation of a FQH ground state at these filling factors. It is known that at $\nu = 1/7$, for example, no FQH ground state develops even though a depression in R_{xx} is present at finite temperatures [43]. A defining feature of an integer or fractional quantum Hall state, and of any topological ground state in general, is the opening of an energy gap in the bulk of the sample. An energy gap Δ is signaled by an activated magnetoresistance R_{xx} with a T-dependence of the form $R_{xx} \propto e^{-\Delta/2k_BT}$. Other hallmark properties of a FQHS are a quantized Hall resistance R_{xy} and a vanishing R_{xx} in the limit of $T = 0$ [2]. While weak indications of FQHSs have been reported at $\nu = 3 + 1/3$ or $3 + 2/3$ in Ref.[39], none of the above described hallmark properties of a FQHS have been observed. A close-up of the USB SLL is shown in Fig.2. We can see that at $\nu = 3 + 1/3$, our $T = 6.9$ mK data exhibit both a vanishingly small R_{xx} as well as an R_{xy} consistent with a plateau quantized to $h/(3 + 1/3)e^2$.

Magnetotransport at $\nu = 3+2/3$, however, is markedly different from that at $\nu = 3 + 1/3$. As seen in Fig.2, R_{xx} develops a local minimum at $\nu = 3 + 2/3$. However, R_{xy} at $\nu = 3 + 2/3$ clearly does not cross the classical

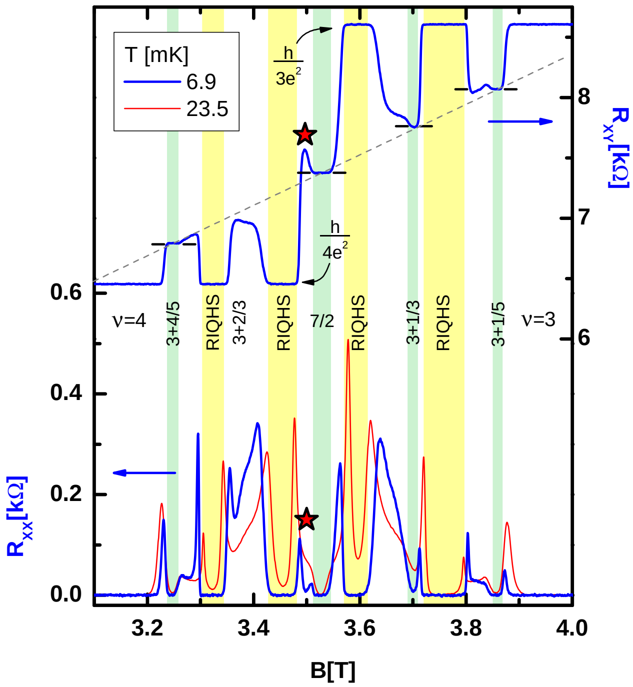

FIG. 2. The magnetoresistance in USB SLL ($3 < \nu < 4$). Blue traces are measured at 6.9 mK, while the red one at 23.5 mK. Numbers mark various filling factors of interest. We note the absence of a FQHS at $\nu = 3 + 2/3$, even though a local minimum is present in R_{xx} at this filling factor. The dashed line is the classical Hall line and the star symbol is indicative of a developing RIQHS of a new type described in the text.

Hall line, it therefore deviates from the quantum value $h/(3 + 2/3)e^2$, the expected value for a FQHS at this filling factor. This deviation casts a doubt on whether the ground state at $\nu = 3+2/3$ is a FQHS. Furthermore, as also shown in Fig.2, R_{xx} at $\nu = 3+2/3$ increases with a decreasing temperature, suggesting that R_{xx} does not vanish as T is lowered.

A detailed temperature dependence of the $\nu = 3+1/3$ and $3 + 2/3$ FQHSs is shown in Fig.3b. Demonstrated by the linear segments in the Arrhenius plots shown in Fig.3b, R_{xx} measured at $\nu = 3 + 1/3$ is found to be activated. The opening of an energy gap $\Delta_{3+1/3} = 37$ mK unambiguously establishes the formation of a new FQHS at $\nu = 3 + 1/3$. From data shown in Fig.3a and Fig.3b, we extract the energy gaps of the other odd-denominator FQHSs in the SLL: $\Delta_{3+1/5} = 104$ mK, $\Delta_{3+4/5} = 113$ mK, $\Delta_{2+1/5} = 210$ mK, and $\Delta_{2+4/5} = 212$ mK. Error due to scatter in the data is $\pm 5\%$.

Fig.3b also reveals that the T-dependence at $\nu = 3 + 2/3$, in contrast to that at $\nu = 3 + 1/3$, is not activated. The FQHS at $\nu = 3 + 2/3$ thus does not develop an energy gap in our sample in spite of the presence of a local minimum in R_{xx}. The ground state at $\nu = 3+2/3$ is therfore not a FQHS. However, the emergence of a fractional quantum Hall ground state at this filling factor in future higher quality samples cannot be ruled out.

Inspecting the energy gaps measured, we notice that $\Delta_{3+1/3} < \Delta_{3+1/5}$. This relationship is very unusual since in all instances, within a given spin branch, the gaps of FQHSs at partial filling 1/3 were found to exceed that at partial filling 1/5. Indeed, $\Delta_{1/3} > \Delta_{1/5}$ is well known in the LSB of the lowest Landau level (LLL) [44–46] and $\Delta_{2+1/3} > \Delta_{2+1/5}$ is widely reported in the LSB SLL [28, 31–34]. Furthermore, there is evidence that in the USB LLL the $\nu = 1+1/3$ FQHS is more prominent than the $\nu = 1 + 1/5$ FQHS [47, 48]. We find, therefore, that in the USB SLL the expected relationship between the gaps of the $\nu = 3 + 1/3$ and $3 + 1/5$ is reversed. The anomalous gap reversal observed in the USB SLL indicates an unanticipated difference between the prominent odd-denominator FQHSs forming in the SLL.

We note that a related contrasting behavior of the FQHSs at partial filling 1/3 and 1/5 can be observed in recent data [49]. When populating the second electrical subband of a quantum well, the $2 + 1/3$ and $2 + 2/3$ FQHSs were strengthened, whereas the $2+1/5$ and $2+4/5$ FQHSs were destroyed [49].

The anomalous $\Delta_{3+1/3} < \Delta_{3+1/5}$ gap reversal may be caused by a suppression of the FQHS at $\nu = 3 + 1/3$ due to a spin transition in this state. Experiments so far have not detected any sign of a spin transition in either the $\nu = 2 + 1/3$ or the $2 + 1/5$ FQHSs and NMR measurements at $\nu = 2 + 1/3$ are consistent with fully spin polarizated state [34, 50, 51]. While a spin transition has recently been observed in a related FQHS at $\nu = 2 + 2/3$ [51], this transition occurs at a magnetic field $B \sim 1.24$ T considerably lower than the field $B = 3.7$ T the $\nu = 3 + 1/3$ FQHS forms in our sample. We thus

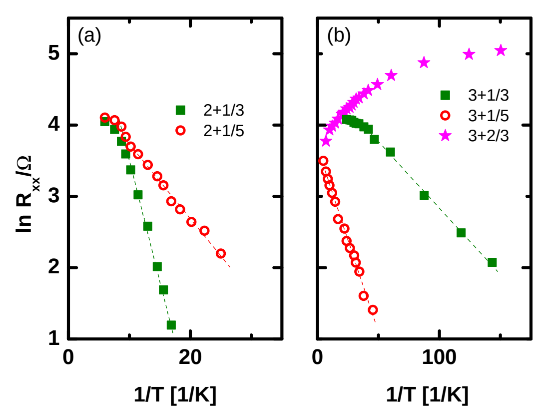

FIG. 3. Arrhenius plots of the R_{xx} minima at several odd-denominator filling factors in the LSB (panel a.) and USB (panel b.) of the SLL. Data at $\nu = 2 + 1/3$ is from Ref.[33].

think spin is not likely to play a significant role in the observed anomalous gap reversal of the prominent odd-denominator FQHSs.

With spin effects being ruled out, we find that the anomalous gap reversal of the $\nu = 3 + 1/3$ and $3 + 1/5$ FQHSs cannot be readily accounted for within the Laughlin-Jain description. Indeed, it is well known from numerical work [18, 19, 24–26] and from experiments [44–46] that the FQHSs of flux-four composite fermions are always more feeble than similar FQHSs of flux-two composite fermions. One possible explanation for the observed anomalous gap reversal is that among the odd-denominator FQHSs in the USB SLL at least one has a different origin than its counterpart state in the LSB SLL. Such a scenario is supported by the diminished overlap of the Laughlin and numerically obtained wavefunctions for the FQHSs in the SLL at partial filling 1/3 [18–27]. The anomalous gaps we found and the contrasting results reported in Ref.[49] highlight the lacunar understanding of the prominent odd-denominator FQHSs of the SLL and even elicit the provocative possibility that some of the FQHSs may not be a conventional Laughlin-Jain type, but rather of an unknown origin [14–16].

It is known that the effective electron-electron interactions affect FQHSs and in special cases may even induce phase transitions [3, 24, 25, 52]. These interactions in the SLL are very different from that in the LLL due to the dissimilar single particle wavefunctions in these two Landau levels. These interactions are also tuned by Landau level mixing (LLM), an effect due to the unoccupied Landau levels above the Fermi energy [53]. We think that the anomalous $\Delta_{3+1/3} < \Delta_{3+1/5}$ gap reversal observed reflects such a sensitivity to LLM tuned electron-electron interactions. The FQHSs at $\nu = 3 + 1/3$ and $3 + 1/5$ develop at lower B-fields, and therefore an enhanced LLM, as compared to the $\nu = 2+1/3$ and $2+1/5$ FQHSs. However, even though LLM likely plays a role in the anomalous gap reversal we observe, the details are not understood. Indeed, LLM can also be tuned for the $\nu = 2 + 1/3$ and $2 + 1/5$ pair of states as well, but a reversal of the gaps has never been detected, not even at large LLM [34, 51]. We thus conclude that the gap reversal of the prominent odd-denominator FQHSs of the SLL was not observed in the LSB at any sample conditions, therefore it is an exclusive characteristic of the USB.

In summary, the upper spin branch of the second Landau level exhibits an increasingly complex structure. Our energy gap measurements of the odd-denominator FQHSs in this region allowed for a test of the symmetry relations between these FQHSs and revealed an unexplained relative magnitudes of these energy gaps. We think that the observed anomalous gap reversal is due to modified electron-electron interactions which likely change the nature of at least one of the FQHSs in the USB SLL.

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