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# Production and Elliptic Flow of Dileptons and Photons in the semi-Quark Gluon Plasma

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We consider a non-perturbative approach to the thermal production of dileptons and photons at temperatures near the critical temperature in QCD. The suppression of colored excitations at low temperature is modeled by including a small value of the Polyakov loop, in a “semi” Quark-Gluon Plasma (QGP). Comparing the semi-QGP to the perturbative QGP, we find a mild enhancement of thermal dileptons. In contrast, to leading logarithmic order in weak coupling there are far fewer hard photons from the semi-QGP than the usual QGP. To illustrate the possible effects on photon and dileptons production in heavy ion collisions, we integrate the rate with a simulation using ideal hydrodynamics. Dileptons uniformly exhibit a small flow, but the strong suppression of photons in the semi-QGP tends to weight the elliptical flow of photons to that generated in the hadronic phase.

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The collisions of heavy nuclei at ultra-relativistic energies can be used to investigate the properties of the Quark-Gluon Plasma (QGP). At both the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), much of the collision takes place at temperatures which are not that far above that for the transition,  $T_c$ . This is a difficult region to study: perturbative methods can be used at high temperature, but not near  $T_c$  [1]. Similarly, hadronic models are valid at low temperature, but break down near  $T_c$  [2, 3]. In this work, we use a novel model of the non-perturbative region near  $T_c$ , termed the “semi”-Quark-Gluon Plasma [4–7]. The dominant effect which the semi-QGP incorporates is that as  $T \rightarrow T_c^+$ , colored fields evaporate, and are replaced by color singlet excitations, or hadrons. How color is suppressed is quantified by the decrease of the expectation value of the Polyakov loop, which we take from numerical simulations on the lattice [8].

A notable property of heavy ion collisions is elliptic flow, how the initial spatial anisotropy of peripheral collisions is transformed into a momentum anisotropy. The large elliptic flow of hadrons can be well modeled by hydrodynamic models in which the QCD medium is close to an ideal fluid [9–11].

Electromagnetic signals, such as dilepton or photon production, are another valuable probe, since they reflect properties of the quark and gluon distributions of the QGP, and once produced, escape without significant interaction [12–32]. The elliptic flow of dileptons and photons [12] are especially important. For example, if most dileptons and photons are emitted at high temperature in the QGP, then since the flow at early times

is small, one would expect a small net elliptic flow for both. However, the PHENIX experiment at RHIC [30] has found a large elliptic flow for photons, comparable to that of hadrons. There are also unpublished results from the ALICE experiment at the LHC [31]. This large photon elliptic flow is most puzzling [12, 24–26, 32], and one of the outstanding problems in the field.

In this paper we present results for the thermal production of hard dileptons and photons in the region near  $T_c$ , using the semi-QGP, and compare them with those of the perturbative QGP. We compute to leading order in the QCD coupling (for photons, only to leading logarithmic order) and give complete results later [33]. We then use a hydrodynamic model [23] to compute the effect on the number of dileptons and photons produced, and on the elliptic flow,  $v_2$ . The effects on thermal dileptons are modest. The suppression of thermal photons in the semi-QGP, though, implies that  $v_2$  favors that generated in the hadronic phase. Our results may help to understand the puzzle of the elliptic flow for photons.

Deconfinement in a  $SU(N_c)$  gauge theory is characterized by the Polyakov loop,  $\ell = (1/N_c)\text{tr} \mathcal{P} \exp(ig \int_0^{1/T} d\tau A_0)$  ( $\mathcal{P}$  denotes path ordering,  $T$  is the temperature,  $g$  the gauge coupling constant, and  $A_0$  the temporal component of the gauge field). At high temperature the theory is perturbative:  $A_0 = 0$  and the loop close to one,  $\langle \ell \rangle \sim 1$  [34]. In a pure gauge theory, there is rigorously a confined phase, as  $\langle \ell \rangle = 0$  below  $T_c$ . When dynamical quarks are present, though, the loop is nonzero at any  $T > 0$ ,  $\langle \ell \rangle > 0$ . Nevertheless, for three colors and three light flavors, lattice simulations find that the value of the

(renormalized) loop is rather small at  $T_c$ ,  $\langle \ell \rangle \approx 0.1$ , and so close to a true confined phase [8]. Thus we often find it useful to compare how signals change between the perturbative and confined phases, remembering that the loop is small at  $T_c$  in QCD.

The simplest way to represent a phase where the expectation value of the Polyakov loop is nonzero, but less than one, is to work in mean field theory, taking  $A_0$  to be a constant, diagonal matrix,  $(A_0^{cl})^{ab} = \delta^{ab} Q^a/g$  [4–7]. The Polyakov loop is then  $\ell = 1/N_c \sum_a e^{iQ^a/T}$ , where the color index  $a = 1 \dots N_c$ . For three colors,  $A_0^{cl} = (Q, -Q, 0)/g$ . The perturbative vacuum is  $Q = 0$ ,  $\ell = 1$ , while the confined vacuum, in which  $\ell = 0$ , is given by  $Q = 2\pi T/3$ . Since the background field  $A_0^{cl}$  is inversely proportional to the QCD coupling,  $g$ , this is manifestly a model of non-perturbative physics.

In Minkowski spacetime, the diagrams are those of ordinary perturbation theory, except that the background field  $A_0^{cl}$  acts like an imaginary chemical potential for color. For a quark with color  $a$ , the Fermi-Dirac distribution function is  $1/(e^{(E-iQ^a)/T} + 1)$ . In the double line basis gluons carry two color indices,  $(ab)$ , and their Bose-Einstein distribution function involves a difference of  $Q$ 's,  $1/(e^{(E-i(Q^a-Q^b))/T} - 1)$ . In the Boltzmann approximation, the distribution function for a single quark (or anti-quark), summed over color, is suppressed by the Polyakov loop,  $\sim \sum_a e^{-(E-iQ^a)/T}/N_c \sim e^{-E/T}\ell$ ; for gluons, it is  $\sim e^{-E/T}\ell^2$ .

In the model of the semi-QGP, one computes to leading order in the QCD coupling with  $Q^a \neq 0$  [4–7]. We first discuss the results for thermal dilepton production. Let the sum of the momenta of the dilepton be  $P^\mu = (E, \vec{p})$ ,  $p = |\vec{p}|$ , where  $E > p$ . To leading order in perturbation theory, this arises from the annihilation of a quark anti-quark pair into a virtual photon, which then decays into a dilepton pair. For three colors and  $Q = 0$ , the production rate [23] is

$$\left. \frac{d\Gamma}{d^4P} \right|_{Q=0} = \frac{\alpha_{em}}{6\pi^4} n(E) \left( 1 - \frac{2T}{p} \ln \frac{1 + e^{-p-/T}}{1 + e^{-p+/T}} \right); \quad (1)$$

$p_\pm = (E \pm p)/2$ , and  $n(E) = 1/(e^{E/T} - 1)$  is the usual Bose-Einstein distribution function. This includes the contributions of (massless) up, down and strange quarks, where  $\alpha_{em} = e^2/4\pi$ , and  $e$  is the electromagnetic coupling constant.

In the semi-QGP, to leading order the result when  $Q \neq 0$  is a simple factor times that for  $Q = 0$  [33],

$$\left. \frac{d\Gamma}{d^4P} \right|_{Q \neq 0} = f_{ll}(Q) \left. \frac{d\Gamma}{d^4P} \right|_{Q=0}, \quad (2)$$

where  $f_{ll}(Q) \equiv \tilde{f}_{ll}(Q)/\tilde{f}_{ll}(0)$ . For three colors, this can be written in terms of the Polyakov loop,

$$\tilde{f}_{ll} = 1 - \frac{2T}{3p} \ln \frac{1 + 3\ell e^{-p-/T} + 3\ell e^{-2p-/T} + e^{-3p-/T}}{1 + 3\ell e^{-p+/T} + 3\ell e^{-2p+/T} + e^{-3p+/T}}. \quad (3)$$

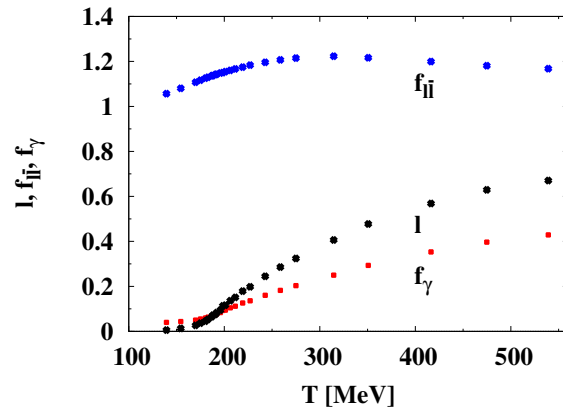


FIG. 1. The ratio of the thermal production of dileptons and photons in the semi-QGP, versus that in perturbation theory, as a function of temperature. For dileptons,  $f_{ll}$  from Eq. (3) is for  $E = 1$  GeV and  $p = 0$ . For photons,  $f_\gamma$  in Eq. (7) is independent of the photon momentum. The Polyakov loop,  $l$ , is extracted from the value of the (renormalized) Polyakov loop in lattice QCD [8], see Ref. [7] for details.

In the special case that the dileptons move back to back,  $p = 0$ , we plot the modification factor at  $E = 1$  GeV as a function of temperature in Fig. (1), taking the  $Q^a$ 's from Ref. [7]. We find that  $f_{ll}(Q)$  is always greater than one.

To understand this, remember that in kinetic theory the production rate for dileptons is the product of statistical distribution functions times the square of an amplitude. When  $p = 0$ , the distribution functions are for a quark and an anti-quark, each with energy  $E/2$  and color  $a$ . If the total energy  $E \gg T$ , we can use the Boltzmann approximation for the  $Q^a$ -dependent Fermi-Dirac distribution functions,

$$e^2 \sum_{a=1}^N e^{-(E/2-iQ^a)/T} e^{-(E/2+iQ^a)/T} |\mathcal{M}_{ll}|^2. \quad (4)$$

As the  $Q^a$ 's are like a chemical potential for color, they have the *opposite* sign for the quark and anti-quark, and so at large energy, cancel identically. That is, the probability for a hard virtual photon to produce a quark anti-quark pair is independent of the  $Q^a$ 's, and so the Polyakov loop. This is in stark contrast to the statistical distribution function for a *single* quark or anti-quark, which is  $\sim \ell$ . Fig. (1) shows that for moderate values of  $E \sim T$ , there are corrections to the Boltzmann approximation which even give a modest *enhancement* above  $T_c$ , by about  $\sim 20\%$ .

Expanding Eq. (3) to quadratic order in the  $Q^a$  is equivalent to considering a condensate  $\sim \langle \text{tr } A_0^2 \rangle$ , and agrees with previous results [16]. Ref. [16] suggested that an enhancement like that which we find could explain the excess of dileptons found below the  $\rho$  meson mass in

heavy ion collisions; see, also, Ref. [14, 21, 22].

We now consider the production of real photons at a large momentum  $P^\mu$ , where  $E = p \gg T$ . To leading order in the QCD coupling, apparently two processes contribute to photon production: Compton scattering of a quark or anti-quark, and the pair annihilation of a quark and an anti-quark. These  $2 \rightarrow 2$  processes [13] are both  $\sim e^2 g^2$ . However, a quark which scatters with an arbitrary number of soft gluons, with  $E_{\text{soft}} \sim gT$ , emits collinear photons at the same order,  $\sim e^2 g^2$ . [17, 18]. This depends crucially upon Bose-Einstein enhancement for the soft gluon, as  $n(E_{\text{soft}}) \sim 1/g$ .

In the semi-QGP, however, there is no Bose-Einstein enhancement for off-diagonal gluons: at small  $E$  the gluon distribution function is  $\sim 1/(e^{-i(Q^a - Q^b)/T} - 1)$ , if  $a \neq b$  and  $Q^a - Q^b \sim T$ . There is Bose-Einstein enhancement for soft, diagonal gluons, where  $a = b$ , but at large  $N_c$  there are only  $\sim N_c$  diagonal gluons to  $\sim N_c^2$  off-diagonal gluons. Consequently up to corrections  $\sim 1/N_c$ , in the semi-QGP the production of real photons is dominated by  $2 \rightarrow 2$  processes. This is a straightforward generalization of the original computations of Ref. [13]. The results for collinear emission at large  $N_c$  will be given later [33].

Computing thermal photon production only to leading logarithmic order, we find [33]

$$E \frac{d\Gamma}{d^3p} \Big|_{Q \neq 0} = f_\gamma(Q) E \frac{d\Gamma}{d^3p} \Big|_{Q=0}. \quad (5)$$

At the same order, the result for  $2 \rightarrow 2$  scattering in the perturbative regime [13] is

$$E \frac{d\Gamma}{d^3p} \Big|_{Q=0} = \frac{\alpha_{em} \alpha_s}{3\pi^2} e^{-E/T} T^2 \ln \left( \frac{E}{g^2 T} \right), \quad (6)$$

where  $\alpha_s = g^2/(4\pi)$ , and

$$f_\gamma(Q) = 1 - 4q + \frac{10}{3} q^2; \quad q = \frac{Q}{2\pi T}, \quad 0 < q < 1. \quad (7)$$

In the perturbative limit,  $f_\gamma(0) = 1$ . This function decreases monotonically as  $Q$  increases, with  $f_\gamma(2\pi T/3) = 1/27$  in the confined phase. In Fig. (1) we plot  $f_\gamma$  versus temperature. For hard photons this result is independent of momentum.

Why photon production from colored fields is strongly suppressed in the confined phase can be understood from the case of pair annihilation. Using kinetic theory in the Boltzmann approximation, photon production is proportional to

$$e^2 g^2 \sum_{a,b} e^{-(E_1 - iQ^a)/T} e^{-(E_2 + iQ^b)/T} |\mathcal{M}_\gamma^{ab}|^2, \quad (8)$$

where  $E_1$  is the energy of the incoming quark with color  $a$ ,  $E_2$  the energy of the anti-quark with color  $b$ , and  $\mathcal{M}_\gamma^{ab}$  a matrix element, which depends upon  $a$  and  $b$ . The quark

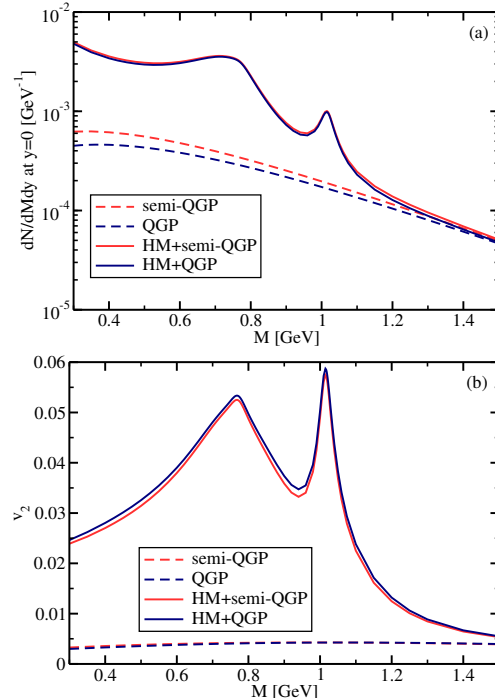


FIG. 2. Dilepton yield (a) and elliptic flow (b) computed using MUSIC, from the semi-QGP and QGP, plus hadronic matter (HM). This calculation is for Au+Au collisions at the top RHIC energy,  $\sqrt{s} = 200 \text{ GeV}/A$ , in the 20-40% centrality class.

and anti-quark then scatter into a gluon, with color indices  $(ab)$ , and a photon. In the perturbative QGP, the rate is  $\sim e^2 g^2 N_c^2$ . In the confined phase, however, to avoid suppression by powers of the Polyakov loop the color charges of the quark and anti-quark must match up, with  $a = b$ . This reduces the result by one factor of  $1/N_c$ . Further, the matrix element  $\mathcal{M}_\gamma^{ab}$  involves the quark-gluon vertex; when  $a = b$ , this gives another factor of  $1/N_c$ , for an overall factor of  $1/N_c^2$ . The same counting in  $1/N_c$  applies for Compton scattering. Explicit computation shows that to leading logarithmic order, in the confined phase  $f_\gamma$  equals a small number,  $1/3$ , times  $1/N_c^2$  [33]. Even for three colors this is a strong suppression, as  $f_\gamma(2\pi T/3) = 1/27$ .

To see if the effects of the semi-QGP might be important for experiment, we perform the following exercise. We take the perturbative results for dilepton and photon emission, computed at leading order (for dileptons, to  $\sim e^2$  [12, 23–25], and for photons, to  $\sim e^2 g^2$  [12, 17, 18]) and multiply by the corresponding suppression factors,  $\tilde{f}_{ii}(Q)$  in Eq. (3) and  $f_\gamma(Q)$  in Eq. (7). We then fold these into a 3+1 D hydrodynamic simulation, MUSIC [10, 11], using ideal hydrodynamics for nucleus-nucleus collisions, with  $A = 200$  at RHIC energies,  $\sqrt{s} = 200 \text{ GeV}/A$ . The hadronic rates for dileptons

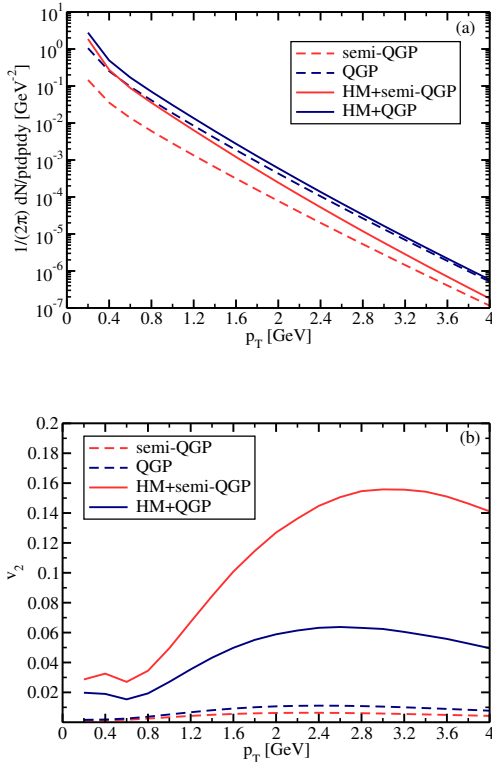


FIG. 3. Photon yield (a) and elliptic flow (b) using MUSIC, from the semi-QGP and QGP, plus hadronic matter (HM). As in Fig. (2), this calculation is for Au+Au collisions at the top RHIC energy, in the 20-40% centrality class.

[23] and photons [19] are also included.

For photon production, to make a meaningful comparison to experiment it is necessary to compute beyond leading logarithmic order, and to include collinear emission [33]. Further, hydrodynamic studies will consider the effects of including primordial flow, bulk viscosity, different possible initial values of the shear pressure tensor, and the temperature dependence of the transport coefficients. For this reason we defer a detailed comparison to experiment [30] until a more thorough analysis is complete. Since our results show that the effects upon thermal photon production are large, we believe that they constitute an important development and will definitely be part, eventually, of a more complete analysis. We note that for dilepton and photon production, there are only a few previous studies of nonperturbative effects near  $T_c$  [21, 26].

In ideal hydrodynamics, fluid dynamics is governed by the conservation equation for the stress-energy tensor,  $\partial_\mu T^{\mu\nu} = 0$ , where  $T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - g^{\mu\nu}P$ ;  $u^\mu$  is the fluid four-velocity,  $\varepsilon$  the energy density,  $P$  the thermodynamic pressure. The latter is connected to  $\varepsilon$  by the equation of state. In this work we use a parametrization of the lattice QCD equation of state [2]. The source of

the lattice data used for this parametrization and for the Polyakov loop to calculate the rates is the same [8]. The details regarding the numerical algorithm being used to solve the hydrodynamic equations along with the initial and freeze-out conditions are presented in Ref. [10].

Fig. (2) shows the results for the dileptons. There are slightly more dileptons from the semi-QGP than the usual QGP, but below an invariant mass of 1.5 GeV, the total yield is dominated by the hadronic matter. It might be possible to detect dileptons from the semi-QGP above 1.5 GeV. The dilepton elliptic flow is small,  $v_2 \sim 0.01 - 0.06$ , and is dominated by that from hadronic matter.

The results for photons, shown in Fig. (3), are very different. The suppression of color in the semi-QGP greatly reduces the photon yield, Fig. (3a). The  $v_2$  of the semi-QGP is also reduced with respect to that of the QGP, Fig. (3b).

However, the *total* thermal photon  $v_2$  is a yield-weighted average of the  $v_2$  from the QGP and hadronic phases. There is a competition between the change in the QGP yield and that of  $v_2$ : lowering the QGP  $v_2$  lowers the thermal photon  $v_2$ , while a decrease in the yield from the QGP biases the thermal photon  $v_2$  towards that from the hadronic phase, which is large. From Fig. (3b), the latter wins, so that using semi-QGP rates significantly increases the total  $v_2$  for thermal photons.

Besides those mentioned above, there are several other issues which we think must also be addressed in order to make a realistic comparison to experiment.

One is the contribution of prompt photons, produced through the collisions of hard partons. In proton-proton collisions these can be computed at next to leading order using perturbative QCD, with the dominant uncertainties the limited knowledge of the parton fragmentation functions into photons and the dependence on the renormalization mass scale [28]. In extrapolating to heavy ion collisions, it is also essential to include how parton fragmentation functions change in the medium [29]. Photons with low  $p_T$  are produced predominantly by parton fragmentation, so that a photon produced by the fragmentation of a hard jet should inherit a good fraction of the elliptic flow of that jet.

In the end, our mechanism for enhancing the photon  $v_2$  is due to the suppression of photons in the semi-QGP. However, in heavy ion collisions at moderate  $p_T$ , the number of photons produced by both perturbative and thermal mechanisms seem to be significantly *below* that observed experimentally. Thus solving the  $v_2$  puzzle appears to exacerbate a problem with the overall rate. Using a virial expansion, however, Ref. [14, 15] claim that previous computations of photon production in the hadronic medium are too low, and find good agreement with the overall rates from PHENIX [30]. Their photon  $v_2$  is still too small [15], which the semi-QGP might explain. Other mechanisms for producing photons, such as from the Color Glass Condensate [26], may also con-

tribute.

Clearly there is much left to do. One obvious matter, which is especially important for dilepton production, is to include the effects of hadrons near  $T_c$ . The other is to compute the effect of the suppression of color on all transport coefficients, as has been done for the shear viscosity [5] and the collisional energy loss of heavy quarks [7]. In the end, it could well be that heavy-ion physics at both RHIC and the LHC is dominated by the non-perturbative region near  $T_c$ .

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- [1] J. O. Andersen, M. Strickland, and N. Su, *Phys.Rev.Lett.* **104**, 122003 (2010), arXiv:0911.0676 [hep-ph]; *JHEP* **1008**, 113 (2010), arXiv:1005.1603 [hep-ph]; J. O. Andersen, L. E. Leganger, M. Strickland, and N. Su, *Phys.Lett.* **B696**, 468 (2011), arXiv:1009.4644 [hep-ph]; *JHEP* **1108**, 053 (2011), arXiv:1103.2528 [hep-ph]; N. Haque, M. G. Mustafa, and M. Strickland, *Phys.Rev.* **D87**, 105007 (2013), arXiv:1212.1797 [hep-ph]; S. Mogliacci, J. O. Andersen, M. Strickland, N. Su, and A. Vuorinen, *JHEP* **1312**, 055 (2013), arXiv:1307.8098 [hep-ph]; N. Haque, A. Bandyopadhyay, J. O. Andersen, M. G. Mustafa, M. Strickland, *et al.*, (2014), arXiv:1402.6907 [hep-ph].
- [2] P. Huovinen and P. Petreczky, *Nucl.Phys.* **A837**, 26 (2010), arXiv:0912.2541 [hep-ph].
- [3] A. Andronic, P. Braun-Munzinger, J. Stachel, and M. Winn, *Phys.Lett.* **B718**, 80 (2012), arXiv:1201.0693 [nucl-th].
- [4] R. D. Pisarski, *Phys.Rev.* **D74**, 121703 (2006), arXiv:hep-ph/0608242 [hep-ph].
- [5] Y. Hidaka and R. D. Pisarski, *Phys.Rev.* **D78**, 071501 (2008), arXiv:0803.0453 [hep-ph]; *Phys.Rev.* **D80**, 036004 (2009), arXiv:0906.1751 [hep-ph]; *Phys.Rev.* **D80**, 074504 (2009), arXiv:0907.4609 [hep-ph]; *Phys.Rev.* **D81**, 076002 (2010), arXiv:0912.0940 [hep-ph].
- [6] A. Dumitru, Y. Guo, Y. Hidaka, C. P. Korthals Altes, and R. D. Pisarski, *Phys.Rev.* **D83**, 034022 (2011), arXiv:1011.3820 [hep-ph]; A. Dumitru, Y. Guo, Y. Hidaka, C. P. K. Altes, and R. D. Pisarski, *Phys.Rev.* **D86**, 105017 (2012), arXiv:1205.0137 [hep-ph]; K. Kashiwa, R. D. Pisarski, and V. V. Skokov, *Phys.Rev.* **D85**, 114029 (2012), arXiv:1205.0545 [hep-ph]; R. D. Pisarski and V. V. Skokov, *Phys.Rev.* **D86**, 081701 (2012), arXiv:1206.1329 [hep-th]; K. Kashiwa and R. D. Pisarski, *Phys.Rev.* **D87**, 096009 (2013), arXiv:1301.5344 [hep-ph]; S. Lin, R. D. Pisarski, and V. V. Skokov, *Phys.Rev.* **D87**, 105002 (2013), arXiv:1301.7432 [hep-ph]; J. Xu, J. Liao, and M. Gyulassy, (2014), arXiv:1411.3673 [hep-ph].
- [7] S. Lin, R. D. Pisarski, and V. V. Skokov, (2013), 10.1016/j.physletb.2014.01.043, arXiv:1312.3340 [hep-ph].
- [8] A. Bazavov, T. Bhattacharya, M. Cheng, N. Christ, C. DeTar, *et al.*, *Phys.Rev.* **D80**, 014504 (2009), arXiv:0903.4379 [hep-lat]; C. DeTar and U. Heller, *Eur.Phys.J.* **A41**, 405 (2009), arXiv:0905.2949 [hep-lat]; Z. Fodor and S. Katz, (2009), arXiv:0908.3341 [hep-ph]; P. Petreczky, *J.Phys.* **G39**, 093002 (2012), arXiv:1203.5320 [hep-lat]; S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, *et al.*, *Phys.Lett.* **B370**, 99 (2014), arXiv:1309.5258 [hep-lat]; T. Bhattacharya, M. I. Buchoff, N. H. Christ, H. T. Ding, R. Gupta, *et al.*, (2014), arXiv:1402.5175 [hep-lat]; S. Sharma, *Adv.High Energy Phys.* **2013**, 452978 (2013), arXiv:1403.2102 [hep-lat].
- [9] U. W. Heinz, (2009), arXiv:0901.4355 [nucl-th]; U. Heinz and R. Snellings, *Ann.Rev.Nucl.Part.Sci.* **63**, 123 (2013), arXiv:1301.2826 [nucl-th]; C. Gale, S. Jeon, and B. Schenke, *Int.J.Mod.Phys.* **A28**, 1340011 (2013), arXiv:1301.5893 [nucl-th].
- [10] B. Schenke, S. Jeon, and C. Gale, *Phys.Rev.* **C82**, 014903 (2010), arXiv:1004.1408 [hep-ph].
- [11] B. Schenke, S. Jeon, and C. Gale, *Phys.Rev.Lett.* **106**, 042301 (2011), arXiv:1009.3244 [hep-ph].
- [12] R. Chatterjee, E. S. Frodermann, U. W. Heinz, and D. K. Srivastava, *Phys.Rev.Lett.* **96**, 202302 (2006), arXiv:nucl-th/0511079 [nucl-th]; R. Chatterjee, D. K. Srivastava, U. W. Heinz, and C. Gale, *Phys.Rev.* **C75**, 054909 (2007), arXiv:nucl-th/0702039 [nucl-th]; S. Turbide, C. Gale, E. Frodermann, and U. Heinz, *Phys.Rev.* **C77**, 024909 (2008), arXiv:0712.0732 [hep-ph]; C. Shen, U. W. Heinz, J.-F. Paquet, I. Kozlov, and C. Gale, (2013), arXiv:1308.2111 [nucl-th]; C. Shen, U. W. Heinz, J.-F. Paquet, and C. Gale, *Phys.Rev.* **C89**, 044910 (2014), arXiv:1308.2440 [nucl-th].
- [13] R. Baier, H. Nakkagawa, A. Niegawa, and K. Redlich, *Z.Phys.* **C53**, 433 (1992); J. I. Kapusta, P. Lichard, and D. Seibert, *Phys.Rev.* **D44**, 2774 (1991).
- [14] J. V. Steele, H. Yamagishi, and I. Zahed, *Phys.Lett.* **B384**, 255 (1996), arXiv:hep-ph/9603290 [hep-ph]; *Phys.Rev.* **D56**, 5605 (1997), arXiv:hep-ph/9704414 [hep-ph]; J. V. Steele and I. Zahed, *Phys.Rev.* **D60**, 037502 (1999), arXiv:hep-ph/9901385 [hep-ph]; K. Dusling, D. Teaney, and I. Zahed, *Phys.Rev.* **C75**, 024908 (2007), arXiv:nucl-th/0604071 [nucl-th]; K. Dusling and I. Zahed, *Nucl.Phys.* **A825**, 212 (2009), arXiv:0712.1982 [nucl-th]; *Phys.Rev.* **C82**, 054909 (2010), arXiv:0911.2426 [nucl-th]; C.-H. Lee and I. Zahed, (2014), arXiv:1403.1632 [hep-ph].
- [15] K. Dusling, C. H. Lee, and I. Zahed, "EM emission and conductivity in chiral reduction approach," (2014).
- [16] C. Lee, J. Wirstam, I. Zahed, and T. Hansson, *Phys.Lett.* **B448**, 168 (1999), arXiv:hep-ph/9809440 [hep-ph].

- [17] P. Aurenche, F. Gelis, and H. Zaraket, *Phys.Rev.* **D62**, 096012 (2000), [arXiv:hep-ph/0003326 \[hep-ph\]](#).
- [18] P. B. Arnold, G. D. Moore, and L. G. Yaffe, *JHEP* **0112**, 009 (2001), [arXiv:hep-ph/0111107 \[hep-ph\]](#); *JHEP* **0206**, 030 (2002), [arXiv:hep-ph/0204343 \[hep-ph\]](#).
- [19] S. Turbide, R. Rapp, and C. Gale, *Phys.Rev.* **C69**, 014903 (2004), [arXiv:hep-ph/0308085 \[hep-ph\]](#).
- [20] P. Staig and E. Shuryak, (2010), [arXiv:1005.3531 \[nucl-th\]](#).
- [21] E. Bratkovskaya and W. Cassing, *Nucl.Phys.* **A619**, 413 (1997), [arXiv:nucl-th/9611042 \[nucl-th\]](#); E. Bratkovskaya, W. Cassing, and O. Linnyk, *Phys.Lett.* **B670**, 428 (2009), [arXiv:0805.3177 \[nucl-th\]](#); J. Manninen, E. Bratkovskaya, W. Cassing, and O. Linnyk, *Eur.Phys.J.* **C71**, 1615 (2011), [arXiv:1005.0500 \[nucl-th\]](#); O. Linnyk, W. Cassing, J. Manninen, E. Bratkovskaya, and C. Ko, *Phys.Rev.* **C85**, 024910 (2012), [arXiv:1111.2975 \[nucl-th\]](#); O. Linnyk, E. Bratkovskaya, V. Ozvenchuk, W. Cassing, and C. Ko, *Phys.Rev.* **C84**, 054917 (2011), [arXiv:1107.3402 \[nucl-th\]](#); O. Linnyk, W. Cassing, J. Manninen, E. Bratkovskaya, P. Gossiaux, *et al.*, *Phys.Rev.* **C87**, 014905 (2013), [arXiv:1208.1279 \[nucl-th\]](#); E. Bratkovskaya, J. Aichelin, M. Thomere, S. Vogel, and M. Bleicher, *Phys.Rev.* **C87**, 064907 (2013), [arXiv:1301.0786 \[nucl-th\]](#).
- [22] R. Rapp, *Adv.High Energy Phys.* **2013**, 148253 (2013), [arXiv:1304.2309 \[hep-ph\]](#); P. M. Hohler and R. Rapp, *Phys.Lett.* **B731**, 103 (2014), [arXiv:1311.2921 \[hep-ph\]](#).
- [23] G. Vujanovic, C. Young, B. Schenke, R. Rapp, S. Jeon, and C. Gale, (2013), [arXiv:1312.0676 \[nucl-th\]](#).
- [24] E. Bratkovskaya, S. Kiselev, and G. Sharkov, *Phys.Rev.* **C78**, 034905 (2008), [arXiv:0806.3465 \[nucl-th\]](#); O. Linnyk, V. Konchakovski, W. Cassing, and E. Bratkovskaya, *Phys.Rev.* **C88**, 034904 (2013), [arXiv:1304.7030 \[nucl-th\]](#); O. Linnyk, W. Cassing, and E. Bratkovskaya, *Phys.Rev.* **C89**, 034908 (2014), [arXiv:1311.0279 \[nucl-th\]](#).
- [25] H. van Hees, C. Gale, and R. Rapp, *Phys.Rev.* **C84**, 054906 (2011), [arXiv:1108.2131 \[hep-ph\]](#); G. Basar, D. Kharzeev, and V. Skokov, *Phys.Rev.Lett.* **109**, 202303 (2012), [arXiv:1206.1334 \[hep-ph\]](#); A. Bzdak and V. Skokov, *Phys.Rev.Lett.* **110**, 192301 (2013), [arXiv:1208.5502 \[hep-ph\]](#); K. Fukushima and K. Mameda, *Phys.Rev.* **D86**, 071501 (2012), [arXiv:1206.3128 \[hep-ph\]](#); F.-M. Liu and S.-X. Liu, *Phys.Rev.* **C89**, 034906 (2014), [arXiv:1212.6587 \[nucl-th\]](#); B. Muller, S.-Y. Wu, and D.-L. Yang, *Phys.Rev.* **D89**, 026013 (2014), [arXiv:1308.6568 \[hep-th\]](#); G. Basar, D. E. Kharzeev, and E. V. Shuryak, (2014), [arXiv:1402.2286 \[hep-ph\]](#); H. van Hees, M. He, and R. Rapp, (2014), [arXiv:1404.2846 \[nucl-th\]](#); A. Monnai, (2014), [arXiv:1403.4225 \[nucl-th\]](#).
- [26] M. Chiu, T. K. Hemmick, V. Khachatryan, A. Leonidov, J. Liao, *et al.*, *Nucl.Phys.* **A900**, 16 (2013), [arXiv:1202.3679 \[nucl-th\]](#); L. McLerran and B. Schenke, (2014), [arXiv:1403.7462 \[hep-ph\]](#).
- [27] M. Dion, J.-F. Paquet, B. Schenke, C. Young, S. Jeon, *et al.*, *Phys.Rev.* **C84**, 064901 (2011), [arXiv:1109.4405 \[hep-ph\]](#).
- [28] M. Klasen, C. Klein-Boesing, F. Knig, and J. Wessels, *JHEP* **1310**, 119 (2013), [arXiv:1307.7034](#); M. Klasen and F. Konig, (2014), [arXiv:1403.2290 \[hep-ph\]](#).
- [29] F. Arleo, *JHEP* **0609**, 015 (2006), [arXiv:hep-ph/0601075 \[hep-ph\]](#); *Eur.Phys.J.* **C61**, 603 (2009), [arXiv:0810.1193 \[hep-ph\]](#); F. Arleo, K. J. Eskola, H. Paukkunen, and C. A. Salgado, *JHEP* **1104**, 055 (2011), [arXiv:1103.1471 \[hep-ph\]](#).
- [30] A. Adare *et al.* (PHENIX Collaboration), *Phys.Rev.Lett.* **109**, 122302 (2012), [arXiv:1105.4126 \[nucl-ex\]](#).
- [31] D. Lohner (ALICE), *J.Phys.Conf.Ser.* **446**, 012028 (2013), [arXiv:1212.3995 \[hep-ex\]](#).
- [32] T. Sakaguchi, (2014), [arXiv:1401.2481 \[nucl-ex\]](#).
- [33] Y. Hidaka, S. Lin, R. D. Pisarski, and D. Satow, in preparation (2014).
- [34] E. Gava and R. Jengo, *Phys.Lett.* **B105**, 285 (1981); Y. Burnier, M. Laine, and M. Vepsalainen, *JHEP* **1001**, 054 (2010), [arXiv:0911.3480 \[hep-ph\]](#); N. Brambilla, J. Ghiglieri, P. Petreczky, and A. Vairo, *Phys.Rev.* **D82**, 074019 (2010), [arXiv:1007.5172 \[hep-ph\]](#).