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The parity-violating nucleon-nucleon force in the $1/N_c$ expansion

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Several experimental investigations have observed parity violation in nuclear systems—a consequence of the weak force between quarks. We apply the $1/N_c$ expansion of QCD to the P-violating T-conserving component of the nucleon-nucleon (NN) potential. We show there are two leading-order operators, both of which affect $\bar{p}p$ scattering at order $N_c$. We find an additional four operators at order $N_c^0 \sin^2 \theta_W$ and six at $O(1/N_c)$. Pion exchange in the PV NN force is suppressed by $1/N_c$ and $\sin^2 \theta_W$, providing a quantitative explanation for its non-observation up to this time. The large-$N_c$ hierarchy of other PV NN force mechanisms is consistent with estimates of the couplings in phenomenological models. The PV observed in $\bar{p}p$ scattering data is compatible with natural values for the strong and weak coupling constants: there is no evidence of fine tuning.

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The strong-nuclear and electromagnetic forces play the most prominent role in proton-proton ($pp$) scattering. There are also parity-violating (PV) $pp$ interactions, which manifest the presence of weak interactions between the quarks inside each proton. Measurements of longitudinal beam asymmetries $\sim 10^{-7}$ at Bonn [1], PSI [2], and TRIUMF [3] demonstrate that PV nucleon-nucleon (NN) forces exist. PV in NN systems is also probed via and TRIUMF [3] demonstrate that PV nucleon-nucleon (NN) forces exist. PV in NN systems is also probed via an asymmetry in the reaction $\bar{n}p \rightarrow d\gamma$ [4, 5]. And ab initio calculations of few-nucleon systems allow us to take models of the PV NN force and predict, e.g., the longitudinal asymmetry in $^3\text{He}(\bar{n}, p)^3\text{H}$ [6], which is soon to be measured [7]. Nuclear parity violation is also observed in, e.g., the radiative decay of the first excited state of $^{19}\text{F}$, but there theoretical uncertainties in the relationship between the observable and the model of the PV NN force are harder to quantify. Much work has gone into constraining the PV NN force from a variety of nuclear experiments, see Refs. [8, 9] for recent reviews.

The prevailing paradigm in such analyses is based on single-meson exchange between nucleons, most commonly in the framework developed by Desplanques, Donoghue, and Holstein (DDH) [10]. The quantum numbers of the exchanged mesons determine the operator structures that contribute, while operator coefficients involve products of strong and weak meson-nucleon-nucleon coupling constants. In this paper we show that Standard Model (SM) couplings and the $1/N_c$ expansion of QCD predict the operators, and the sizes of the associated coefficients, which appear in the PV NN potential.

An alternative framework—suitable for studying PV at very low energies—that systematizes pioneering studies [11, 12] has recently emerged [13–15], but has, as yet, been applied to far fewer experiments. The extension of chiral perturbation theory to few-nucleon systems, $\chi$EFT [16] has also been invoked [17–21]. In $\chi$EFT the one-pion-exchange piece of the PV NN force dominates, with all other effects suppressed by two orders in the chiral expansion.

One-pion exchange gives the long-distance parity-conserving potential, and drives many of the properties of light nuclei. But, thus far, experimental data show no evidence for pion exchange in the PV NN force: only upper bounds on its impact on observables have been obtained. We will show that the smallness of the PV NN operator associated with one-pion exchange is a consequence of the large-$N_c$ expansion.

Originally suggested by ’t Hooft [22], this technique notes that QCD has a “hidden”, perhaps small, parameter in $1/N_c$. Multiple simplifications of QCD occur in the limit $N_c \rightarrow \infty$. In particular, the expansion in powers of $1/N_c$ provides insights about baryons [23, 24]. In the context of nuclear forces the $1/N_c$ expansion was used to study the NN potential [25, 26]. These works analyzed the NN potential for momenta of order $N_c^0$, i.e., $p \sim N_c^{1/2}$, and found that it is an expansion in $1/N_c^2$. This hierarchy between different contributions to the NN potential is roughly borne out in the Nijm93 [27] NN potential. This analysis was extended to the 3N potential [28]; here we tackle the PV component of the NN force. Some of our results have been obtained within the chiral soliton model [29–31], or from consistency relations for PV pion-nucleon scattering [32]. But a model-independent derivation of all pertinent scalings and comparison with experimental data and phenomenological potentials appears here for the first time.

The fact that $\sin^2 \theta_W \approx 0.23$ [33] is key to the hierarchy of PV NN operators. The SM effective Lagrangian for the...
PV four-quark operators involving $u$ and $d$ quarks [34] is:

$$\mathcal{L}_{4q}^{\text{eff}} = \frac{G_F}{\sqrt{2}} \cos^2 \theta_C \sum_{a=1,2} \left( V_{\mu}^a A_{\mu}^a \right)^2 - \frac{G_F}{\sqrt{2}} \left( \cos 2\theta_W V_{\mu}^3 A_{\mu}^3 - 2 \sin^2 \theta_W I_{\mu} \right)^2 = \sqrt{2} G_F \left\{ \cos^2 \theta_C \sum_{a=1,2} V_{\mu}^a A_{\mu}^a + \cos 2\theta_W V_{\mu}^3 A_{\mu}^3 \right\} - 2\sin^2 \theta_W I_{\mu} A_{\mu}^3 + \cdots,$$

(1)

where we kept only the PV terms. Here $G_F = 1.16 \times 10^{-5} \text{ GeV}^{-2}$, $\cos^2 \theta_C = 0.946$ and we dropped Cabibbo suppressed terms. The currents are $V_{\mu}^a = \frac{1}{2} \bar{q} \gamma_{\mu} t^a q$, $A_{\mu}^a = \frac{1}{2} \bar{q} \gamma_{\mu} \gamma^5 t^a q$ and $I_{\mu} = \frac{1}{2} \bar{q} \gamma_\mu q$ respectively. Importantly, the factor $\sin^2 \theta_W$ accompanies the product of isoscalar and axial currents, which is the only source of $\Delta I = 1$ operators. $\Delta I = 0$ and $\Delta I = 2$ operators have pre-factors of $\mathcal{O}(1)$. Running from the $Z^0$-mass down to the strong scale $\Lambda_N \sim 1 \text{ GeV}$ does not significantly modify this hierarchy [35, 36].

Now we estimate the NN matrix elements of quark operators in Eq. (1) using the Hartree expansion for the nuclear Hamiltonian in the large-$N_c$ limit [26, 37]

$$H = N_c \sum_{s,t,u} \sum_{\nu} v_{stu} \left( \frac{S}{N_c} \right)^s \left( \frac{I}{N_c} \right)^t \left( \frac{G}{N_c} \right)^u.$$

(2)

The explicit factors of $1/N_c$ ensure that an $n$-body interaction scales as $1/N_c^{n-1}$ [38]. The coefficients are $\mathcal{O}(1)$ functions of the momenta. We take a quark basis for the operators in Eq. (1) using the Hartree expansion for the to the strong scale $\Lambda_N$

$$S^i = q^i \frac{\sigma^i}{2} q, \quad I^a = q^a \frac{\tau^a}{2} q, \quad G^{ia} = q^i \frac{\tau^a}{4} q,$$

(3)

$$U_{\nu}^{N_0} = N_c \left( U_{\nu}^3 (p^2) \left[ p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \right] + U_{\nu}^2 (p^2) \left[ p_- \cdot (\sigma_1 \times \sigma_2) (\tau_1 \times \tau_2) \right] \right) + U_{\nu}^4 (p^2) \left[ p_- \cdot (\sigma_1 \times \sigma_2) (\tau_1 + \tau_2)^2 \right] + U_{\nu}^5 (p^2) \left[ (p_+ \times p_-)^i p^i_{\nu \nu} \cdot (\sigma_1^i \sigma_2^i) \right] (\tau_1 \times \tau_2)^z.$$

(5)

At $\mathcal{O}(1/N_c)$ there are a number of additional $\Delta I = 0, 2$ operators that arise:

$$U_{\nu}^{N^{-1}_0} = N_c^{-1} \left( U_{\nu}^3 (p^2) \left[ p_- \cdot (\sigma_1 \times \sigma_2) \right] + U_{\nu}^2 (p^2) \left[ p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \right] + U_{\nu}^4 (p^2) \left[ p_+ \cdot (\sigma_1 \times \sigma_2) \right] \right) + U_{\nu}^5 (p^2) \left[ p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \right] + U_{\nu}^{10} (p^2) \left[ p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \right] + U_{\nu}^{11} (p^2) \left[ p_+ \cdot (\sigma_1 \times \sigma_2) \tau_1 \cdot \tau_2 \right].$$

(6)

while at $\mathcal{O}(1/N_c^2)$ corrections to the coefficient functions in Eq. (5) and additional $\Delta I = 1$ operators appear. Note that the $1/N_c$ expansion says very little about the coefficient functions $U_{\nu}^3-U_{\nu}^{11}$ and $U_{\nu}^{11}$; the only constraint on them is that they should be $\mathcal{O}(1)$.

Within each isospin sector the expansion is thus in $1/N_c^2$, as for the strong NN and NNN force. Since $\Delta I = 1$ operators are suppressed by $\sin^2 \theta_W$ and $1/N_c$, the two operators in Eq. (4) give the entire PV NN force up to corrections that are formally of relative order $1/N_c^2$, $\sin^2 \theta_W/N_c$. Below we will argue, though, that the numerical suppression is not quite the $\approx 10\%$ this implies.

An expansion in momenta would reduce Eqs. (4)–(6) to the five operators that describe the S-P transitions [11, 13, 39]. Here we do not take the low-energy limit as we want to compare with the full DDH potential. Fur-
thermore, at the 221 MeV energy of the TRIUMF $\bar{p}p$ experiment that we also seek to describe, an expansion in powers of momenta is not trustworthy.

We now compare our result to the PV NN potential of DDH. The relevant expressions can be found in [10] and [9]. The one-meson-exchange diagrams from the weak and strong meson-nucleon Hamiltonians given there yield the DDH potential as a set of operators, each of which is multiplied by one strong and one weak meson-nucleon-nucleon coupling. Up to $\mathcal{O}(N_c \sin^2 \theta_W)$ only one spin-flavor structure is produced by the $1/N_c$ analysis that does not appear in the DDH potential. It is the operator multiplied by the coefficient function $U^\lambda_1$ and corresponds to a tensor constructed from $L$ and $p_- \pi$ coupled to the rank-two spin tensor. This structure is not generated straightforwardly in the meson-exchange picture.

The rest of the DDH structures can each be matched to one structure in the LO or $\mathcal{O}(N^0_c \sin^2 \theta_W), \mathcal{O}(1/N_c)$ potentials. DDH made a prediction for the strength of these operators based on standard values for the strong meson-nucleon-nucleon couplings and estimates of the “best values” for the weak couplings. We will use these weak-coupling estimates as our point of comparison (but see Ref. [40]). In order to extract values for the weak couplings from our $1/N_c$ analysis, we recall the $N_c$-scaling rule of the strong couplings from Ref. [26]:

$$g_\omega \sim \sqrt{N_c}, \quad g_\rho \sim \frac{1}{\sqrt{N_c}}, \quad \xi_V \sim N_c, \quad \xi_S \sim \frac{1}{N_c}. \quad (7)$$

We count the pion’s coupling as $\bar{g}_\pi N_N \sim \sqrt{N_c}$. This, together with the Goldberger-Treiman relation, means that the pseudoscalar $\pi N N$ coupling which appears in the DDH potential, $g_{\pi NN} = \frac{m_N}{m_\pi} g_{\pi NN}$, scales as $N_c^{3/2}$. In a similar vein, we replaced DDH’s parameters $\chi_{V,S}$ by $m_{N_{V,S}/\Lambda_\chi}$ and $h^1_\rho$ by $m_N h^1_\rho / \Lambda_\chi$, so that there are no spurious factors of $N_c$ appearing in the coefficients of operators via the nucleon mass. $\Lambda_\chi \sim 1$ GeV suppresses higher dimensional operators and is independent of $N_c$.

We then extract the $N_c$ and $\sin^2 \theta_W$ scalings of the weak couplings in DDH potential as:

$$h^0_\rho \sim \sqrt{N_c}, \quad h^2_\rho \sim \sqrt{N_c}, \quad h^1_\rho \sim \sqrt{N_c}, \quad h^3_\rho \sim \sqrt{N_c}, \quad h^1_\rho \sim \frac{1}{\sqrt{N_c}}, \quad h^2_\rho \sim \frac{1}{\sqrt{N_c}}. \quad (8)$$

Since they arise from the $I_\mu A_{\mu\nu}$ product in the effective four-quark Lagrangian, the $\Delta I = 1$ couplings must all include a factor of $\sin^2 \theta_W$. The bounds on the scalings of $h^1_\rho, h^2_\rho, h^3_\rho$ follow from requiring that the large-$N_c$ scaling is not violated, while the scalings of $h^0_\rho, h^1_\rho$ are needed in order that the $U$’s in Eqs. (4,5,6) scale as $\mathcal{O}(N^0_c)$. Some of these results in Eq. (8) were previously derived in soliton models [29–31]. And Ref. [32] computed the large-$N_c$ scaling of $h^1_\rho$, but did not account for its $\sin^2 \theta_W$ suppression.

The two operators in Eq. (4) give the entire LO PV NN potential. When written in terms of the DDH couplings they are proportional to $g_\rho h^0_\rho, h^1_\rho$. Taking the product of all scalings in this expression verifies that the potential in Eq. (4) is $\mathcal{O}(N_c)$, but also shows that one of the factors of $N_c$ is associated with the factor of $m_N / \Lambda_\chi \sim 1$ that (implicitly) occurs in the DDH coupling $\chi_{V,S}$. This effectively demotes the LO piece of the PV NN potential to a numerical size typical of a $\mathcal{O}(N^0_c)$ contribution. We therefore conclude that the two operators in Eq. (4) determine the parity-violating NN force up to $\approx 30\%$ corrections.

Although the DDH ranges are large, the preferred values fall in the relatively narrow bands predicted by the $1/N_c$ hierarchy, except for $h^1_\rho$ and $h^1_\pi$, see Fig. 1. Notably, the two LO operators are associated with the largest couplings, $h^0_\rho, h^2_\rho$. The DDH best value for the coupling $h^1_\rho$ is also within $30\%$ of the natural value once the $\sin^2 \theta_W$ suppression is taken into account. The $\sin^2 \theta_W / N_c$ suppressed couplings include $h^1_\pi$. This is in contrast to DDH, who have a $h^1_\pi$ glaringly larger than the large-$N_c$ prediction. A much smaller $h^1_\pi$ is found in soliton models [29, 30], and appears to be borne out by experiment (see, e.g. Fig. 3 in Ref. [9]). Lastly, Ref. [41] used the quark model to argue that the coupling $h^1_\rho$ was small, and as a consequence it has been neglected in many subsequent analyses. In contrast, large-$N_c$ gives no reason that this coupling is any less important than, say, $h^1_\rho$: both generate the operator structure $(\sigma_1 + \sigma_2)(\tau_1 \times \tau_2)^2$ in Eq. (5) and consistency with the $N^0_c$ scaling of $U^\lambda_1$ only requires that at least one of $h^1_\rho, h^1_\pi$ saturates the bounds given in Eq. (8).

None of this, though, is a comparison at the level of observables. As already alluded to, there are many problems with the extraction of weak meson-nucleon-nucleon
couplings from data, e.g. extracted weak coupling constants depend on the strong coupling constants used [9]. Constraining the products of weak and strong couplings from experiment may be a better choice. Therefore we conclude our discussion by considering the dominant combinations $g_\rho h^0_\rho, g_\rho h^2_\rho, g_\omega h^0_\omega \sim \mathcal{O}(1)$ and $g_\omega h^1_\omega \sim \mathcal{O}(N_c \sin^2 \theta_W)$ asking what they predict for experiments. All four contribute to $pp$ scattering. The $\bar{p}p$ asymmetry has been measured at 15 [1], 45 [2], and 221 MeV [3]. In the main, the first two experiments constrain the PV-induced mixing between $^1S_0$- and $^3P_0$-waves, while the third constrains mixing between $^3P_2$- and $^1D_2$-waves. In plane-wave Born approximation the information can be parameterized by $A_{SP}$ and $A_{PD}$ [9, 42], the pertinent combinations of coupling constants governing these mixings. In the DDH approach they are:

$$A_{SP} \equiv g_\rho h^0_\rho (2 + \chi V) + g_\omega h^0_\omega (2 + \chi S),$$

$$A_{PD} \equiv g_\rho h^0_\rho \chi V + g_\omega h^0_\omega \chi S,$$

with $h^0_\rho$ the combination of $\Delta I = 0, 1, 2$ couplings relevant for $pp$ scattering \footnote{Note that two leading couplings $h^0_\rho$ and $h^2_\rho$ affect the $\bar{p}p$ asymmetry only via this combination. A lattice QCD calculation of the isovector piece of the PV NN interaction would help break this degeneracy.}. The data from Refs. [1–3] were analyzed in Ref. [42], resulting in the constraints on $A_{SP}$ and $A_{PD}$ shown in Fig. 2. (See also the recent $\chi$EFT analysis [20, 21].) While the variables for the ellipse are motivated using plane-wave Born approximation, the calculation is not done that way. It accounts for all initial- and final-state (strong) $pp$ interactions, via a CD-Bonn potential calculation of the corresponding wave functions [43].

To make a prediction for $A_{SP}$ and $A_{PD}$ we take $G_F f_\pi A_\chi \sim 1.0 \times 10^{-6}$ (with $f_\pi = 92.4$ MeV $\sim \sqrt{N_c}$) as the naturalness estimate for the LO weak couplings $h^{0,2}_\rho$. Assuming a natural value for $g_\omega \approx 4\pi$ [44], we determine other couplings by the $N_c, \sin^2 \theta_W$ scalings of Eqs. (7) and (8), yielding $\{-1.0, -0.077, -1.0, -0.33, -0.23\} \times 10^{-6}$ for the weak couplings \{h^0_\rho, h^1_\rho, h^2_\rho, h^0_\omega, h^1_\omega\} and \{12, 4.0, -0.33, 3.0\} for the strong couplings \{g_\omega, g_\rho, \xi_S, \xi_V\}. This is denoted by the red point in Fig. 2. All nine couplings should be assigned a 30% error, due to omitted terms in the $1/N_c$ expansion. Uncorrelated variation over this range produces the blue shaded area in the figure. The yellow shaded area is the result found from solely varying the five weak couplings. The prediction for $A_{SP}$ and $A_{PD}$ from large-$N_c$ and naturalness is thus consistent with the constraints extracted in Ref. [42] within the combined theoretical and experimental uncertainties. It shows no evidence of fine tuning. The black dot is obtained with DDH “best values” for the weak and strong couplings. Those values are consistent with large-$N_c$ and naturalness, but such consistency will not occur in observables where $h^1_\rho$ contributes.

The 1/$N_c$ expansion for hadronic matrix elements, superimposed on suppressions by factors of $\sin^2 \theta_W$ predicted by the Standard Model, provides a new benchmark for PV NN couplings. This approach not only estimates the couplings, it also gives plausible ranges for them based on $1/N_c$ scaling. The results for the only non-zero measurements of parity-violating effects in the NN system are consistent with data. It also naturally predicts a small $h^1_\rho$: $|h^1_\rho| \lesssim (0.8 \pm 0.3) \times 10^{-7}$ in agreement with the bound obtained from $^{18}$F experiments [45–50], $|h^1_\rho| \lesssim 1.3 \times 10^{-7}$. It is also consistent with the first lattice calculation of $h^1_\rho$ [51]. The 1/$N_c$ expansion thus explains the otherwise puzzling failure of pion effects to yet manifest themselves in hadronic parity-violation experiments. Finally, it also suggests a new $\Delta I = 1$ spin-flavor structure $\{U^5_{12}\}$ at $\mathcal{O}(N_c^0 \sin^2 \theta_W)$ should be included in analyses that examine the subleading piece of the NN force generated by the weak interaction.

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