Three Loop Cusp Anomalous Dimension in QCD

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The three-loop cusp anomalous dimension in QCD

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We present the full analytic result for the three-loop angle-dependent cusp anomalous dimension in QCD. With this result, infrared divergences of planar scattering processes with massive particles can be predicted to that order. Moreover, we define a closely related quantity in terms of an effective coupling defined by the light-like cusp anomalous dimension. We find evidence that this quantity is universal for any gauge theory, and use this observation to predict the non-planar n_f-dependent terms of the four-loop cusp anomalous dimension.

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Understanding the structure of soft and collinear divergences is of great theoretical interest in quantum field theory. It is also relevant for phenomenological applications such as the production of heavy particles at the LHC, where effects from soft gluon radiation need to be resummed in order to improve theoretical predictions.

It is well known that the infrared (or long-distance) asymptotics of scattering amplitudes is described by correlation functions of Wilson lines pointing along the momenta of the scattered particles [1, 2]. The latter satisfy evolution equations with the corresponding anomalous dimension being in general a matrix in color space. In the planar limit, this matrix is expressed in terms of the two-line cusp anomalous dimension [3]. The two-loop result for this fundamental quantity has been known for more than 25 years [4], see also ref. [5]. Here we report on the full result for the cusp anomalous dimension in QCD at three loops.

To compute the cusp anomalous dimension, we consider the vacuum expectation value of the Wilson line operator

$$W = \frac{1}{N}\langle 0|\,\mathrm{tr}\left[P\exp\left(i\oint_C dx\cdot A(x)\right)\right]|0\rangle\,, \qquad (1)$$

with $A_\mu(x) = A^a_\mu(x)T^a$ and T^a being the generators of the fundamental representation of the $SU(N)$ gauge group. Here the integration contour C is formed by two segments along directions v_1^μ and v_2^μ (with $v_1^2 = v_2^2 = 1$), with (Euclidean space) cusp angle ϕ,

$$\cos\phi = v_1\cdot v_2\,, \qquad (2)$$

cf. Fig. 1. Perturbative corrections to the Wilson loop (1) contain both ultraviolet (cusp) and infrared divergences.

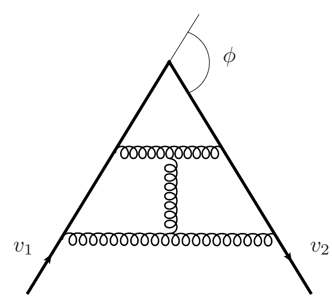

FIG. 1: Sample Feynman diagram producing a contribution to the three-loop cusp anomalous dimension in QCD. Thick lines denote two semi-infinite segments forming a cusp of angle ϕ, and wavy lines represent gauge fields.

We employ dimensional regularization with $D = 4-2\epsilon$ to regularize the former and introduce an infrared cut-off using the heavy quark effective theory (HQET) framework. The cusp anomalous dimension $\Gamma_{\rm cusp}$ is extracted as the residue at the simple pole $1/\epsilon$ in the corresponding renormalization factor.

It depends on the cusp angle ϕ, the strong coupling constant $\alpha_s = g^2_{\rm YM}/(4\pi)$, and on $SU(N)$ color factors. It is convenient to introduce the complex variable

$$x = e^{i\phi}\,, \qquad 2\cos\phi = x + 1/x\,. \qquad (3)$$

In Euclidean space $|x| = 1$, whereas for Minkowskian angles $\phi = i\theta$ (with θ real) the variable x can take arbitrary nonnegative values. Due to the symmetry $x\to 1/x$ of the definition (3), we can assume $0 < x < 1$ without loss of generality.

We chose to perform the calculation in momentum space. We generated all Feynman diagrams contributing to W up to three loops, in an arbitrary covariant gauge. This was done with the help of the computer programs QGRAF and FORM [6]. Using integration by parts relations [7], we found that a total of 71 master integrals was required. We derived differential equations for them in the complex variable x defined in (3). Switching to a basis of master integrals $\vec{f}(x,\epsilon)$ as suggested in ref. [8], we found the expected canonical form of the differential equations [26],

$$\partial_x \vec{f}(x,\epsilon) = \epsilon \left[\frac{a}{x} + \frac{b}{x+1} + \frac{c}{x-1} \right] \vec{f}(x,\epsilon)\,, \qquad (4)$$

with constant ($x-$ and $\epsilon-$independent) matrices a, b, c.

Eq. (4) has four regular singular points in x, namely $0, 1, -1$, and ∞. Thanks to the $x \to 1/x$ symmetry of the definition (3), only the first three are independent. They correspond, in turn, to the light-like limit (infinite Minkowski angle), to the zero angle limit, and to the antiparallel lines limit. Requiring that the integrals be nonsingular in the straight-line case $x = 1$ allowed us to fix all except one boundary conditions, and we obtained the remaining one from ref. [9].

It follows from (4) that the solution for $\vec{f}$ in the $\epsilon-$expansion can be written in terms of iterated integrals with integration kernels $dx/x, dx/(x-1), dx/(x+1)$. The latter integrals are known as harmonic polylogarithms $H_{n_1\ldots n_k}(x)$ [10]. The indices n_i can take values $0, 1, -1$, corresponding to the three integration kernels, respectively.

To express our results up to three loops, we introduce the following functions [27],

$$\begin{aligned}
A_1(x) =& \xi\, \frac{1}{2} H_1(y)\,, \qquad A_2(x) = \left[\frac{\pi^2}{3} + \frac{1}{2} H_{1,1}(y)\right] + \xi \left[-H_{0,1}(y) - \frac{1}{2} H_{1,1}(y)\right]\,,\\
A_3(x) =&\ \xi \left[-\frac{\pi^2}{6} H_1(y) - \frac{1}{4} H_{1,1,1}(y)\right] + \xi^2 \left[\frac{1}{2} H_{1,0,1}(y) + \frac{1}{4} H_{1,1,1}(y)\right]\,,\\
A_4(x) =& \left[-\frac{\pi^2}{6} H_{1,1}(y) - \frac{1}{4} H_{1,1,1,1}(y)\right] +\\
&+ \xi \left[\frac{\pi^2}{3} H_{0,1}(y) + \frac{\pi^2}{6} H_{1,1}(y) + 2H_{1,1,0,1}(y) + \frac{3}{2} H_{0,1,1,1}(y) + \frac{7}{4} H_{1,1,1,1}(y) + 3\zeta_3 H_1(y)\right]\\
&+ \xi^2 \left[-2H_{1,0,0,1}(y) - 2H_{0,1,0,1}(y) - 2H_{1,1,0,1}(y) - H_{1,0,1,1}(y) - H_{0,1,1,1}(y) - \frac{3}{2} H_{1,1,1,1}(y)\right]\,,\\
A_5(x) =& \xi \left[\frac{\pi^4}{12} H_1(y) + \frac{\pi^2}{4} H_{1,1,1}(y) + \frac{5}{8} H_{1,1,1,1,1}(y)\right] + \xi^2 \left[-\frac{\pi^2}{6} H_{1,0,1}(y) - \frac{\pi^2}{3} H_{0,1,1}(y) - \frac{\pi^2}{4} H_{1,1,1}(y)\right.\\
&\left. - H_{1,1,1,0,1}(y) - \frac{3}{4} H_{1,0,1,1,1}(y) - H_{0,1,1,1,1}(y) - \frac{11}{8} H_{1,1,1,1,1}(y) - \frac{3}{2}\zeta_3 H_{1,1}(y)\right]\\
&+ \xi^3 \left[H_{1,1,0,0,1}(y) + H_{1,0,1,0,1}(y) + H_{1,1,1,0,1}(y) + \frac{1}{2} H_{1,1,0,1,1}(y) + \frac{1}{2} H_{1,0,1,1,1}(y) + \frac{3}{4} H_{1,1,1,1,1}(y)\right]\,,\\
B_3(x) =& \left[-H_{1,0,1}(y) + \frac{1}{2} H_{0,1,1}(y) - \frac{1}{4} H_{1,1,1}(y)\right] + \xi \left[2H_{0,0,1}(y) + H_{1,0,1}(y) + H_{0,1,1}(y) + \frac{1}{4} H_{1,1,1}(y)\right]\,,\\
B_5(x) =& \frac{x}{1-x^2} \Big[-\frac{\pi^4}{60} H_{-1}(x) - \frac{\pi^4}{60} H_1(x) - 4H_{-1,0,-1,0,0}(x) + 4H_{-1,0,1,0,0}(x) - 4H_{1,0,-1,0,0}(x)\\
&+ 4H_{1,0,1,0,0}(x) + 4H_{-1,0,0,0,0}(x) + 4H_{1,0,0,0,0}(x) + 2\zeta_3 H_{-1,0}(x) + 2\zeta_3 H_{1,0}(x)\Big]\,,
\end{aligned} \qquad (5)$$

where $\xi = (1+x^2)/(1-x^2)$ and $y = 1-x^2$. The subscript of A indicates the (transcendental) weight of the functions. Moreover, we introduce the abbreviation $\tilde{A}_i = A_i(x) - A_i(1)$, and similarly for $\tilde{B}_i$.

Performing the three-loop computation, we reproduced the expected structure of UV divergences of W in the $\overline{\mathrm{MS}}$ scheme, as well as the HQET wavefunction renormalization [9], for arbitrary values of the gauge parameter in the covariant gauge. As yet another check, the dependence on the gauge parameter disappeared for the cusp anomalous dimension.

Let us write the expansion in the coupling constant as

$$\Gamma_{\rm cusp}(\alpha_s, x) = \sum_{k\geq 1} \left(\frac{\alpha_s}{\pi}\right)^k \Gamma^{(k)}_{\rm cusp}(x)\,. \tag{6}$$

The previously known one- and two-loop [4] results can be written as

$$\Gamma^{(1)}_{\rm cusp} = C_F\,\tilde{A}_1\,, \tag{7}$$

$$\begin{aligned}\Gamma^{(2)}_{\rm cusp} =& \frac{1}{2} C_F C_A \left[\tilde{A}_3 + \tilde{A}_2\right] \\ &+ \left(\frac{67}{36} C_F C_A - \frac{5}{9} C_F T_F n_f\right) \tilde{A}_1\,.\end{aligned} \tag{8}$$

At three loops we find

$$\begin{aligned}\Gamma^{(3)}_{\rm cusp} =& c_1\, C_F C_A^2 + c_2\, C_F (T_F n_f)^2 \\ &+ c_3\, C_F^2 T_F n_f + c_4\, C_F C_A T_F n_f\,,\end{aligned} \tag{9}$$

with

$$\begin{aligned}c_1 =& \frac{1}{4}\left[\tilde{A}_5 + \tilde{A}_4 + \tilde{B}_5 + \tilde{B}_3\right] \\ &+ \frac{67}{36}\tilde{A}_3 + \frac{29}{18}\tilde{A}_2 + \left(\frac{245}{96} + \frac{11}{24}\zeta_3\right)\tilde{A}_1\,,\end{aligned} \tag{10}$$

$$c_2 = -\frac{1}{27}\tilde{A}_1\,, \quad c_3 = \left(\zeta_3 - \frac{55}{48}\right)\tilde{A}_1\,, \tag{11}$$

$$c_4 = -\frac{5}{9}\left[\tilde{A}_3 + \tilde{A}_2\right] - \frac{1}{6}\left(7\zeta_3 + \frac{209}{36}\right)\tilde{A}_1\,. \tag{12}$$

Here $C_F = (N^2-1)/(2N)$ and $C_A = N$ are the quadratic Casimir operators of the $SU(N)$ gauge group in the fundamental and adjoint representation, respectively, n_f is the number of quark flavors, and $T_F = 1/2$.

The following comments are in order. The cusp anomalous dimension has a branch cut for x lying on the negative real axis. The results given in (9) are valid for $0 < x < 1$ and can be analytically continued to other regions according to this choice of branch cuts [28].

The leading n_f^2 term in (9) is in agreement with the known result [12]. We reported on the n_f-dependent part of (9) in [13]. The expression for the coefficient c_1 is new.

As a check of our result, we can consider Minkowskian angles and take the light-like limit, $x = e^{-\theta}$ with $\theta \to \infty$, of eq. (9), where one expects the behavior [14]

$$\Gamma_{\rm cusp}(\alpha_s, x) \overset{x\to 0}{=} K(\alpha_s)\log(1/x) + \mathcal{O}(x^0)\,, \tag{13}$$

with $K(\alpha_s)$ being the light-like cusp anomalous dimension. To three loops, it is given by [15]

$$\begin{aligned}K^{(1)} =& C_F\,,\\ K^{(2)} =& C_A C_F \left(\frac{67}{36} - \frac{\pi^2}{12}\right) - \frac{5}{9} n_f T_F C_F\,,\\ K^{(3)} =& C_A^2 C_F \left(\frac{245}{96} - \frac{67\pi^2}{216} + \frac{11\pi^4}{720} + \frac{11}{24}\zeta_3\right) \\ &+ C_A C_F n_f T_F \left(-\frac{209}{216} + \frac{5\pi^2}{54} - \frac{7}{6}\zeta_3\right) \\ &+ C_F^2 n_f T_F \left(\zeta_3 - \frac{55}{48}\right) - \frac{1}{27} C_F (n_f T_F)^2\,,\end{aligned} \tag{14}$$

where $K(\alpha_s) = \sum_{m\geq 1}(\alpha_s/\pi)^m K^{(m)}$. We found perfect agreement for all terms.

Finally, if the conformal symmetry of (massless) QCD were not broken, one would expect that the cusp anomalous dimension should be related in the antiparallel lines limit $\phi = \pi - \delta$, $\delta \to 0$, to the quark-antiquark potential [16] (at one loop order lower compared to $\Gamma_{\rm cusp}$). Starting from eq. (9) we indeed find perfect agreement with the result quoted in the second ref. of [17], up to conformal symmetry breaking terms proportional to the QCD β function.

Our result for the cusp anomalous dimension is valid in the $\overline{\rm MS}$ (dimensional regularisation) scheme. Going to the $\overline{\rm DR}$ (dimensional reduction) scheme amounts to a finite renormalisation of the coupling constant. We can introduce a quantity Ω which is the same in both schemes by switching from α_s to an "effective coupling" a,

$$\Omega(a, x) := \Gamma_{\rm cusp}(\alpha_s, x)\,, \qquad a := \pi/C_F K(\alpha_s)\,, \tag{15}$$

where $\Gamma_{\rm cusp}$ and $K(\alpha_s)$ are evaluated in the same scheme (and for the same theory). By construction, Ω has the universal limit

$$\Omega(a, x) \overset{x\to 0}{=} \frac{a}{\pi} C_F \log(1/x) + \mathcal{O}(x^0)\,, \tag{16}$$

as one can easily verify by comparing to eq. (13).

Using the results up to three loops given in eqs. (7), (8), (9) and (14), and expanding both sides of the first relation in (15) to third order in α_s, we find

$$\begin{aligned}\Omega(a,x) =& \frac{a}{\pi} C_F \tilde{A}_1 + \left(\frac{a}{\pi}\right)^2 \frac{C_A C_F}{2}\left[\tilde{A}_3 + \tilde{A}_2 + \frac{\pi^2}{6}\tilde{A}_1\right] \\ &+ \left(\frac{a}{\pi}\right)^3 \frac{C_F C_A^2}{4}\Big[\tilde{A}_5 + \tilde{A}_4 - \tilde{A}_2 + \tilde{B}_5 + \tilde{B}_3 \\ &+ \frac{\pi^2}{3}\tilde{A}_3 + \frac{\pi^2}{3}\tilde{A}_2 - \frac{\pi^4}{180}\tilde{A}_1\Big] + \mathcal{O}(a^4)\,.\end{aligned} \tag{17}$$

Remarkably, this quantity is independent of n_f to three loops! Comparing to eq. (15) we see that this means that e.g. all n_f dependent terms in $\Gamma^{(3)}_{\rm cusp}$ are generated from lower-loop terms, when expanding $K(\alpha_s)$ in α_s.

In Fig. 2 we plot the one-, two- and three-loop coefficients of Ω in an expansion of a/π, for Minkowskian

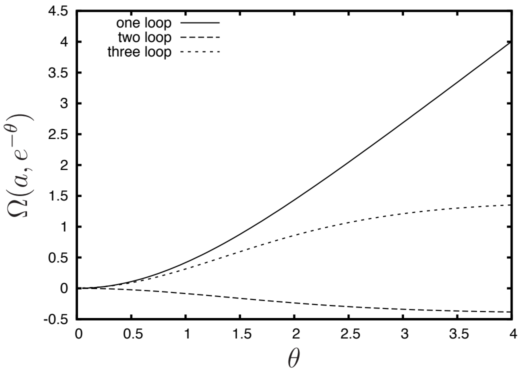

FIG. 2: θ dependence of the cusp anomalous dimension $\Omega(a, e^{-\theta})$ at one (solid), two (dashed), and three (dotted) loops.

angles θ, i.e. $x = e^{-\theta}$ for the range $\theta \in [0, 4]$, and with the number of colors set to $N = 3$. Note that the n_f-dependence in QCD can be obtained from eq. (15), and amounts to a rescaling of the coupling. At large θ, the one-loop contribution displays the linear behavior of eq. (16), while the two- and three-loop contributions go to a constant, as expected. In the small angle region, we have,

$$\Omega(a, e^{-\theta}) = C_F \left[\left(\frac{a}{\pi}\right) \frac{1}{3} + \left(\frac{a}{\pi}\right)^2 \frac{C_A}{4} \left(1 - \frac{\pi^2}{9}\right) + \left(\frac{a}{\pi}\right)^3 \frac{C_A^2}{12} \left(-\frac{5}{3} - \frac{\pi^2}{6} + \frac{\pi^4}{20} - \zeta_3\right) + \mathcal{O}(a^4) \right] \theta^2 + \mathcal{O}(\theta^4) . \tag{18}$$

The observed n_f-independence of $\Omega(a, x)$ leads us to conjecture that the latter quantity is universal in gauge theories, i.e. independent of the specific particle content of the theory. Assuming this conjecture leads to a number of non-trivial predictions.

First, let us recall the known value for K in $\mathcal{N} = 4$ super Yang-Mills (in the $\overline{\mathrm{DR}}$ scheme) [18],

$$K_{\mathcal{N}=4}(\alpha_s) = C_F \left[\left(\frac{\alpha_s}{\pi}\right) - \frac{\pi^2}{12} C_A \left(\frac{\alpha_s}{\pi}\right)^2 + \frac{11}{720} \pi^4 C_A^2 \left(\frac{\alpha_s}{\pi}\right)^3 + \mathcal{O}(\alpha_s^4) \right] . \tag{19}$$

Plugging this formula and the result for Ω given in eq. (17) into eq. (15) then gives the previously unknown three-loop result for the cusp anomalous dimension for the Wilson loop operator of eq. (1) in that theory,

$$\Gamma_{\mathcal{N}=4}(\alpha_s, x) = \frac{\alpha_s}{\pi} C_F \tilde{A}_1 + \frac{C_A C_F}{2} \left(\frac{\alpha_s}{\pi}\right)^2 \left[\tilde{A}_3 + \tilde{A}_2\right] + \frac{C_F C_A^2}{4} \left(\frac{\alpha_s}{\pi}\right)^3 \left[\tilde{A}_5 + \tilde{A}_4 - \tilde{A}_2 + \tilde{B}_5 + \tilde{B}_3\right] + \mathcal{O}(\alpha_s^4) . \tag{20}$$

The two-loop terms agree with ref. [13]. As a test of the three-loop prediction, we take the antiparallel lines limit and obtain

$$\Gamma_{\mathcal{N}=4}(\alpha_s, x) \overset{\delta \to 0}{=} -\frac{C_F \alpha_s}{\delta} \left\{ 1 - \left(\frac{\alpha_s}{\pi}\right) C_A + \left(\frac{\alpha_s}{\pi}\right)^2 C_A^2 \left[\frac{5}{4} + \frac{\pi^2}{4} - \frac{\pi^4}{64}\right] + \mathcal{O}(\alpha_s^3) \right\} + \mathcal{O}(\delta^0) , \tag{21}$$

as expected from the direct calculation of the quark antiquark potential [19].

Second, the conjecture of the n_f-independence of Ω can be used to predict the form of the non-planar n_f corrections that can first appear at four loops. The latter involve quartic Casimir operators of $SU(N)$, whose contribution we abbreviate by $C_4 = d_F^{abcd} d_F^{abcd} / N_A = (18 - 6N^2 + N^4)/(96N^2)$ (with N_A the number of the $SU(N)$ generators) [20]. Consider a term in $\Gamma_{\mathrm{cusp}}(\alpha_s, x)$ of the form $n_f (\alpha_s/\pi)^4 g(x) C_F C_4 / 64$, for some $g(x)$. Assuming that Ω defined in eq. (15) is independent of n_f then implies $g(x) = g_0 \tilde{A}_1$. Moreover, we can determine g_0 by comparing to the antiparallel lines limit. The expected relation to the known quark antiquark potential computed (numerically) in ref. [21] then yields $g_0 = -56.83(1)$.

In conclusion, we presented the full three-loop result for the cusp anomalous dimension in QCD. The latter allows to predict the infrared divergent part of planar scattering amplitudes of massive particles in QCD to that order. Moreover, our result can be applied to reduce theoretical uncertainties both in describing the scale dependence of heavy meson form factors [1, 2] and in computing cross sections of top-antitop pair production in electron-positron annihilation and in hadronic collisions [5, 22] (for a recent review see [23]).

We observed that the result has a surprisingly simple dependence on the number of quark flavors n_f, which led us to define a quantity Ω, independent of n_f to three loops. If the latter is the same in any gauge theory it could be studied using powerful integrability techniques that have been developed in $\mathcal{N} = 4$ super Yang-Mills, see [24] for more details.

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[26] The expert reader may find the explicit basis choice $\vec{f}$, as well as the matrices appearing in eq. (4), in ancillary files available in the online version of this paper. There we also provide the explicit result for $\vec{f}$ in the ϵ expansion, to the order required for this calculation.
[27] The function A_5 (and A_1 and A_3) appeared previously in the result for the cusp anomalous dimension in $\mathcal{N} = 4$ super Yang-Mills (SYM) for a locally supersymmetric Wilson line operator [11].
[28] Note that $\Gamma_{\rm cusp}(\alpha_s, -x)$ is expected to be related to $\Gamma_{\rm cusp}(\alpha_s, x)$ by crossing symmetry, i.e. the two functions should be equal, up to terms picked up by the analytic continuation. It turns out that all functions except B_5 contain rational factors that are symmetric under $x \to -x$, and therefore the harmonic polylogarithms in these expressions can be written with argument $1 - x^2$, and positive indices only. In contrast, the factor $x/(1-x^2)$ contained in B_5 is antisymmetric under $x \to -x$, and so is the transcendental function multiplying it (up to terms coming from branch cuts).