Classifying Directional Gaussian Entanglement, Einstein-Podolsky-Rosen Steering, and Discord

Q. Y. He, Q. H. Gong, and M. D. Reid

Phys. Rev. Lett. **114**, 060402 — Published 12 February 2015

DOI: 10.1103/PhysRevLett.114.060402

Classifying directional Gaussian entanglement, EPR-steering and discord

Q. Y. He[1,*], Q. H. Gong[1] and M. D. Reid[2,†]
[1] *State Key Laboratory of Mesoscopic Physics, School of Physics, Peking University, Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*
[2] *Centre for Quantum Atom Optics, Swinburne University of Technology, Melbourne, Australia and*
[*] *qiongyihe@pku.edu.cn*, [†] *mdreid@swin.edu.au*

Using Venn diagrams, we classify the different types of two-mode Gaussian continuous variable quantum correlation including directional entanglement and Einstein-Podolsky-Rosen (EPR) steering. We establish unified signatures for one- and two-way quantum steering, entanglement, and discord beyond entanglement in terms of an EPR-type variance. By focusing on Gaussian states, we link an optimised condition for entanglement based on an EPR variance to the Simon-Peres condition. This allows us to quantify the asymmetry of the Gaussian entanglement, and to relate the asymmetry to a directional quantum teleportation protocol where Alice and Bob possess asymmetrically noisy channels. Our analysis enables a determination of the type and direction of quantum correlation in a way that is easily measured in experiment. We also find that for symmetric states, when discord exceeds a certain threshold, the states are necessarily steerable.

The topic of quantum correlations has received much attention in modern physics [1, 2]. Entanglement is a distinctive feature of quantum correlations [3] and it is considered that all entangled states are useful for quantum information processing (QIP) [4]. Einstein-Podolsky-Rosen (EPR) correlations enable error-free predictions for the position − and the momentum − of one particle given some type of measurement on another. EPR correlations are especially useful [5]. As one example, the fidelity of the quantum teleportation (QT) of a coherent state is directly related to the strength of EPR correlation available in the quantum resource [6].

Very recently, there has been an appreciation of the importance of asymmetry and direction in quantum correlations [7–11]. Entanglement is a property shared between two parties, and measures of it have not been sensitive to differences between the quantum parties involved [12]. Yet, the original EPR argument was expressed asymmetrically between the two systems. The analysis by Schrodinger introduced the asymmetric term "steering" to describe the EPR idea of one party apparently adjusting the state of another by way of local measurements [13]. This aspect has been beautifully captured in two recent alternative definitions for quantum correlations: *quantum discord* [7, 8] and *EPR-steering* [9, 10]. Besides being of intrinsic fundamental interest, these asymmetrical nonlocalities are attracting a great deal of attention [14–17] for special tasks in quantum information processing (QIP) e.g. cloning of correlations [18], quantum metrology [19], quantum state merging [20], remote state preparation [21], one-sided device-independent quantum key distribution [22] and entanglement verification [23]. Surprisingly, for mixed states, quantum discord can emerge without entanglement and recent experiments [24, 25] have used discord to distribute entanglement using separable states only [26]. Despite the potential value of directional quantum correlation, relatively little is known about the quantitative link between discord and steering, and methodologies to simply characterize quantum states for their asymmetrical correlation [27].

Our aim in this Letter is to provide a method to distinguish the type and direction of correlation of a given state, with the aid of a parameter involving an EPR-type variance. By focusing on the subclass of bipartite quantum systems called Gaussian states [28] which have enabled experimental milestones such as deterministic QT [29], we find a condition for entanglement based on an EPR-type variance that is equivalent to the Simon's positive partial transpose (PPT) condition [30]. This allows us to quantify the asymmetry of the Gaussian entanglement, and to show this is directly related to the amplification of an optimal teleportation protocol. Asymmetrical Gaussian entanglement is not fully understood, yet the feasibility of using discord for quantum tasks involving asymmetrically noisy channels is already of experimental interest [17, 25, 31–33].

Here, we address this gap in knowledge by introducing a means to quantify and characterize directional entanglement, via a symmetry parameter $g_{sym}^{A|B}$, even where there is no EPR-steering. By further introducing an EPR-steering parameter, we provide a simple experimental signature to distinguish the states of different classes, whether EPR-steering, entanglement, or discord beyond entanglement. Moreover, we arrive at conditions to identify symmetric correlation, where the roles of Alice and Bob are interchangeable, and in this way arrive at an inequality sufficient to identify two-way EPR-steering. We show how one can produce a desired two-mode squeezed EPR state to fulfill a given directional quantum task, by adding asymmetric amounts of thermal noise to each sub-system. Finally, we find that in the parameter region where the states are highly discordant, they are also highly steerable.

We begin by considering two-party Gaussian systems where Alice and Bob make position/ momentum (or

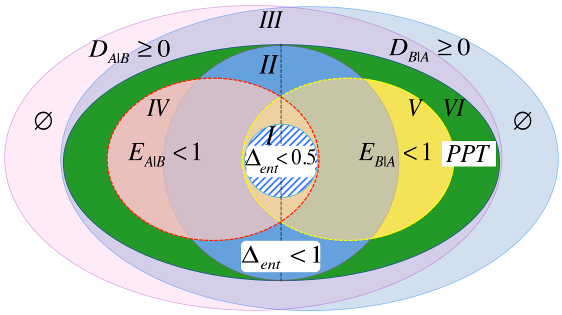

Figure 1. (Color online) The Venn diagram relations classifying the different types of quantum correlation for two-mode, two-party Gaussian states. The larger blue circle II contains states satisfying the Tan-Duan criterion for entanglement $\Delta_{ent} < 1$. The inner blue circle I contains states with the symmetric two-way EPR steering correlation given by $\Delta_{ent} < 0.5$. The set of all entangled states quantified by the Simon-PPT criterion $Ent_{PPT} < 0$ (equivalent to $Ent < 1$) are contained in the larger green ellipse VI. The smaller orange IV and yellow V ellipses enclose states that display the directional steering, given by the Reid-EPR paradox condition $E_{A|B} < 1$ and $E_{B|A} < 1$, respectively. Their intersection (colored yellow) is the set of two-way steerable states, which is a strict superset of the states in I. All two-way steerable states are a subset of the entangled states quantified by the Tan-Duan condition $\Delta_{ent} < 1$. One-way steering states are a strict subset of the PPT entangled states, and are strictly not contained in the Tan-Duan circle $\Delta_{ent} < 1$. The symmetric states with $g_{sym}^{A|B} = 1$ $(n = m)$ are depicted at the very centre of the diagram (dashed line). Those with $g_{sym}^{A|B} > 1$ $(n > m)$ are to the right of the centre line; those with $g_{sym}^{A|B} < 1$ $(n < m)$ to the left. The outer ellipse III contains the set of Gaussian states with non-zero quantum A and B discord.

field quadrature) measurements X_A, P_A and X_B, P_B respectively, on two well-separated modes denoted A and B. All Gaussian properties can be determined from the symplectic form of the covariance matrix (CM) defined as $C_{ij} = \langle (X_i X_j + X_j X_i) \rangle /2 - \langle X_i \rangle \langle X_j \rangle$ where $X \equiv (X_A, P_A, X_B, P_B)$ is the vector of the field quadratures [30, 34–36]:

$$C = \begin{pmatrix} n & 0 & c_1 & 0 \\ 0 & n & 0 & c_2 \\ c_1 & 0 & m & 0 \\ 0 & c_2 & 0 & m \end{pmatrix}. \tag{1}$$

The symplectic invariants are defined by $I_1 = n^2$, $I_2 = m^2$, $I_3 = c_1 c_2$, $I_4 \equiv det(C) = (nm - c_1^2)(nm - c_2^2)$, and the symplectic eigenvalues of the CM of a generic two-party Gaussian state are given as $d_\pm = \sqrt{\left(\Delta \pm \sqrt{\Delta^2 - 4det(C)}\right)/2}$ with $\Delta = I_1 + I_2 + 2I_3$ [31, 35, 36]. Our particular interest will be the subclass $c_1 = -c_2 = c$, which includes the major experimentally realized CV EPR resources [29] such as the two-mode squeezed thermal state (STS) and the two-mode EPR state with phase-insensitive losses. The covariance matrix elements in the STS case are $n = (n_A + n_B + 1)cosh(2r) + (n_A - n_B)$, $m = (n_A + n_B + 1)cosh(2r) - (n_A - n_B)$, $c = (n_A + n_B + 1)sinh(2r)$, where n_A, n_B are the average number of thermal photons for each system and r denotes the squeezing parameter. We stress however our classification is for all two-mode Gaussian systems, and does not restrict to this case.

Entanglement: In this paper, we normalize the vacuum fluctuations so that $\Delta X \Delta P \geq 1$. Simon's PPT criterion for *entanglement* is [30]

$$\begin{aligned} Ent_{PPT} = (nm - c_1^2)(nm - c_2^2) + 1 \\ - \left(n^2 + m^2 + 2|c_1 c_2|\right) < 0. \end{aligned} \tag{2}$$

This is a necessary and sufficient condition for the entanglement of two-mode, two-party Gaussian systems. Using the PPT criterion (2), we see that a two-mode STS is entangled iff r exceeds the threshold value r_{ent}: $cosh^2(r_{ent}) = (n_A + 1)(n_B + 1)/(n_A + n_B + 1)$ [15]. The complete set of PPT entangled states is depicted as contained within the green ellipse of Fig. 1. This set is not exhaustive for Gaussian states as seen by the values for Ent_{PPT} versus the thermal noises n_A and n_B shown in Fig. 2a [31, 32].

Entanglement can also be determined using an EPR-type variance [34, 37–40]. On considering the *weighted* difference variances $[\Delta(X_A - g_x X_B)]^2 = n - 2g_x c_1 + g_x^2 m$, $[\Delta(P_A + g_p P_B)]^2 = n + 2g_p c_2 + g_p^2 m$, it is straightforward to prove that entanglement between modes A and B is confirmed if [40]

$$\begin{aligned} Ent_g^{A|B} = & \Delta(X_A - g_x X_B)\Delta(P_A + g_p P_B)/(1 + g_x g_p) \\ & < 1. \end{aligned} \tag{3}$$

We use the notation $(\Delta x)^2 \equiv \langle x^2 \rangle - \langle x \rangle^2$. This condition does not assume Gaussian states, and is sufficient to confirm entanglement for all states. Here g_x, g_p are arbitrary real constants that can be optimally chosen to minimize the value of $Ent_g^{A|B}$. For the restricted subclass of Gaussian EPR resources where $c_1 = -c_2 = c$, there is symmetry between the X and P moments and a single $g = g_x = g_p$ is optimal. This choice of g that minimises $Ent_g^{A|B}$ is readily found to be $g = g_{sym}^{A|B}$ where

$$g_{sym}^{A|B} \equiv \left(n - m + \sqrt{(n-m)^2 + 4c^2}\right)/2c. \tag{4}$$

Manipulation shows that the EPR-type variance bound $Ent_g^{A|B} < 1$ and the Simon-Peres PPT bound $Ent_{PPT} < 0$ are equivalent, as shown in Fig. 2a. The optimal gains for $c_1 \neq c_2$ are given in the supplemental materials [41]. The minimum EPR variance is defined $Ent = Ent_g^{A|B}$ where $g = g_{sym}^{A|B}$, and its smallness gives a quantification of the Gaussian entanglement. Note that the entanglement between modes A and B can be also confirmed by $Ent_{g'}^{B|A} < 1$. The quantification of entanglement is symmetric with respect to A and B: That is,

$Ent_{g'}^{B|A} = Ent_g^{A|B}$ where $g' = g_{sym}^{B|A} = 1/g_{sym}^{B|A}$. This is to be expected: Entanglement is by definition a quantity shared between two systems, and its PPT threshold does not account for the directional properties associated with quantum correlation (see Fig. 2a).

Symmetric "Tan-Duan" entanglement: Where one has either pure states or else complete symmetry between the systems so that $c_1 = -c_2 = c$ and $n = m$, we find that the symmetry parameter is $g_{sym}^{A|B} = 1$. The PPT criterion (3) for entanglement then reduces to $Ent_{g=1}^{A|B} < 1$, which (for $c_1 = -c_2$) is equivalent to the Tan-Duan entanglement condition [34, 37]

$$\Delta_{ent} = \left[[\Delta(X_A - X_B)]^2 + [\Delta(P_A + P_B)]^2 \right]/4 < 1. \quad (5)$$

Resources with the property (5) are required for the CV quantum teleportation (QT) of a coherent state via the protocol of Braunstein and Kimble [6, 35]. These states are depicted as enclosed within the dark blue circle II of Fig. 1. This type of entanglement can also be created from asymmetric mixed states in special cases: For example, the STS squeezing threshold for Tan-Duan entanglement is $r > r_{QT} = ln\sqrt{n_A + n_B + 1}$.

Asymmetric entanglement: The directional correlation happens for asymmetric mixed states, which create the ellipses of Fig. 1 *outside* the blue circle II (symmetry parameter $g_{sym} \neq 1$). Sufficiently asymmetric systems (where $n \gg m$) arise for example when coupling massive objects to laser pulses, and require the full PPT entanglement test (outside the blue circle II, but within the green ellipse) as illustrated in Fig. 1 [49].

Discord: Quantum discord is by definition a measure of asymmetric quantum correlation between the two subsystems [7]. The "quantum A discord" that considers the conditional information for Alice's system A based on measurements on system B by Bob, has been derived for a Gaussian state by Giorda and Paris [31] and Adesso and Datta [32] as

$$D_{A|B} = f(m) - f(d_+) - f(d_-) + f(z), \quad (6)$$

where $z = (n + mn + c_1c_2)/(m + 1)$ and $f(x) = [(x + 1)/2]ln[(x + 1)/2] - [(x - 1)/2]ln[(x - 1)/2]$. With the exchanging $m \leftrightarrow n$ and hence $I_1 \leftrightarrow I_2$, we obtain the result for the B discord $D_{B|A}$. Quantum A discord is obtained for all bipartite Gaussian states that are not product states, although there are non-entangled states that have nonzero discord [31]. The quantum discord is the difference between two classically-equivalent definitions of conditional entropy [7, 8, 31]. Denoting the von Neumann entropy of the quantum state ρ by $S(\rho)$, the first $S(\rho_{A|B}) \sim f(d_+) + f(d_-) - f(m)$ arises from using the definition of mutual information based on the bipartite state ρ_{AB}. The second arises from quantization of the expressions for the conditional entropy: $\mathcal{H}(\rho_{A|B}) = \sum_k p_B(k)S(\rho_{A|k}) \sim f(\sqrt{z})$ where $p_B(k)$ is the probability of result k for a measurement at B, and

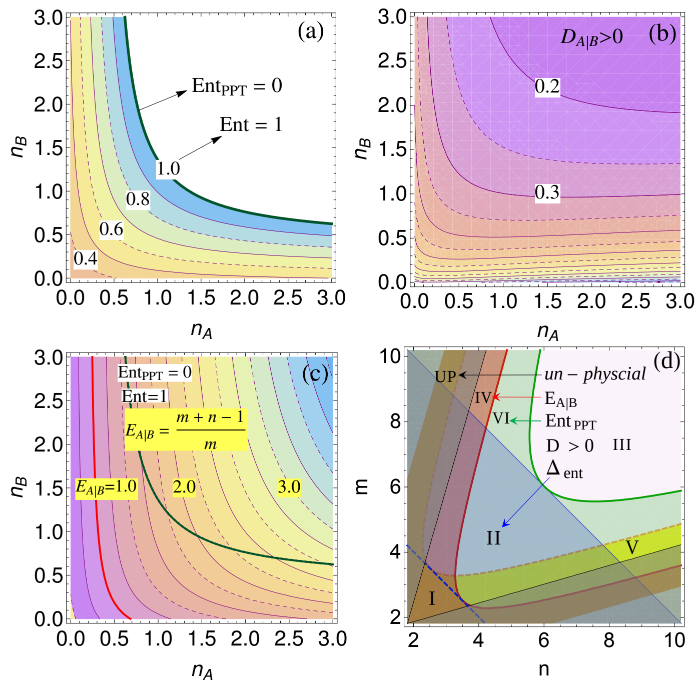

Figure 2. (Color online) Contour plots show the effect of asymmetric noises n_A and n_B on quantum correlation, for the two-mode STS with $r = 0.6$: (a) entanglement measured by Ent, (b) discord measured by $D_{A|B}$, and (c) the steering parameter $E_{A|B}$. In (c), the states can be used as quantum resources with EPR steering (below red curve), entanglement (below green curve), or discord beyond entanglement (above green curve) as explained in text. (d) shows the different regions defined in Fig. 1 certified by criteria of steering, entanglement, discord, and unphysical CMs (light gray area UP) versus n and m.

$S(\rho_{A|k}) = \sum_i p(i|k)S(\rho_{i|k})$ where $p(i|k)$ is the conditional probability of outcome i at A given the result k at B. The discord (6) is obtained by minimizing the mismatch over all Gaussian measurements. The terms in the quantum A discord $\mathcal{H}$ quantify the available information for the conditional state of A after measurement on B, and also reflect uncertainty in measurements of Alice when Bob's outcome k is known.

EPR Steering: Interestingly, this reminds us of the other asymmetric nonlocality, EPR steering from B to A ($B \to A$) [1, 9, 10], which is realized if the Reid-EPR paradox condition [38]

$$E_{A|B} \equiv \Delta_{inf} X_{A|B} \Delta_{inf} P_{A|B} < 1 \quad (7)$$

is satisfied [10]. The condition becomes necessary and sufficient for steering $B \to A$ for two-mode Gaussian systems [9]. Here $[\Delta_{inf} X_{A|B}]^2 = \sum_k p_B(k)[\Delta(X_A|k)]^2$ where $[\Delta(X_A|k)]^2$ is the variance of the conditional distribution for Alice's X_A conditional on the result k. The measurement at B is selected to minimize the quantity $[\Delta_{inf} X_{A|B}]^2$. The $[\Delta_{inf} P_{A|B}]^2 = \sum_{k'} p_B(k')[\Delta(P_A|k')]^2$ is defined similarly, for the momentum P_A. The states with the steering property (7) ($B \to A$) are depicted by the small orange-pink ellipse of Fig. 1. The EPR steering condition (7) also holds if we define $[\Delta_{inf} X_{A|B}]^2 = [\Delta(X_A - g'_x X_B)]^2$ and $[\Delta_{inf} P_{A|B}]^2 = [\Delta(P_A + g'_p P_B)]^2$ where $g'_{x,p}$ are real constants, adjusted to minimize the variances [39]. For Gaussian states the optimization ensures the equivalence of the two definitions [39]. The quantity $E_{A|B}$ is minimized to $E_{A|B} = \sqrt{(n - c_1^2/m)(n - c_2^2/m)}$ by the optimal factors $g'_x = c_1/m$, $g'_p = -c_2/m$ [35, 38], and its smallness gives a measure of the degree of the steering nonlocal correlations. Ideally, $E_{A|B}$ becomes zero in the limit of large r. As with discord, we obtain the result for the steering from A to B ($A \to B$) by interchanging parameters: $E_{B|A}$ is minimised to $E_{B|A} = \sqrt{(m - c_1^2/n)(m - c_2^2/n)}$ where $g'_x = c_1/n$, $g'_p = -c_2/n$ (small yellow ellipse of Fig. 1). Returning to the two-mode STS example, to satisfy steering $E_{A|B} < 1$ or $E_{B|A} < 1$ requires the squeezing r to exceed the threshold value given by $r_{A|B}$ and $r_{B|A}$ respectively, where $cosh^2(r_{A|B}) = (2n_A + 1)(n_B + 1)/(1 + n_B + n_A)$ or $cosh^2(r_{B|A}) = (n_A + 1)(2n_B + 1)/(1 + n_B + n_A)$.

Two-way EPR steering: Two-way steering is confirmed when both $E_{A|B} < 1$ and $E_{B|A} < 1$, as given by the yellow intersection of the two smaller ellipses of Fig. 1. The STS state with $r > \{r_{A|B}, r_{B|A}\}_{max}$ can be used to produce two-way steering, which is only possible when $|n_A - n_B| < 1/2$. A single criterion sufficient to certify two-way steering (without the assumption of Gaussian states) is $Ent_g^{A|B} < 0.5$ where $g = g_x = g_p = 1$, or $\Delta_{ent} < 0.5$. This is seen on noting that $Ent_{g=1}^{A|B} < 0.5$ becomes $E_{A|B} < 1$ and $E_{B|A} < 1$ when we take $g'_x = g'_p = 1$, and that algebraically $\Delta_{ent} \geq Ent_{g=1}^{A|B}$. The condition $\Delta_{ent} < 0.5$ is also the Grosshans and Grangier condition required of an EPR resource for the secure no-cloning teleportation (ST) of a coherent state [47]. For symmetric states, $g_{sym} = 1$, the condition reduces to $Ent < 0.5$. The states with this strongly symmetric two-way EPR steering correlation are depicted by the inner light blue circle I of Fig. 1. For STS states, this requires the squeezing r to exceed the threshold value $r > r_{ST} = ln\sqrt{2(n_A + n_B + 1)}$. We see that $\Delta_{ent} < 0.5$ is not a necessary condition for two-way steering (nor $Ent < 0.5$ in the symmetric case): Two-way steering is possible when $\{r_{A|B}, r_{B|A}\}_{max} < r < r_{ST}$, as shown by the yellow region not contained in I (Fig. 1), and for the symmetric pure two-mode squeezed state ($n_A = n_B = 0$) for all $r \neq 0$ (corresponding to all values of entanglement, including $Ent \to 1$).

Unified signature and application of asymmetric correlation: We note that the inequality (3) with $g = g_{sym}^{A|B}$ will determine the Gaussian entanglement for the subset of states where $c_1 = -c_2 = c$. The inequality (3) and (2) both then require $nm - c^2 + 1 - n - m < 0$. This can be written as a bound on the steering parameter:

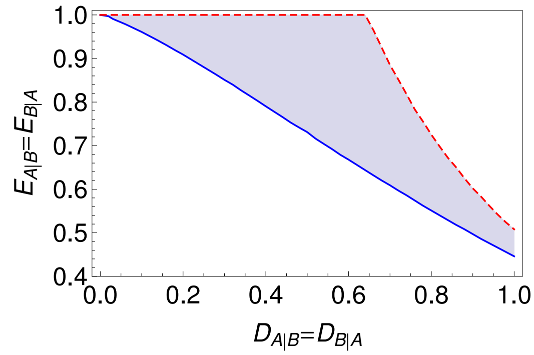

Figure 3. (Color online) The allowed Gaussian steerability (shaded region) versus discord for the symmetric Gaussian case restricting to $c_1 = -c_2 = c$ and $n = m$.

$$E_{A|B} \equiv E_{A|B}(g) < \frac{m + n - 1}{m}, \quad (8)$$

with factor $g = c/m$. This can be also written as $E_{B|A} \equiv E_{B|A}(g') < (m + n - 1)/n$ with the optimal gain factor $g' = c/n$. Hence, we establish a unified experimental measure of quantum correlation for this subset: EPR steering if $E_{A|B} < 1$ is satisfied (below the red curve in Fig. 2c); entanglement if $E_{A|B} < (m + n - 1)/m$ (below the green curve in Fig. 2c) and discord beyond entanglement if $E_{A|B} > (m + n - 1)/m$ (above the green curve in Fig. 2c). By a single steering measure $E_{A|B}$, one can quantify quantum correlation of a given Gaussian state. Note however that our classification uses criteria that are sufficient (but not necessary) to confirm entanglement and steering for an arbitrary quantum state.

Generally, the presence of asymmetric noises creates the possibility of asymmetric steering/ discord, making steering/ disturbance from A to B more difficult than that from B to A, as illustrated in Fig. 2 for the STS with $r = 0.6$. Entanglement is absent for $Ent_{PPT} \geq 0$, as shown in Fig. 2a. All regions show "quantum A discord", given by $D_{A|B} > 0$ (Fig. 2b) [31]. Thermal noises tend to suppress entanglement, for which the dependence on n_A and n_B is symmetric. However, the effect on the discord is more complex and asymmetrical. We can see that $D_{A|B}$ is maximized when most of thermal noise is placed on the unmeasured system A. The sensitivity of the steering parameter $E_{A|B}$ to the noises is asymmetrical and "one-way steering" (the states contained in the smallest left ellipse of Fig. 1 but exclusive of the right one) is evident. The value of $E_{A|B}$ is minimized (and steering increased) when most of thermal noise is placed on the system B, since $E_{A|B} < (m + n - 1)/m \sim 1$ when $m \gg n$. With the knowledge of these different sensitivities, one can prepare states with the desired type of correlation.

The behavior of discord is strongly related to steering. We note from Fig. 3, which is general for the subset of symmetric states, that in the parameter region where the states are highly discordant, they are also highly steerable. We also notice that a state is always steerable provided that the discord exceeds a certain threshold. This is consistent with the picture of EPR-type disturbances to Alice's system because of Bob's measurements [41].

Finally, we emphasize potential applications of asymmetric quantum correlation. We show in the Supplementary Materials [41] that the directional entangled states are useful as a resource for the quantum amplified teleportation of a coherent state $|\alpha\rangle \rightarrow |g_{sym}^{B|A}\alpha\rangle$ from Alice to Bob (if $g_{sym}^{B|A}\geq 1$), or from Bob to Alice (if $g_{sym}^{B|A} \leq 1$). In conclusion, our results offer a unified signature to examine the type and direction of correlation for a given quantum state, and suggest asymmetric correlations to be promising candidates for quantum tasks requiring a directional operation.

We acknowledge fruitful discussions with G. Adesso. This work was supported by the Australian Research Council via Discovery and DECRA grants and the National Natural Science Foundation of China under grants Nos. 11274025, 1112109 and 61475006. MDR thanks the support of the NSF Grant No. PHYS-1066293 and the hospitality of the Aspen Center for Physics.

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] J. S. Bell, Physics **1**, 195 (1965).
[3] E. Schrodinger, Naturwissenschaften **23**, 844 (1935).
[4] L. Masanes, Phys. Rev. Lett. **96**, 150501 (2006); M. Piani and J. Watrous, *ibid.* **102**, 250501 (2009).
[5] C. H. Bennett *et al.,* Phys. Rev. Lett. **70**, 1895 (1993).
[6] S. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998). T. C. Zhang *et al.,* Phys. Rev. A **67**, 033802 (2003).
[7] H. Ollivier and W. H. Zurek, Phys. Rev. Lett. **88**, 017901 (2001).
[8] H. Henderson and V. J. Vedral, Phys. A **34**, 6899 (2001).
[9] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007); S. J. Jones, H. M. Wiseman, and A. C. Doherty, Phys. Rev. A **76**, 052116 (2007).
[10] E. G. Cavalcanti *et al.,* Phys. Rev. A **80**, 032112 (2009).
[11] S. L. W. Midgley, A. J. Ferris, and M. K. Olsen, Phys. Rev. A **81**, 022101 (2010). V. Handchen *et al.,* Nature Photonics **6**, 598 (2012); J. Bowles *et al.*, arXiv:1402.3607 [quant-ph].
[12] R. F. Werner, Phys. Rev. A **40**, 4277 (1989).
[13] E. Schroedinger, Proc. Cambridge Philos. Soc. **31**, 555 (1935); Proc. Cambridge Philos. Soc. **32**, 446 (1936).
[14] A. Datta, A. Shaji, and C. M. Caves, Phys. Rev. Lett. **100**, 050502 (2008); A. Datta and S. Gharibian, Phys. Rev. A **79**, 042325 (2009).
[15] P. Marian, T. A. Marian, and H. Scutaru, Phys. Rev. A **68** 062309 (2003).
[16] D. J. Saunders *et al.,* Nature Physics **6**, 845 (2010). A. J. Bennett *et al.,* Phys. Rev. X **2**, 031003 (2012); D. Smith *et al.,* Nature Commun. **3**, 625 (2012); B. Wittmann *et al.,* New J. Phys. **14**, 053030 (2012).
[17] M. Gu *et al.,* Nature Phys. **8**, 671 (2012); L. S. Madsen, A. Berni, M. Lassen, and U. L. Andersen, Phys. Rev. Lett. **109**, 030402 (2012); R. Blandino *et al.,* Phys. Rev. Lett. **109**, 180402 (2012).
[18] M. Piani, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **100**, 090502 (2008); S. Luo and W. Sun, Phys. Rev. A **82**, 012338 (2010).
[19] K. Modi, H. Cable, M. Williamson, and V. Vedral, Phys. Rev. X **1**, 021022 (2011).
[20] M. Horodecki, J. Oppenheim, and A. Winter, Nature, **436**, 673 (2005); D. Cavalcanti *et al.,* Phys. Rev. A **83**, 032324 (2011); V. Madhok and A. Datta, Phys. Rev. A **83**, 032323 (2011).
[21] B. Dakić *et al.,* Nature Phys. **8**, 666 (2012).
[22] C. Branciard *et al.,* Phys. Rev. A **85**, 010301(R) (2012).
[23] B. Opanchuk, L. Arnaud, and M. Reid, Phys. Rev. A**89**, 062101 (2014).
[24] A. Fedrizzi *et al.*, Phys. Rev. Lett. **111**, 230504 (2013).
[25] C. Peuntinger *et al.*, Phys. Rev. Lett. **111**, 230506 (2013). C. Vollmer *et al.*, Phys. Rev. Lett. **111**, 230505 (2013).
[26] T. S. Cubitt *et al.*, Phys Rev. Lett. **91**, 037902 (2003). A. Streltsov, H. Kampermann and D. Bruss, Phys Rev Lett. **108**, 250501 (2012). T. Chuan *et al.*, Phys Rev. Lett. **109**, 070501 (2012). L. Mista, Jr. and N. Korolkova, Phys. Rev. A**80**, 032310.
[27] A. C. S. Costa, R. M. Angelo, and M. W. Beims, arXiv:1311.5702.
[28] C. Weedbrook *et al.*, Rev. Mod. Phys. **84**, 621 (2012).
[29] A. Furusawa *et al.,* Science **282**, 706 (1998).
[30] R. Simon, Phys. Rev. Lett. **84**, 2726 (2000).
[31] P. Giorda and M. Paris, Phys. Rev. Lett. **105**, 020503 (2010).
[32] G. Adesso and A. Datta, Phys. Rev. Lett. **105**, 030501 (2010).
[33] C. M. Caves and T. Ralph, Phys. Rev. A **87**, 012119 (2013).
[34] Lu-Ming Duan *et al.,* Phys. Rev. Lett. **84**, 2722 (2000).
[35] D. Buono *et al.,* Phys. Rev. A **86**, 042308 (2012).
[36] F. A. S. Barbosa *et al.,* Phys. Rev. A **84**, 052330 (2011).
[37] S. M. Tan, Phys. Rev. A **60**, 2752 (1999).
[38] M. D. Reid, Phys. Rev. A **40**, 913 (1989).
[39] M. D. Reid *et al.,* Rev. Mod. Phys. **81**, 1727 (2009).
[40] V. Giovannetti *et al.,* Phys. Rev. A **67**, 022320 (2003). S. Mancini *et al.*, Phys. Rev. Lett. **88**, 120401 (2002).
[41] See supplemental materials, which includes Refs. [42-48], for details of the optimal gains, steering-discord correlation, and the asymmetric teleportation protocol.
[42] T. C. Zhang *et al.,* Phys. Rev. A **67**, 033802 (2003).
[43] G. Chiribella and J. Xie, Phys. Rev. Lett. **110**, 213602 (2013).
[44] K. Hammerer *et al.,* Phys. Rev. Lett. **94**, 150503 (2005).
[45] G. Adesso and F. Illuminati, Phys. Rev. Lett. **95**, 150503 (2005).
[46] A. Mari and D. Vitali, Phys. Rev. A **78**, 062340 (2010).
[47] F. Grosshans and P. Grangier, Phys. Rev. A **64**, 010301(R) (2001).
[48] M. D. Reid and Q. Y. He, arXiv:1410.1041.
[49] Q. Y. He and M. D. Reid, Phys. Rev. A **88**, 052121 (2013). S. Kiesewetter *et al.*, Phys. Rev. A **90**, 043805 (2014).