Exact Adler Function in Supersymmetric QCD

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Exact Adler Function in Supersymmetric QCD

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The Adler function D is found *exactly* in supersymmetric QCD. Our exact formula relates $D(Q^2)$ to the anomalous dimension of the matter superfields $\gamma(\alpha_s(Q^2))$. *En rout* we prove another theorem: the absence of the so-called singlet contribution to D. While such singlet contributions are present in individual supergraphs, they cancel in the sum.

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FORMULATION OF THE PROBLEM AND RESULTS

The celebrated ratio

$$R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$

plays a special role in QCD-based phenomenology. For instance, it can be used for a precise determination of the gauge coupling α_s from accurate data on $e^+e^- \to$ hadrons in an appropriate energy range. It is also one of the key objects in various theoretical analyses in QCD, both in perturbation theory and beyond. In perturbation theory the ratio R is defined as a normalized cross section

$$\sigma(e^+e^- \to \text{quarks} + \text{gluons} \to \text{hadrons})\,.$$

It is directly reducible to the imaginary part of the photon polarization operator Π (see (4)),

$$R_{\rm QCD} = 12\pi\,{\rm Im}\,\Pi_{\rm QCD}\,. \qquad (1)$$

Alternatively, one can define $R_{\rm QCD}$ through a certain analytic continuation (see e.g. [1]) of the Adler function [2],

$$D(Q^2) \equiv -12\pi^2 \left(Q^2\, d/dQ^2\right)\Pi(Q^2)\,, \qquad (2)$$

In QCD the Adler function and the ratio R are calculated [3] up to $O(\alpha_s^4)$. Supersymmetric QCD (SQCD) is only a cousin of QCD since there is still no indication to the existence of supersymmetry in our world. Nevertheless, SQCD is known to be a unique theoretical laboratory in many aspects of gauge dynamics. The $O(\alpha_s)$ correction to R in SQCD was calculated in [4].

In this paper we will derive an exact relation between $D_{\rm SQCD}$ and the anomalous dimension γ of the matter superfield(s), valid to all orders in α_s,

$$D(Q^2) = \frac{3}{2} N \sum_f q_f^2 \left[1 - \gamma\left(\alpha_s(Q^2)\right)\right]\,, \qquad (3)$$

where f is the flavor index, and q_f is the corresponding electric charge (in units of e). Equation (3) assumes that all matter fields are in the fundamental representation of $SU(N)$, although their electric charges can be different. In calculating $\gamma\left(\alpha_s(Q^2)\right)$ one should remember that $\alpha_s(Q^2)$ runs according to the Novikov-Vainshtein-Shifman-Zakharov (NSVZ) β function [5, 6]. Our derivation of Eq. (3) refers to the renormalization group functions defined in terms of the bare coupling constant and uses the higher covariant derivative regularization [7].

From the practical side our result means, among other things, that for this renormalization prescription $O(\alpha_s^n)$ calculation of the Adler function $D(Q^2)$ in SQCD exactly reduces to a much simpler $O(\alpha_s^{n-1})$ calculation of the anomalous dimension γ.

The photon polarization operator $\Pi(Q^2)$ is defined as

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iqx}\,\langle T\left\{j_\mu(x)\, j_\nu(0)\right\}\rangle \\ \equiv \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi(Q^2)\,, \qquad (4)$$

where $Q^2 = -q^2$ and j_μ is the electric current. In the case of QCD $j^\mu = \sum_f q_f\, \bar\psi_f \gamma^\mu \psi^f$. In the supersymmetric case it is also necessary to take into account the quarks' superpartners, squarks.

$\Pi(Q^2)$ consists of two parts. The so-called singlet part of Π is determined by graphs with at least two matter loops, with photons attached to different loops, and is proportional to $\left(\sum_f q_f\right)^2$, see Fig. 1. In the nonsinglet part both external photon lines are attached to one and the same matter loop; therefore, the nonsinglet part is proportional to $\left(\sum_f q_f^2\right)$. Correspondingly,

$$D(\alpha_s) = \sum_f q_f^2\, D_1(\alpha_s) + \Big(\sum_f q_f\Big)^2 D_2(\alpha_s)\,. \qquad (5)$$

In deriving Eq. (3), *en rout* we explicitly establish the following theorem: it turns out that the singlet contribution, symbolically depicted in Fig. 1, vanishes, $D_2 \equiv 0$, once all relevant supergraphs are summed. Only the nonsinglet part D_1 survives. (This theorem was implicit in [8].) Hereafter, we will focus exclusively on D_1 keeping in mind that $D_2 \equiv 0$.

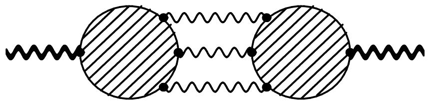

FIG. 1. An example of the singlet contribution to D. The shaded circles represent matter loops with all possible α_s corrections. Thin wavy lines denote the gauge superfield V while thick wavy lines denote external photons.

A general derivation of the formula (3), relating D and γ, which is conceptually similar to the NSVZ β function [5], is based on an examination of a certain "hybrid" β function in SQCD, to be explained below, and parallels the Shifman-Vainshtein nonrenormalization theorem [6].

When one deals with higher order corrections one must be careful since higher-order terms in the perturbative expansion are regularization and scheme dependent, generally speaking. Equation (3) implies supersymmetric regularization as well as renormalization scheme necessary for the NSVZ β function. The both elements were worked out in detail in [9] in the case of $\mathcal{N}=1$ SQED. The appropriate regularization is based on the higher derivative method [7] supplemented by the Pauli-Villars regularization for one-loop divergent (sub)diagrams [10].

Our general analysis of the Adler function is followed by a direct supergraph calculation (in terms of the bare quantities) and comparison of $D(Q^2)$ and γ which runs in parallel to that in [11]. This highly nontrivial calculation fully confirms Eq. (3) – a considerable technical achievement in itself.

THE MODEL

We will consider $\mathcal{N}=1$ SQCD with N colors and N_f flavors, assuming $N_f > N+1$. The latter condition is needed in order to avoid nonperturbative quantum deformations of the moduli space [12]. This will allow us to work at the origin of the moduli space.

Each flavor is described by two chiral superfields Φ^i and $\widetilde{\Phi}_i$ (i is the color index) in the fundamental (antifundamental) representations of $SU(N)$, respectively.

$$S = S_{\rm gauge} + S_{\rm matter} = \frac{1}{2g_0^2}\mathrm{Re}\,\mathrm{tr}\int d^4x\, d^2\theta\, W^2 + \frac{1}{4e_0^2}\mathrm{Re}\int d^4x\, d^2\theta\, \boldsymbol{W}^2 + \sum_{f=1}^{N_f}\frac{1}{4}\int d^4x\, d^4\theta\, \Big(\Phi_f^+ \times e^{2q_f\boldsymbol{V}+2V}\Phi_f + \widetilde{\Phi}_f^+ e^{-2q_f\boldsymbol{V}-2V^t}\widetilde{\Phi}_f\Big). \qquad (6)$$

The gauge sector consists of the dynamical $SU(N)$ part and an auxiliary $U(1)$ part. The $U(1)$ gauge superfield $\boldsymbol{V}$ (containing the photon field) is treated as an external field and is present only in the external lines, as in Fig. 1. The $SU(N)$ and $U(1)$ gauge couplings are denoted by g and e, respectively; the subscript 0 marks their bare (unrenormalized) values, i.e. the values at the ultraviolet cut-off. The $U(1)$ field strength tensor corresponding to $\boldsymbol{V}$ is $\boldsymbol{W}$,

$$\boldsymbol{W}_a = \frac{1}{4}\bar{D}^2 D_a \boldsymbol{V}\,, \qquad W_a \equiv \frac{1}{8}\bar{D}^2(e^{-2V}D_a e^{2V})\,. \qquad (7)$$

$\boldsymbol{V}$ is coupled to (s)quarks in the standard way, see the last line in (6). Our notation is similar to that in [13]. Moreover,

$$\int \theta^2 d^2\theta = 2\,, \qquad \int \theta^2\bar{\theta}^2 d^4\theta = 4\,,$$
$$V = V^A t^A\,, \qquad \mathrm{tr}(t^A t^B) = \delta^{AB}/2\,. \qquad (8)$$

We will discuss the β function for $\alpha = e^2/4\pi$, ignoring all orders in the electromagnetic coupling higher than the leading order, while all orders in $\alpha_s = g^2/4\pi$ will be taken into account. This β function (referred to above as hybrid) is defined and parametrized as follows:

$$\alpha_0^{-2}\beta = -\frac{d\left(\alpha(M_0)^{-1}\right)}{d\log M_0} \equiv \frac{1}{\pi}\left[b + b_1\frac{\alpha_{0s}}{\pi} + b_2\left(\frac{\alpha_{0s}}{\pi}\right)^2 + ...\right]. \qquad (9)$$

Here M_0 is the ultraviolet cut-off. In differentiating with respect to $\log M_0$ we keep the renormalized couplings α_s and the normalization point μ fixed. We will say that in this case β is defined in terms of the bare coupling constant. Alternatively, one can keep $\alpha_{0s}(M_0)$ fixed and differentiate over $\log\mu$. Then we obtain β defined in terms of the renormalized coupling constants. Generally speaking, these are two distinct schemes. The difference between these definitions are discussed in [9] in detail. Following [5, 6], we will use the former procedure.

In the leading order in α_s

$$b = \begin{cases} \frac{2N}{3}\,, & \text{one Dirac spinor} \\ \frac{N}{6}\,, & \text{one complex scalar} \\ N\,, & \text{one supersymmetric flavor} \end{cases}\,. \qquad (10)$$

Our task is to determine $b_{1,2,...}$.

β VERSUS D AND COMMENTS ON DERIVATION

The β function in (9) is obtained in a conventional way starting from the two-point Green function of the superfield $\boldsymbol{V}$. Due to the $U(1)$ background gauge invariance it is transversal,

$$\Delta\Gamma^{(2)} = -\frac{1}{16\pi}\int \frac{d^4q}{(2\pi)^4}\, d^4\theta\, \boldsymbol{V}(\theta,-q)\,\partial^2\Pi_{1/2}\boldsymbol{V}(\theta,q) \times \left(d^{-1}(\alpha_0,\alpha_{0s},M_0/Q) - \alpha_0^{-1}\right), \qquad (11)$$

where $\partial^2\Pi_{1/2} = -D^a\bar{D}^2D_a/8$ denotes the supersymmetric transversal projection operator. In our notation

$$d^{-1}(\alpha_0, \alpha_{0s}, M_0/Q) - \alpha_0^{-1} = 4\pi\Pi(\alpha_{0s}, M_0/Q). \quad (12)$$

Differentiating this equation with respect to $\log M_0$ and taking into account that d^{-1} (as a function of the renormalized coupling constants) is independent of M_0, we obtain

$$\alpha_0^{-2}\beta = 4\pi\frac{d}{d\log M_0}\Pi\Big(\alpha_{0s}(\alpha_s, M_0/\mu), M_0/Q\Big), \quad (13)$$

where the limit $Q/M_0 \to 0$ is assumed. Let us define the function $\alpha_{0s}(Q) \equiv \alpha_{0s}(\alpha_s, M_0/Q)$ by replacing $\mu \to Q$. Then Eq. (13) can be rewritten as a relation between the functions β and D:

$$\alpha_0^{-2}\beta = -4\pi\frac{d}{d\log Q}\Pi\Big(\alpha_{0s}(Q), M_0/Q\Big) = \frac{2}{3\pi}D(\alpha_{0s}). \quad (14)$$

Hence, the hybrid β and the Adler D functions coincide modulo the overall normalization. In this way we arrive at (3).

In the mid-1980s an exact relation for the NSVZ β function in SQCD was obtained [5],

$$\alpha_{0s}^{-2}\beta = -\frac{1}{2\pi}\left[3N - \sum_f T(R_f)(1-\gamma_f)\right]\left(1 - \frac{N\alpha_{0s}}{2\pi}\right)^{-1}. \quad (15)$$

Here γ_f's are the anomalous dimensions of the matter superfields in the representation R_f,

$$\gamma = -\frac{d\log Z}{d\log M_0} \quad (16)$$

and the coefficients $T(R_f)$ are related to the quadratic Casimir operators $C(R_f)$,

$$T(R) = C(R)\,\frac{\dim(R)}{N^2-1}\,. \quad (17)$$

A similar formula in supersymmetric quantum electrodynamics (SQED) with one electron was obtained in [8],

$$\alpha_0^{-2}\beta_{\rm SVZ} = \frac{1}{\pi}\left[1 - \gamma(\alpha_0)\right]. \quad (18)$$

Superficially, Eqs. (18) and (3) have the same factors in the square brackets; in fact, they are different: $\gamma(\alpha_{0s})$ is calculated in SQCD while $\gamma(\alpha_0)$ in SQED.

In all the above cases the arguments of [6, 14] tell us that only the first loop is "normal," the Wilsonean β function is exhausted by one loop. In other words, the coefficient $b = N$ in (9), (10) is "normal", while $b_{1,2,...}$ are due to the matter operator in the effective action. Naively it vanishes by virtue of the equations of motion, but the Konishi anomaly [15] converts it into the $U(1)$ gauge kinetic term, and, therefore, all higher orders come from γ's.

VERIFYING AT ORDER $O(\alpha_S)$

One can easily verify the match of the coefficient b_1. To this end let us compare our prediction (3) with the results of [4]

$$R_{\rm SQCD} = \frac{3}{2}N\sum_f q_f^2\left[1 + \frac{N^2-1}{2N}\frac{\alpha_s}{\pi} + O(\alpha_s^2)\right]. \quad (19)$$

Using the fact that [14]

$$\gamma(\alpha_s) = -\frac{N^2-1}{2N}\frac{\alpha_s}{\pi} + O(\alpha_s^2), \quad (20)$$

and that in the first order in α_s the Adler function D coincides with R we reproduce (3) to order $O(\alpha_s)$. The first coefficient in (20) is scheme-independent and, therefore, is the same for the renormalization group functions defined in terms of the bare or renormalized coupling constant. In the latter case the coefficient b_1 is scheme-independent while $b_{2,3,...}$ will depend on the renormalization scheme.

SCHEME DEPENDENCE IN HIGHER ORDERS

In direct perturbative derivation of Eq. (3) one should understand that all coefficients starting from b_2 are scheme dependent. To obtain the NSVZ β functions by using dimensional reduction [16] one has to ensure a specially tuned finite renormalization. It was verified that in three- and four-loop orders such a renormalization exists [17], to be referred to as the NSVZ scheme. The NSVZ scheme (in which the NSVZ relations are valid in all orders) was explicitly constructed in [9, 18] by using the higher derivative regularization.

As was already mentioned, in this paper we also calculate supergraphs using the higher derivatives method [7] supplemented by the Pauli-Villars regularization for one-loop divergent (sub)diagrams [10]. This procedure can be formulated in a manifestly supersymmetric way [19]. A possible version of the higher derivative term is as follows. We introduce superfield Ω related to the gauge superfield V as

$$e^{2V} \equiv e^{\Omega^+}e^{\Omega}. \quad (21)$$

The superfield Ω allows one to construct the gauge covariant supersymmetric derivatives

$$\nabla_a = e^{-\Omega^+}D_a e^{\Omega^+}; \qquad \nabla_{\dot a} = e^{\Omega}\bar{D}_{\dot a}e^{-\Omega}. \quad (22)$$

Using the superfield Ω and the above covariant derivatives we construct an appropriate higher derivative term,

$$S_\Lambda = \frac{1}{2g_0^2}\mathrm{tr}\int d^4x\, d^2\theta\,(e^{\Omega}W^a e^{-\Omega}) \times\left[R\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big) - 1\right](e^{\Omega}W_a e^{-\Omega}), \quad (23)$$

where Λ is a parameter with the dimension of mass, which plays the role of the ultraviolet cutoff (later we set $\Lambda = M_{\rm PV} = M_0$). The regulator R should obey the constraints $R(0) - 1 = 0$ and $R(x) \to \infty$ for $x \to \infty$. For example, one can choose $R(x) = 1 + x^n$. Needless to say, it is necessary to fix a gauge by adding the term $S_{\rm gf}$ to the action and introduce the corresponding ghosts with the action $S_{\rm ghosts}$. The one-loop divergences which remain after introducing the higher derivative term are removed by inserting the Pauli–Villars determinants into the generating functional [10].

The Adler function defined in terms of the bare coupling constant

$$D(\alpha_{0s}) \equiv -\frac{3\pi}{2}\frac{d}{d\log\Lambda}\alpha_0^{-1}(\alpha, \alpha_s, \Lambda/\mu), \qquad (24)$$

can be obtained from the expression (11) by making a substitution $\boldsymbol{V} \to \theta^4$:

$$\frac{1}{3\pi^2}\mathcal{V}_4 \cdot D(\alpha_{0s}) = \frac{d(\Delta\Gamma^{(2)})}{d\log\Lambda}\Big|_{\boldsymbol{V}=\theta^4}, \qquad (25)$$

where $\mathcal{V}_4 \to \infty$ is the space-time volume. (Certainly, it should be properly regularized, see [20] for details.)

By definition, the function (24) is scheme-independent for a fixed regularization [9, 18]. Here we argue that it is related to the anomalous dimension (16) (where M_0 should be replaced by Λ), which is also defined in terms of the bare coupling constant. The anomalous dimension defined by Eq. (16) also does not depend on the subtraction scheme for a fixed regularization.

In this paper we argued that, if the higher derivative regularization is used, the functions D in (24) and γ are related as

$$D(\alpha_{0s}) = \frac{3}{2}N\sum_f q_f^2\Big[1 - \gamma(\alpha_{0s})\Big] \qquad (26)$$

in all orders independently of the subtraction scheme. This statement is an analog of a similar statement proved for the β-function of $\mathcal{N}=1$ SQED in [11] and of $\mathcal{N}=1$ SQED with N_f flavors in [20].

The scheme dependent renormalization group functions are defined in terms of the renormalized coupling constant. In this case the derivatives with respect to $\log\mu$ are calculated at fixed values of the bare coupling constant. Then the exact expression for the D function is valid only in a certain subtraction scheme which seemingly can be constructed by imposing boundary conditions similar to the ones considered in [9, 21].

SUMMATION OF SUPERGRAPHS

To prove Eq. (26) we note that momentum integrals giving the function D are integrals of double total derivatives if the higher derivative method is used for regularization of supersymmetric theories. This implies that they have the same structure as integrals giving the NSVZ β functions in supersymmetric theories which was first noted in [22] and subsequently confirmed by other calculations [23]. Hence, one of the momentum integrals can be calculated analytically and the function D in the n-th loop can be written as an integral over $(n-1)$ loop momenta. This integral does not vanishes due to singularities of the integrand, which appear due to the identity

$$\left[\frac{\partial}{\partial Q_\mu}, \frac{Q^\mu}{Q^4}\right] = 2\pi^2\delta^4(Q), \qquad (27)$$

where Q^μ denotes the Euclidian momentum. The sum of the singularities gives the term with the anomalous dimension in the exact expression for the Adler function. Details of our calculation will be given elsewhere [24]. Here we outline only main stages.

We will use the notation

$$* \equiv \frac{1}{1 - (e^{2V}-1)\bar{D}^2D^2/16\partial^2},$$
$$\widetilde{*} = \frac{1}{1 - (e^{-2V^t}-1)\bar{D}^2D^2/16\partial^2}. \qquad (28)$$

These expressions encode sequences of vertices and propagators on the matter line (for Φ and $\widetilde{\Phi}$, respectively). Then the singlet contribution to the Adler function (after the substitution $\boldsymbol{V} \to \theta^4$) is proportional to

$$\frac{d}{d\log\Lambda}\Big\langle\Big[i\sum_f q_f\,\mathrm{Tr}\Big(\bar\theta^c(\gamma^\mu)_c{}^d\theta_d[x_\mu, \log(*) - \log(\widetilde{*})]\Big)$$
$$+(PV)\Big]^2\Big\rangle = 0, \qquad (29)$$

where (PV) denotes the contribution of the Pauli–Villars superfields. The commutator with x_μ corresponds to the integral over the total derivative in the momentum space, which vanishes because the integrand does not contain singularities. As a consequence, the singlet contribution is given by integrals of total derivatives and vanishes. (The Pauli–Villars contribution has a similar structure and also vanishes for the same reason.)

The remaining contribution is proportional to

$$i\frac{d}{d\log\Lambda}\sum_f q_f^2\mathrm{Tr}\Big\langle\theta^4\Big[x_\mu, \Big[x^\mu, \log(*) + \log(\widetilde{*})\Big]\Big]\Big\rangle + (PV)$$
$$-\text{terms with } \delta\text{-functions}. \qquad (30)$$

Again, this is an integral of a total derivative. However, it does not vanish due to singularities of the integrand. The contribution of these singularities can be found repeating the calculations made in [11]. It turns out that it is proportional to the anomalous dimension of the matter superfields and gives the second term in Eq. (26). Details of this proof will be published elsewhere [24].

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