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# Thermometry via light shifts in optical lattices

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For atoms or molecules in optical lattices, conventional thermometry methods are often unsuitable due to low particle numbers or a lack of cycling transitions. However, a differential spectroscopic light shift can map temperature onto the lineshape with a low sensitivity to trap anharmonicity. We study narrow molecular transitions to demonstrate precise frequency-based lattice thermometry, as well as carrier cooling. This approach should be applicable down to nanokelvin temperatures. We also discuss how the thermal light shift can affect the accuracy of optical lattice clocks.

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Neutral atoms in tight optical traps have proven to be indispensable for time and frequency metrology [1–5] and studies of many-body quantum phenomena [6]. Recently, molecules have been similarly trapped and used for precision measurements of two-body and many-body physics [7–9]. These state-of-the-art experiments require ultracold temperatures to maximize control over the external degrees of freedom. However, few reliable thermometry techniques exist aside from time-of-flight (TOF) expansion imaging [10]. This technique is unreliable with low particle numbers or a lack of cycling transitions, as is often the case for molecules [11]. Moreover, existing methods often break down at ultralow temperatures in the nK regime, and new thermometry tools are needed, particularly those not relying on complex modeling [12].

In this Letter, we show that if atoms or molecules can be trapped and probed in the Lamb-Dicke (LD) and resolved-sideband (RSB) regimes [13] of an optical lattice, the temperature  $T$  can be determined from the spectrum of the carrier line (“C” in Fig. 1) with a precision that is roughly an order of magnitude higher than for conventional sideband (SB) based thermometry [14]. This temperature determination requires only the polarizability ratio  $\alpha'/\alpha$  for the excited and ground states at the trap conditions, and the full-width-at-half-maximum (FWHM)  $\Gamma_C$  of the carrier lineshape. We show that

$$T_C \approx \frac{0.295\Gamma_C}{|\sqrt{\alpha'/\alpha} - 1|} \frac{h}{k_B}, \quad (1)$$

where  $h = 2\pi\hbar$  and  $k_B$  are the Planck and Boltzmann constants,  $\alpha', \alpha > 0$ , and  $T_C$  refers to carrier-based thermometry. A light shift measurement can directly yield  $\alpha'/\alpha = 1 - 2W_0/(M\lambda^2 f_x^2)$  where  $M$  is the particle mass,  $\lambda$  is the lattice wavelength,  $f_{x,r} \equiv \omega_{x,r}/(2\pi)$  are the trap frequencies along the axial and transverse directions defined relative to the probe beam, and  $W_0$  is the temperature-independent light shift as described below. Expression (1) is valid (i) for Boltzmann particle statistics in deep lattices, (ii) in the RSB regime  $\Gamma_0 < f_x$ , where  $\Gamma_0$  is the unbroadened carrier linewidth that would be measured in a ‘magic’ trap with  $\alpha'/\alpha = 1$ , (iii) if  $\Gamma_0 \ll \Gamma_C$ , and (iv) if the transverse SBs are not excited,

either by ensuring  $\Gamma_0 < f_r$  or by strictly controlling the probe-lattice alignment [14]. Importantly,  $T_C$  is insensitive to leading-order trap anharmonicities. This carrier thermometry should be valid at very low temperatures provided that  $\alpha'/\alpha$  can be tuned sufficiently far from unity. For example, if  $\Gamma_0 \sim 1$  Hz and  $\alpha'/\alpha \gtrsim 1.2$ , the sub-nK regime is accessible, provided a sufficient signal-to-noise ratio to detect the few particles in excited trap levels.

Carrier thermometry is applicable in the radio-frequency (RF), microwave, and optical regimes, which includes experiments with alkali and alkaline-earth metal atoms as well as simple molecules. We apply the technique to accurately characterize the temperature of molecules created via photoassociation in an optical lattice. We identify and quantify the heating mechanisms, and furthermore use the temperature dependence of the carrier for reducing the molecule temperature by a factor of 1.5.

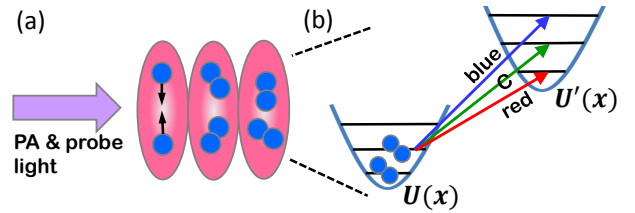


FIG. 1: (a)  $^{88}\text{Sr}$  atoms are trapped and cooled in a 1D optical lattice, and subsequently photoassociated on the narrow 689 nm intercombination line to create ultracold diatomic molecules. The molecules are then probed along the lattice axis in the LD and RSB regimes. (b) The carrier (C) and blue and red SB transitions between long-lived electronic states in an approximately harmonic trap are indicated.

To demonstrate carrier thermometry, we create  $\text{Sr}_2$  molecules via photoassociation (PA) [15] from laser-cooled  $^{88}\text{Sr}$  atoms in a one-dimensional (1D) optical lattice [16], as illustrated in Fig. 1. The lattice is tunable by 10’s of nm around a central wavelength of 910 nm, and has a beam waist  $w_0 \sim 25 \mu\text{m}$ . The probing is performed on an optical transition to a subradiant excited state of  $\text{Sr}_2$  [7], with  $\Gamma_0 < 200$  Hz limited by the

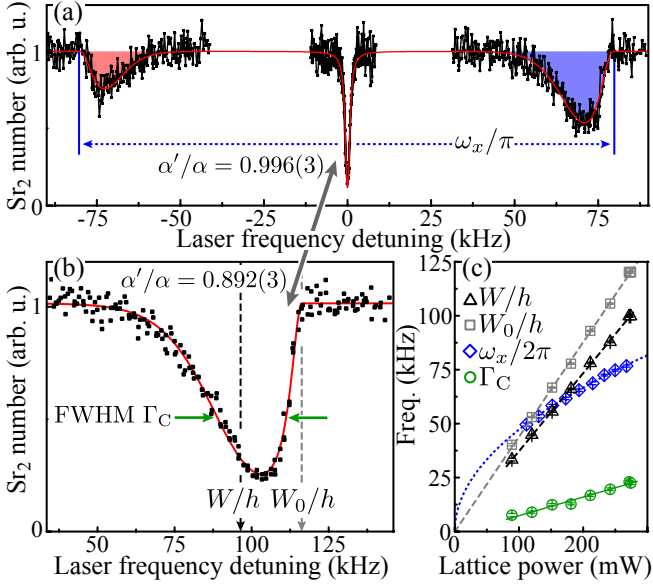


FIG. 2: (a) An optical spectrum of  $\text{Sr}_2$  molecules in a state-insensitive lattice. The central carrier transition and the first-order red and blue SBs are visible (the SB signals are enhanced via longer probing times). The axial trap frequency  $f_x \sim 80$  kHz is found from the SB spacing [26], while  $f_r \sim 0.6$  kHz. (b) The carrier lineshape in a state-sensitive lattice, including light-induced shift and broadening. The average light shift,  $W/h$ , and the temperature-independent contribution to the light shift,  $W_0/h$ , are indicated. (The natural logarithm of the data was taken prior to fitting, to account for linear probe absorption.) Zero detuning of the probe laser on the horizontal axes in (a,b) corresponds to zero lattice light shift. (c) The dependence of  $W/h$ ,  $W_0/h$ ,  $f_x$ , and  $\Gamma_C$  on the lattice light power.

laser linewidth. Figure 2(a) shows an optical spectrum taken along the lattice axis, including the narrow carrier transition and the first-order axial SBs. The trap frequency  $f_x$  is found from the SB spacing with a state-insensitive lattice ( $\alpha'/\alpha = 0.996(3)$ ) [7]. As  $\alpha'/\alpha$  is tuned via wavelength or polarization (the latter is possible via tensor shifts, and experimentally simpler), a differential light shift moves the line center, and an asymmetric  $T$ -dependent line broadening develops. The carrier spectrum in Fig. 2(b) corresponds to  $\alpha'/\alpha = 0.892(3)$ . The line is thermally broadened, with the FWHM  $\Gamma_C$  yielding  $T_C$  in Eq. (1).

The measured temperatures  $T_C$  are plotted in Fig. 3 for various optical lattice light powers, for  $\text{Sr}_2$  molecules in the two least-bound vibrational levels of the electronic ground state X ( $v = -1$  and  $v = -2$ ). Also shown are axial temperatures independently determined from the ratios of the blue to red SB areas [14],

$$T_x \approx \frac{\hbar\omega_x}{k_B} \left( \ln \frac{A_{\text{blue}}}{A_{\text{red}}} \right)^{-1}. \quad (2)$$

Equation (2) holds for Boltzmann statistics in deep lat-

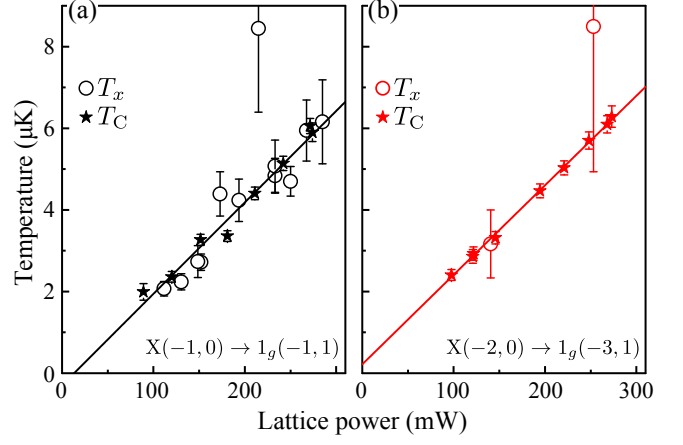


FIG. 3: Carrier thermometry of ultracold molecules (stars) is compared with an alternative technique that uses SB areas (circles), for (a)  $v = -1$  and (b)  $v = -2$  molecules. The notation  $(v, J)$  specifies the vibrational level and total angular momentum of the molecule. The molecular potentials X and  $1_g$  are separated by an optical frequency.

tices (in our experiment, trap depth  $U_0 \sim 10k_B T$ ). While the methods are in close agreement, carrier thermometry benefits from the larger signal-to-noise ratio for the carrier relative to the SBs. Its uncertainties are smaller by almost an order of magnitude, reaching the percent level. The smaller number of  $v = -2$  molecules in Fig. 3(b) hampers the SB but not the carrier measurements. Moreover, at colder temperatures than are reached here, the carrier method is expected to be superior due to the large relative uncertainty of determining the area of a vanishingly small red SB.

We apply carrier thermometry to systematically characterize the temperature of photoassociated molecules. Since PA is the optimal method to create a variety of dimers and can yield molecules in the absolute ground state [17, 18], any heating that occurs during PA must be understood and suppressed if such molecules are to reach a high phase-space density. PA into an electronically excited molecular level is usually followed by a spontaneous or stimulated decay to the electronic ground state. From kinetic considerations, the temperature of homonuclear dimers created in this way is expected to be nearly the same as for the precursor atoms [19]. However, for typical conditions we observe heating by roughly a factor of 2, as shown in Fig. 4(a). This heating is caused by incoherent photon scattering and can be partially suppressed by using shorter PA pulse durations, as shown in Fig. 4(a,b). Heating of the molecules by the optical lattice (Fig. 4(c)) [20] and with higher PA pulse powers is also present, but occurs at a much slower rate.

While SB cooling is an established technique for reducing axial temperatures [21], the method described here suggests a ‘carrier cooling’ procedure that can reduce the 3D temperature of a cold gas. Figure 4(d) illustrates the

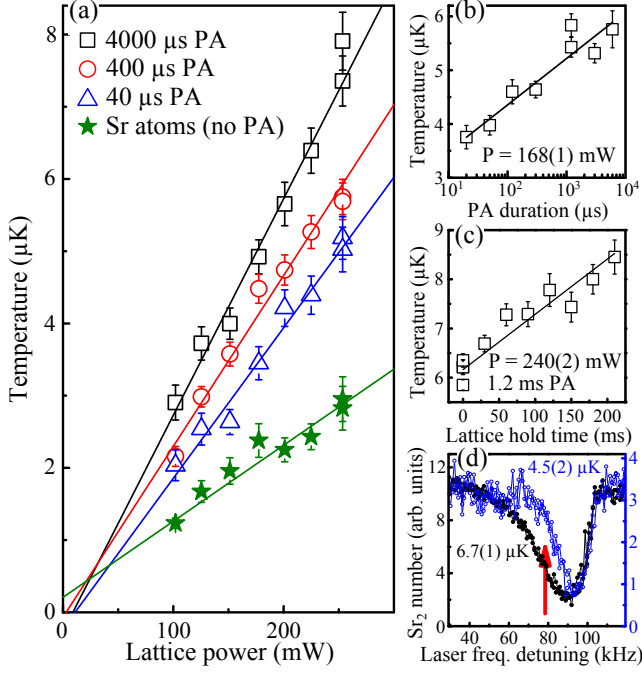


FIG. 4: (a) Molecule temperatures at various lattice light powers via carrier thermometry, along with the initial atom temperatures via TOF. (b) Molecule temperature versus the PA pulse duration. (c) Molecule temperature versus the lattice hold time. (d) ‘Carrier cooling’ of molecules in a weakly state-sensitive lattice.

reduction of the molecule temperature by a factor of 1.5 via weakly exciting the hotter molecules in the tail of the lineshape. In the case of open transitions such as for Sr<sub>2</sub>, the lowering of  $T$  is due to energy selection [22] and is not accompanied by a phase-space density increase.

The experiments in this work utilize a narrow optical transition, but carrier thermometry can be performed in any frequency regime. We have confirmed the results for a two-photon 1.3 GHz vibrational transition in ground-state Sr<sub>2</sub>. In microwave and RF regimes, the LD and RSB conditions for carrier thermometry may be satisfied in optical traps not utilizing a lattice configuration.

To obtain Eq. (1), we consider the case of a 1D lattice to reflect our setup as well as that of many lattice-clock type experiments, but note that the method is general and extends to 2D and 3D lattices. Near the center of a lattice well, the potential energy is nearly harmonic,

$$U(\mathbf{r}) \approx \frac{1}{2}M\omega_x^2x^2 + \frac{1}{2}M\omega_r^2(y^2 + z^2) - U_0, \quad (3)$$

where

$$\omega_x = (2\pi/\lambda)\sqrt{2U_0/M} \text{ and } \omega_r = (2/w_0)\sqrt{U_0/M}, \quad (4)$$

$U_0 = 4\alpha P/(\pi w_0^2 c \epsilon_0)$ ,  $P$  is the lattice light power,  $c$  is the speed of light, and  $\epsilon_0$  is the permittivity of vacuum. If  $\alpha'/\alpha \neq 1$ , the potential for the final internal state differs

from that for the initial state,  $U'(\mathbf{r}) \neq U(\mathbf{r})$ . The light shift (or differential AC Stark shift) of a carrier transition by the lattice is the difference in final and initial expectations,

$$W = \langle H' \rangle - \langle H \rangle. \quad (5)$$

Using (3), the energy expectation values are of the form

$$\langle H \rangle = \hbar\omega_x \langle n_x + 1/2 \rangle + \hbar\omega_r \langle n_r + 1 \rangle - U_0, \quad (6)$$

where  $H$  is the single-particle Hamiltonian,  $n_i$  are harmonic oscillator occupation numbers, and  $n_r \equiv n_y + n_z$ . Evaluating the net light shift (5) using (6), under the assumption that SBs are not excited at an appreciable rate such that  $n'_i = n_i$  [23], we find

$$W = W_0 + W_x + W_r = \left(1 - \frac{\alpha'}{\alpha}\right) U_0 + \left(\sqrt{\frac{\alpha'}{\alpha}} - 1\right) \times \left(\frac{\hbar\omega_x}{2} \coth\left[\frac{\hbar\omega_x}{2k_B T}\right] + \hbar\omega_r \coth\left[\frac{\hbar\omega_r}{2k_B T}\right]\right). \quad (7)$$

Note that if equipartition is valid ( $\hbar\omega_{x,r} \ll k_B T$ ), Eq. (7) simplifies to

$$W \approx (1 - \alpha'/\alpha) U_0 + 3 \left(\sqrt{\alpha'/\alpha} - 1\right) k_B T, \quad (8)$$

highlighting the non-thermal and thermal contributions to  $W$ . The carrier lineshape described below permits a clean extraction of  $W_0 = (1 - \alpha'/\alpha)U_0$ , and therefore of  $\alpha'/\alpha$ , if the trap depth or axial trap frequency (4) is known. The measured shifts  $W$  and  $W_0$  are marked in Fig. 2(b) and plotted versus the lattice light power in Fig. 2(c).

The  $T$ -dependent light shifts  $W_x$  and  $W_r$  in (7) cause asymmetric line broadening [24]. The carrier transition from the trap state  $|n_x n_r\rangle$  experiences a differential light shift

$$\delta E = \delta E_x + \delta E_r, \quad (9)$$

where the axial and radial contributions are

$$\delta E_x = (\sqrt{\alpha'/\alpha} - 1)\hbar\omega_x(n_x + 1/2), \quad (10)$$

$$\delta E_r = (\sqrt{\alpha'/\alpha} - 1)\hbar\omega_r(n_r + 1), \quad (11)$$

and  $\langle \delta E_{x,r} \rangle = W_{x,r}$ . The Boltzmann probability distribution for the discrete variable  $\delta E_x$  is

$$p_x(\delta E_x) = \frac{1}{Z_x} e^{-u(\delta E_x)}. \quad (12)$$

The partition function is  $Z_i = \frac{1}{2} \text{csch} \frac{\hbar\omega_i}{2k_B T}$ , and the dimensionless function

$$u(\delta E_i) = \frac{\delta E_i}{k_B T (\sqrt{\alpha'/\alpha} - 1)} \geq 0 \quad (13)$$

parametrizes the Boltzmann exponent with discrete step size of  $\Delta_i = \hbar\omega_i/(k_B T)$  for  $i = x, r$ . Similarly, for the radial shift,

$$p_r(\delta E_r) = \frac{1}{Z_r^2} \frac{1}{\Delta_r} u(\delta E_r) e^{-u(\delta E_r)}. \quad (14)$$

The discrete probability for the energy  $\delta E$  is then the convolution

$$p(\delta E) = \sum_{\{n_x, n_r\}_{\delta E}} p_x(n_x) p_r(n_r) \quad (15)$$

over the pairs of  $n_x$  and  $n_r$  satisfying  $\delta E(n_x, n_r) = \delta E$ .

If  $\Delta_x \lesssim 1$ , which is the case here, the discrete expression (15) may be simplified in the continuum limit of  $\Delta_i \rightarrow 0$ . Noting that the probability density

$$\bar{p}_i[u(\delta E_i)] = \lim_{\Delta_i \rightarrow 0} p_i(\delta E_i)/\Delta_i, \quad (16)$$

we obtain  $\bar{p}_x = e^{-u(\delta E_x)}$  and  $\bar{p}_r = u(\delta E_r) e^{-u(\delta E_r)}$ . The probability  $p(\delta E)$  then reduces to a Gamma distribution

$$\bar{p}[u(\delta E)] = \int_0^\infty \bar{p}_r(u - u_x) \bar{p}_x(u_x) du_x = \frac{1}{2} u^2 e^{-u}. \quad (17)$$

The probability density (17) directly yields the spectroscopic lineshape since carrier transition rates are nearly independent of  $n_i$  (this assumption may need to be modified if  $\alpha'/\alpha$  is far from unity). The lineshape has the form of a Boltzmann distribution in a 3D harmonic trap [25], as can be expected from (9-11). In a special case where only the ground axial state is occupied ( $\Delta_x \gtrsim 5$ ), the lineshape (17) is replaced by the 2D Boltzmann result  $\bar{p}[u(\delta E)] = u e^{-u}$ . We have assumed  $T_x = T_r \equiv T$  [14], but the analysis can be adapted to other situations, including non-Boltzmann distributions.

The FWHM of the function (17) is nearly 3.395, which together with (13) yields Eq. (1). Figure 2(b) shows a fit of lineshape (17) to a carrier spectrum, with its FWHM  $\Gamma_C$  directly yielding temperatures in Figs. 3 and 4. Note that the width of the carrier is much smaller than its light shift, hence it was necessary to stabilize  $P$ . This was done with minimal error from interference of the forward and retroreflected lattice beams by using a pellicle beamsplitter to sample the forward beam produced by an optical fiber with an angled output face.

It is a property of harmonic oscillator eigenstates that the dominant anharmonic corrections (proportional to  $x^4$ ,  $x^2 r^2$ ,  $r^4$ ) shift their energy by an amount that is independent of trap depth [26]. Hence Eqs. (17) and (1) are largely unaffected by anharmonic corrections. However, these corrections allow a determination of temperature from the shape of first-order axial SBs in a 1D lattice, using spectra with  $\alpha'/\alpha = 1$  as in Fig. 2(a) [14]. As for carriers, this approach yields 3D temperatures (versus 1D temperatures for Eq. (2)), where sensitivity to  $T$

now arises from the anharmonicity of the lattice trap. Introducing leading-order corrections to the harmonic approximation of a sinusoidal potential, and adapting the approach used to derive the carrier line shape [26], we find the temperature in a state-insensitive lattice

$$T_{\text{SB}} \approx 0.484 M \lambda^2 f_x \Gamma_{\text{SB}} / k_B, \quad (18)$$

where  $\Gamma_{\text{SB}}$  is the FWHM of the SB lineshape given by  $\bar{p}[v(\delta E)] = (1/6) v^3 e^{-v}$ , and  $v$  is a function similar to  $u$  [26]. Using this approach on the data of Fig. 3(a) yields  $T_{\text{SB}}$  that is too high by  $\sim 1 \mu\text{K}$  compared to  $T_C$  and  $T_x$ . There are several reasons for Eq. (18) to be less reliable than Eqs. (1) and (2). First, SBs are more sensitive to distortion by other broadening mechanisms than the carrier, since there is no tunability of  $\Gamma_{\text{SB}}$ , unlike for  $\Gamma_C$ . Additionally, any axial displacement from the Gaussian lattice beam waist produces new leading-order anharmonic corrections to Eqs. (3) and (18) that could strongly affect the SB result. More generally,  $T_C$  in Eq. (1) depends only on the polarizability ratio  $\alpha'/\alpha$ , a fundamental property of the atom or molecule that can be measured with a high accuracy. In contrast,  $T_{\text{SB}}$  depends on  $f_x$  that varies slightly across the sample.

Note that the thermal contribution  $(W - W_0)/h$  in Fig. 2(c) and Eqs. (7,8) is a significant fraction of the lattice light shift. This could affect optical lattice clocks [2–5] if the atom temperature versus trap depth (Fig. 4(a), stars) does not linearly extrapolate to exactly zero at  $P = 0$  [27, 28]. For example, if it extrapolates to just  $\pm 0.1 \mu\text{K}$ , then for  $\alpha'/\alpha \sim 1 \pm 3 \times 10^{-7}$  as in Ref. [4], the residual thermal line pulling from (8) is  $\sim 3 \times 10^{-18}$ , comparable to the total uncertainty budget. Furthermore, for typical clock experiment conditions, the full clock shift (7) must be used, where  $W$  is not linear in  $T$ . This non-linearity leads to an effective offset  $|T(P = 0)| \sim 0.1 \mu\text{K}$  in an experiment with our parameters, again leading to line pulling. Counterintuitively, if the temperature is kept fixed at all lattice depths, this thermal pulling is even more problematic. In general, the variation of temperature with lattice depth depends on the cooling and trapping procedure.

In conclusion, we have shown that narrow spectral lines of atoms or molecules tightly trapped in optical lattices allow highly precise 3D temperature determinations, and are not limited by low particle numbers, a lack of cycling transitions, or ultralow temperatures. The method is purely frequency-based, requiring only measurements of the carrier linewidth, light shift, and axial trap frequency, and is mostly immune to trap anharmonicities. We experimentally demonstrate complete control over molecular external and internal degrees of freedom in the LD and RSB regimes of a weakly state-sensitive optical lattice, use ‘carrier cooling’ to reduce the temperature of the ultracold molecules, and identify the significant heating processes of photoassociated molecules. Furthermore,

the result (1) can be inverted to accurately predict light-shift-induced thermal dephasing.

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