Measurement of the Single Top Quark Production Cross Section and $|V_{tb}|$ in Events with One Charged Lepton, Large Missing Transverse Energy, and Jets at CDF
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We report a measurement of single top quark production in proton-antiproton collisions at a center-of-mass energy of $\sqrt{s} = 1.96$ TeV using a data set corresponding to 7.5 fb$^{-1}$ of integrated luminosity collected by the Collider Detector at Fermilab. We select events consistent with the single top quark decay process $t \to Wb \to \ell b$ by requiring the presence of an electron or muon, a large imbalance of transverse momentum indicating the presence of a neutrino, and two or three jets including at least one originating from a bottom quark. An artificial neural network is used to discriminate the signal from backgrounds. We measure a single top quark production cross section of 3.04$^{+0.57}_{-0.53}$ pb and set a lower limit on the magnitude of the coupling between the top quark and bottom quark $|V_{tb}| > 0.78$ at the 95% credibility level.

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In the standard model (SM) of fundamental interactions, top quarks are produced in hadron collisions primarily as top-antitop ($tt$) pairs via the strong interaction. The top quark was first observed in this production mode in 1995 [1]. The top quark is also produced singly via weak charged-current interactions. At the Fermilab Tevatron proton-antiproton ($pp$) collider, single top quark production proceeds via the exchange of a virtual $W$ boson in the $t$ channel, via the decay of an intermediate $W$ boson in the $s$ channel, or in association with a $W$ boson (Wt) [2]. The respective SM production cross sections at the Tevatron, calculated at approximate next-to-next-to-leading-order (NNLO) accuracy in the strong coupling $\alpha_s$, are $\sigma_t \approx 2.10$ pb [3], $\sigma_s \approx 1.06$ pb [4], and $\sigma_{Wt} \approx 0.25$ pb [5] for a top quark mass of 172.5 GeV/$c^2$.

The measurement of the single top quark production cross section provides a test of the SM via a direct determination of the magnitude of the Cabibbo-Kobayashi-Maskawa (CKM) [6] matrix element $|V_{tb}|$, as the cross section is proportional to $|V_{tb}|^2$. The strength of the coupling $|V_{tb}|$ governs the decay rate of the top quark and its decay width into $Wb$. As this measurement assumes only that $|V_{tb}|^2 \gg |V_{ts}|^2 + |V_{td}|^2$ and does not rely on an assumption about the unitarity of the CKM matrix, it can constrain various extensions of the SM, namely models with fourth-generation quarks, models with flavor-changing neutral currents, and other phenomena not predicted by the SM [7].

Single top quark production in the combined $s+t$ channels was first observed independently by the CDF and D0 experiments in 2009 [8, 9]. The D0 collaboration updated its measurement in 2011 [10] and reported the observation of single top quark production in the $t$ channel [11]. The ATLAS and CMS experiments at the Large Hadron Collider (LHC) reported measurements of single top quark production in the $t$ channel in 2012 [12]. More recently, these experiments presented evidence of single top quark production via the Wt process [13], and CMS recently reported the observation of single top quark production via this process [14]. In addition, the CDF and D0 experiments separately reported evidence for $s$-channel production [15] and jointly reported the observation of $s$-channel single top quark production in 2014 [16]. The $s$-channel process is difficult to observe at the LHC due to the small signal-to-background ratio.

In this Letter we report precise measurements of the single top quark production cross section for (i) the sum of the $s$-channel, $t$-channel, and Wt processes, (ii) the $s$-channel process alone, and (iii) the sum of the $t$-channel and Wt processes, using more than twice the data of the previous CDF measurement [8, 17]. Using the measured single top quark cross section for the sum of $s$-channel, $t$-channel, and Wt processes, we also set a lower limit on the coupling $|V_{tb}|$. The data sample was collected at the Tevatron at a center-of-mass energy of $\sqrt{s} = 1.96$ TeV. The data sample corresponds to an integrated luminosity of 7.5 fb$^{-1}$ collected with the CDF II detector, which includes a solenoidal magnetic spectrometer surrounded by projective-geometry sampling calorimeters and muon detectors [18].

Since the magnitude of the top-bottom quark coupling is much larger than that of the top-down and top-strange quark couplings, we assume that every top quark decays into a $W$ boson and a bottom ($b$) quark. We identify sin-
gle top quark candidates by searching for the decay of a W boson to a neutrino and either an electron (e) or muon (µ). Candidate events are required to have an e or µ with large transverse momentum $p_T$ [19], a large imbalance in the event’s total transverse momentum (missing energy) $E_T$ [20] indicating a neutrino, and two or three hadronic jets.

Events are collected by three sequential levels of online selection requirements (triggers). We include events collected by high-$p_T$ lepton triggers, where the candidate e (µ) has $E_T (p_T) > 20$ GeV (18 GeV/c) and pseudorapidity $|\eta| < 1.0$ [19]. We also utilize novel triggers that require either $E_T > 35$ GeV plus two jets or $E_T > 45$ GeV, which increase the acceptance by adding new types of identified muon candidates [17]. Based on the type of lepton identified, events are grouped into two mutually exclusive categories called the tight lepton category and the extended muon category.

The final event selection requires a single isolated charged lepton with $|\eta| < 1.6$ and $p_T > 20$ GeV/c, consistent with the leptonic decay of a W boson. After correcting $E_T$ for the presence of jets and muons in the event, we require $E_T > 25$ GeV to reduce the background from multijet events that do not contain a W boson, referred to as the non-W background. Jets are reconstructed using a fixed-cone algorithm [21] with radius $\Delta R = 0.4$ in $\eta-\phi$ space [22]. We select events with either two or three jets having $E_T > 20$ GeV and $|\eta| < 2.8$. In order to improve the separation of signal from background, at least one of the jets must be identified as originating from a $b$ quark (“$b$-tagged”) using the secvtx algorithm [23].

Backgrounds that mimic the single top quark signal originate from events in which a W boson is produced in association with one or more heavy-flavor jets ($W + HF$), events with light-flavor jets that are mistakenly $b$-tagged ($W + LF$), multijet (non-W) events, $tt$ events, diboson ($WW$, $WZ$, $ZZ$) events, and events with a Z boson and jets. In addition to the $E_T$ requirement, we further reduce the non-W background by using a dedicated selection that exploits the W boson transverse mass $M_T^W$ [24] and the missing transverse energy significance [17]. Events with a reconstructed muon in the tight lepton category are required to have $M_T^W > 10$ GeV/$c^2$, while the remaining muon events and events triggered by an electron must have $M_T^W > 20$ GeV/$c^2$.

Backgrounds are estimated using both data-driven algorithms and simulated data from Monte Carlo (MC) samples [17]. The diboson and $tt$ processes are modeled using PYTHIA [25], and the production of a W or Z boson associated with jets is modeled using ALPGEN [26]. The single top quark signal is modeled using POWHEG [27] at next-to-leading-order (NLO) accuracy in $\alpha_s$. This measurement uses a NLO generator for the first time to model $s$- and $t$-channel single top quark production with the proper inclusion of the $Wt$ contribution [28]. A top quark mass of 172.5 GeV/$c^2$ is assumed, which is fully consistent with recent measurements [29]. Each of the event generators uses the CTEQ5L parton distribution functions [30] as input except for POWHEG, which uses the CTEQ6.1 parton distribution functions [31]. Parton showering and hadronization is simulated using PYTHIA tuned to underlying event data from the Tevatron [32]. The CDF II detector response is modeled using GEANT3 [33].

The probability for a light-flavor jet to be mistakenly $b$-tagged is estimated using a mistag matrix extracted from data control samples and parametrized as a function of the jet and event properties [17]. The kinematic properties of non-W events are determined using data samples obtained with less stringent requirements applied to lepton identification and isolation. The sample of events prior to $b$-tagging, referred to as the pretag sample, is dominated by non-W and $W +$ jets events. As non-W events typically have smaller $E_T$ than W boson events, the normalization for both non-W and $W +$ jets events is determined by fitting the $E_T$ distribution with the $E_T > 25$ GeV requirement removed.

Table I shows the expected sample composition for events with either two or three jets and either one or two $b$ tags, corresponding to a total of four statistically independent signal regions. Events originating from $s$-channel single top quark production frequently populate the two-tag signal region, while $t$-channel and $Wt$ events predominantly populate the one-tag signal region.

The number of expected signal events is much smaller than the uncertainty on the predicted background, and further separation of signal and background is required. We use artificial neural networks (NN) [34] to separate signal events from background events. Two dedicated NNs are used for each of the four signal regions, one for each of the two lepton categories. A number of kinematic variables are studied, and the most significant for distinguishing signal from background are used as inputs to build the NN discriminants. Although the NN inputs with the greatest discriminating power vary for the four signal regions, examples of the best inputs include $Q \times \eta$, the product of the charge of the electron or muon and the pseudorapidity of the light-quark jet, and $M_{\ell\nu b}$, the reconstructed top quark mass based on the electron or muon, the reconstructed neutrino, and the $b$-tagged jet. Descriptions of the variables and the full optimization procedure can be found in Ref. [17].

In order to maximize signal sensitivity, the NN discriminant is trained using only $s$-channel events as signal in the two-jet, two-tag signal region. In the remaining jet and tag signal regions, the NN is trained assuming only $t$-channel events as signal. To further improve the precision of the cross section measurement, we use training samples that contain additional events in which the jet energy scale, renormalization scale, and factorization scale are varied within their systematic uncertainties. By training the NN with a broader set of events with features
that more closely resemble data, the NN better accommodates certain systematic variations, enhancing its ability to discriminate signal from background. Simulations predict that this new procedure improves the accuracy of the final cross section measurement by approximately 3% [28].

The measurement of the single top quark cross section requires substantial input from theoretical models, Monte Carlo simulations, and extrapolations from control samples in data. We assign systematic uncertainties to the predictions and we investigate the effects of these uncertainties on the measured cross section. Three different classes of uncertainty are considered: the uncertainty in the predicted rates of signal and background processes, the uncertainty in the shapes of the distributions of the discriminant variables, and the uncertainty arising from the simulated sample size in each bin of each discriminant distribution. In the pretag sample, discrepancies between data and the MC predictions are visible in certain regions of distributions such as jet $\eta$ and $\Delta R(j_1^-, j_2^-) = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}$, where $j_1^-$ and $j_2^-$ are the momentum vectors of the two most energetic jets [17] (see Fig. 1). The inaccurate modeling of these distributions is potentially significant because jet-related variables are important inputs to the NN. We determine that the mismodeling is mainly due to $W + LF$ events, and we account for the mismodeling by creating a modified $W + LF$ background template in which events are reweighted to match pretag data in the jet $E_T$, jet $\eta$, and $\Delta \phi(j_1^-, j_2^-)$ distributions [28]. The difference between the unweighted and weighted distributions is taken as a one-sided systematic uncertainty. All of the systematic uncertainties are thoroughly discussed in Ref. [17].

The NN output distribution of the combined two-jet and three-jet signal regions is shown in Fig. 2. The predicted output distributions of s-channel, t-channel, and $Wt$ events are combined into one signal distribution, with proportions based on the SM predictions. The measure-
The posterior probability density calculated using the likelihood between the production cross section at 172.5 GeV/s is parametrized using Gaussian prior-density distributions. The effects associated with all sources of systematic uncertainties are integrated over the parameters of non-negative values of the cross section and integrate the posterior probability density over the parameters of the single top quark cross section. We assume a uniform prior probability density for all output distributions of the statistically independent bins. The NN discriminant variables. Regions of 68.3%, 95.5%, and 99.7% credibility regions are shown as shaded areas. The standard model (SM) predictions are also indicated with their theoretical uncertainties.

In conclusion, we study single top quark production in $t\bar{t}$ + jets final state using CDF data. The black circle shows the best-fit value, and the 68.3%, 95.5%, and 99.7% credibility regions are shown as shaded areas. The uncertainties on these measurements are correlated because signal from both the $s$-channel and $t$-channel + $Wt$ processes populate the signal-like bins of each of our discriminant variables. Regions of 68.3%, 95.5%, and 99.7% credibility are derived by evaluating the small region in area that contains the corresponding fractional integrals of the posterior probability density distribution. The best-fit values, the credibility regions, and the SM predictions are shown in Fig. 3. These measurements are fully compatible with the SM predictions of $\sigma_s = 1.06 \pm 0.06$ pb and $\sigma_{t+Wt} = 3.24 \pm 0.30$ pb [3–5].

In conclusion, we study single top quark production in the $W$ + jets final state using $pp$ collision data collected by the CDF experiment, corresponding to 7.5 fb$^{-1}$ of integrated luminosity. We measure a single top quark cross section for the combined $s$-channel + $t$-channel + $Wt$ processes of $3.04^{+0.57}_{-0.58}$ pb and we set a lower limit $|V_{tb}| > 0.78$ at the 95% credibility level, assuming $m_t = 172.5$ GeV/c$^2$. Using a two-dimensional fit.

FIG. 2. Comparison of the data with the sum of the predictions of the NN output for the combined two-jet and three-jet signal regions. The signal + background model is fit to the data. The uncertainty associated with the sum of the predictions (after fitting) is indicated by the grey shaded region in each bin. The inset shows a magnification of the region for which the NN discriminant ranges from 0.8 to 1.0, where the single top quark contribution is larger.

FIG. 3. Results of the two-dimensional fit for $\sigma_s$ and $\sigma_{t+Wt}$. The black circle shows the best-fit value, and the 68.3%, 95.5%, and 99.7% credibility regions are shown as shaded areas. The standard model (SM) predictions are also indicated with their theoretical uncertainties.
for $\sigma_s$ and $\sigma_{tW_1}$, we obtain $\sigma_s = 1.81^{+0.63}_{-0.58}$ pb and $\sigma_{tW_1} = 1.66^{+0.53}_{-0.47}$ pb. All of the measurements are consistent with SM predictions, and the lower limit on $|V_{td}|$ places improved bounds on various extensions of the SM and new phenomena.

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[2] When referring to single top quark processes, we implicitly include the charge-conjugate modes.
[19] CDF uses a cylindrical coordinate system where the z axis is along the proton beam direction, $\phi$ is the azimuthal angle, and $\theta$ is the polar angle. Pseudorapidity is $\eta = -\ln(\tan(\theta/2))$. Transverse momentum is $p_T = |p| \sin \theta$ and transverse energy is $E_T = E \sin \theta$.
[20] The calorimeter missing transverse energy $E_T^\text{cal}$ is defined as the sum over calorimeter towers $E_T^\text{cal} = -\sum_i E_T^i \rho_i$, where $i$ is an index over calorimeter towers with $|\eta| < 3.6$, and $\rho_i$ is a unit vector perpendicular to the beam axis and pointing at the ith calorimeter tower. The reconstructed missing energy $E_T$ is derived by correcting $E_T^\text{cal}$ for muon energy deposition and jet energy adjustments. We define $E_T^\text{cal}$ and $E_T$ to be the scalar magnitudes of $E_T^\text{cal}$ and $E_T$, respectively.
The distance between the reconstructed trajectories of particles or combinations thereof in the $\eta$-$\phi$ plane is defined as $R = \sqrt{\Delta \eta^2 + \Delta \phi^2}$.


The $W$ boson transverse mass is defined as $M_{W^T} = \frac{1}{c^2} \sqrt{2E_T^\ell E_T \left(1 - \cos \phi_{\nu} \right)}$, where $E_T^\ell$ is the transverse energy of the charged lepton and $\phi_{\nu}$ is the angle between the direction of the charged lepton and the $E_T$.


