Soft Gluon Resummations in Dijet Azimuthal Angular Correlations in Hadronic Collisions

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Soft Gluon Resummations in Dijet Azimuthal Angular Correlations at the Collider

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We derive all order soft gluon resummation in dijet azimuthal angular correlation in hadronic collisions at the next-to-leading logarithmic level. The relevant coefficients for the Sudakov resummation factor, the soft and hard factors, are calculated. The theory predictions agree well with the experimental data from D0 Collaboration at the Tevatron. This provides a benchmark calculation for the transverse momentum dependent QCD resummation for jet productions in hadron collisions.

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Introduction. Jet production in high energy scattering is one of the basic hard QCD processes in modern particle and nuclear physics [1]. It has been used as a golden probe for perturbative QCD studies and non-trivial hadron physics. In addition, jet physics has played an important role in electro-weak physics and searching for new physics beyond the standard model at high energy colliders, such as the Tevatron and the CERN Large Hadron Collider (LHC). Meanwhile, formulating jet production in hadronic processes is also one of the most challenging perturbative computations, and has attracted strong theoretical interests in the last few decades [2–5].

Among jet production processes, dijet production is one of the special processes. This is not only because it is easy to be identified experimentally, but also because it is an important channel to search for new physics [6]. In dijet events, the two jets are produced mainly in the back-to-back configuration in the transverse plane,

$$A + B \to Jet_1 + Jet_2 + X \ , \tag{1}$$

where A and B represent the two incoming hadrons with momenta P and $\overline{P}$, respectively, the azimuthal angle between the two jets is defined as $\phi = \phi_1 - \phi_2$ with $\phi_{1,2}$ being the azimuthal angles of the two jets. There have been comprehensive analyses of the azimuthal angular correlation (or decorrelation) in dijet events produced at hadron colliders [7–9]. In the leading order naive parton picture, the Born diagram contributes to a Delta function at $\phi = \pi$. One gluon radiation will lead to a singular distribution around $\phi = \pi$, which will persist at even higher orders. This divergence arises when the total transverse momentum of dijet (imbalance) is much smaller than the individual jet momentum, $q_\perp = |\vec{P}_{1\perp} + \vec{P}_{2\perp}| \ll |P_{1\perp}| \sim |P_{2\perp}| \sim P_J$, where large logarithms appear in every order of perturbative calculations. Here, P_J denotes the transverse momentum of the leading jet in the final state. These large logs are normally referred as the Sudakov logarithms, $\alpha_s^i \ln^{2i-1}(P_J^2/q_\perp^2)$. Therefore, a QCD resummation has to be included in order to have a reliable theoretical prediction. Since it involves a transverse momentum $q_\perp$, the resummation formalism adopted this process is similar to that for low transverse momentum electroweak boson (or Higgs boson) production: the transverse momentum dependent (TMD) or Collins-Soper-Sterman (CSS) resummation formalism [10]. However, due to the fact that the colored final state of dijet will induce additional soft gluon radiations, the resummation of dijet production is much more complicated. In the literature, the leading double logarithmic contribution for dijet azimuthal correlation has been derived [11, 12], where it was found that each incoming parton contributes half of its color charge to the leading double logarithmic Sudakov resummation factor. In this paper, we will go beyond the double logarithmic approximation to perform the resummation calculation at the next-to-leading logarithm (NLL) level. More importantly, we will construct a theoretical framework in the TMD (or CSS) resummation formalism to describe jet production processes which will have great impact on the LHC physics. The NLL resummation for this observable has been considered in Ref. [11, 13] from different perspective. We also note that a Mont Carlo event generator has been developed to study this observable [14], where a k_t-dependent parton shower was applied. In this work, we discuss the azimuthal angular correlation of the two leading jets with large transverse momenta in the central rapidity region produced in hadron collisions. This is in contrast to the study in Ref. [12] which concerns the small-x dynamics and the case that both jets are boosted into large rapidity region, or in Ref. [15] which concerns jets produced with large rapidity gap.

Our derivation is based on a complete one-loop calculation of soft and/or collinear gluon contributions to the dijet production in hadron collisions, from which the NLL resummation coefficients are determined. We show that the factorization in the low $q_\perp$ region, as found in the complete one-loop calculation, provides a solid foundation for applying the TMD resummation formalism in this production process. The factorization of soft gluon radiation follows the original studies on the threshold resummation of colored final state particle production in hadronic collisions by Sterman *et al.* [16], where it was found that a matrix form of the resummation formula has to be applied [17]. We find that the same conclusion also holds for studying dijet production using the TMD resumamtion formlism. Recent studies for the TMD resummation for heavy quark pair production in hadronic collisions also found similar matrix

form [18, 19].

Our resummation formula can be summarized as

$$\frac{d^4\sigma}{dy_1 dy_2 dP_J^2 d^2 q_\perp} = \sum_{ab} \sigma_0 \left[\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} W_{ab\to cd}(x_1, x_2, b_\perp) + Y_{ab\to cd} \right] ,\tag{2}$$

where the first term W contains all order resummation and the second term Y comes from the fixed order corrections; σ_0 represents normalization of the differential cross section, y_1 and y_2 are rapidities of the two jets, P_J is the leading jet transverse momentum, and $q_\perp$ the imbalance transverse momentum between the two jets as defined above. All order resummation for W from each partonic channel $ab \to cd$ can be written as

$$\begin{aligned} W_{ab\to cd}(x_1, x_2, b) &= x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(Q^2, b_\perp)} \\ &\times \text{Tr}\left[\mathbf{H}_{ab\to cd} \exp\left[-\int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^{s\dagger}\right] \mathbf{S}_{ab\to cd} \exp\left[-\int_{b_0/b_\perp}^Q \frac{d\mu}{\mu} \gamma^s\right] \right] , \end{aligned}\tag{3}$$

where $Q^2 = \hat{s} = x_1 x_2 S$, which represents the hard momentum scale, $b_0 = 2e^{-\gamma_E}$, $f_{a,b}(x, \mu)$ are parton distributions for the incoming partons a and b, $x_{1,2} = P_J (e^{\pm y_1} + e^{\pm y_2})/\sqrt{S}$ are momentum fractions of the incoming hadrons carried by the partons. In the above equation, the hard and soft factors $\mathbf{H}$ and $\mathbf{S}$ are expressed as matrices in the color space of partonic channel $ab \to cd$, and γ^s are the associated anomalous dimensions for the soft factor (defined below). The Sudakov form factor $\mathcal{S}_{Sud}$ resums the leading double logarithms and the universal sub-leading logarithms,

$$S_{\text{Sud}}(Q^2, b_\perp) = \int_{b_0^2/b_\perp^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln\left(\frac{Q^2}{\mu^2}\right) + B + D_1 \ln\frac{Q^2}{P_J^2 R_1^2} + D_2 \ln\frac{Q^2}{P_J^2 R_2^2} \right] ,\tag{4}$$

where $R_{1,2}$ represent the cone sizes for the two jets. Here the parameters A, B, D_1, D_2 can be expanded perturbatively in α_s. At one-loop order, $A = C_A \frac{\alpha_s}{\pi}$, $B = -2C_A\beta_0 \frac{\alpha_s}{\pi}$ for gluon-gluon initial state, $A = C_F \frac{\alpha_s}{\pi}$, $B = \frac{-3C_F}{2} \frac{\alpha_s}{\pi}$ for quark-quark initial state, and $A = \frac{(C_F + C_A)}{2} \frac{\alpha_s}{\pi}$, $B = (\frac{-3C_F}{4} - C_A\beta_0) \frac{\alpha_s}{\pi}$ for gluon-quark initial state. These coefficients A, at the one-loop order, agree with those found in the leading double logarithmic analysis [11, 12]. In our numeric calculations, we will also include $A^{(2)}$ contributions since they are associated with the incoming parton distributions and are the same (after dividing by a factor of two) as those for vector boson and Higgs particle productions [20].

At the next-to-leading logarithmic level, the jet cone size enters as well [13]. That is the reason we have two additional factors in Eq. (4): $D = C_A \frac{\alpha_s}{2\pi}$ for gluon jet and $D = C_F \frac{\alpha_s}{2\pi}$ for quark jet. The cone size R is introduced to regulate the collinear gluon radiation associated with the final state jets. Only the soft gluon radiation outside the jet cone contributes to the imbalance $q_\perp$ between the two jets, and yields the logarithmic contribution in the form of $\ln\left(\frac{Q^2}{P_J^2 R^2}\right)$. This conclusion is independent of the jet algorithm [21]. It is interesting to note that the similar cone size dependence in Eq. (4) has also been found in the threshold resummation for jet production in hadronic collisions [5, 16].

In the following, we will explain briefly how we derive the above resummation results and present the numeric comparison with experimental data. The detailed derivations will be left for a separate publication. We will also examine α_s expansion of the resummed result against the full one-gluon radiation perturbative calculations in the limit of $q_\perp \ll P_J$, i.e., close to the back-to-back configuration $\phi = \pi$. This serves as an important cross check of our resummation formula.

TMD Distributions. The total transverse momentum of the dijet $q_\perp$ is perpendicular to the beam direction of incoming hadrons. The most important contributions come from the collinear and soft gluon radiations in the limit of $q_\perp \ll P_J$. The collinear gluon radiations associated with the incoming partons are factorized into the relevant parton distributions. In this paper, we assume this factorization is valid for dijet production. By comparing to the experimental data, we will be able to test the factorization and investigate the factorization breaking effects in this process, which has been extensively discussed in the literature in the last few years [22–26]. The factorization breaking effects found in these studies emerge at order α_s^3. Hence, we only include up to α_s^2 contribution in the the resummation coefficient A of Eq. (4).

To evaluate the gluon radiation contribution associated with the parton distributions, we introduce the TMD parton distributions, following the Ji-Ma-Yuan scheme. For example, for the gluon distribution from hadron A, we have [27],

$$\begin{aligned} xg(x, k_\perp) &= \int \frac{d\xi^- d^2\xi_\perp}{P^+(2\pi)^3} e^{-ixP^+\xi^- + i\vec{k}_\perp \cdot \vec{\xi}_\perp} \\ &\times \left\langle P | F_a^{+}{}_\mu(\xi^-, \xi_\perp) \mathcal{L}_{vab}^\dagger(\xi^-, \xi_\perp) \mathcal{L}_{vbc}(0, 0_\perp) F_c^{\mu+}(0) | P \right\rangle , \end{aligned}\tag{5}$$

where $F^{\mu\nu}$ is the gauge field strength tensor, and $\mathcal{L}_v(\xi) = P \exp\left(-ig \int_0^{-\infty} d\lambda v \cdot A(\lambda v + \xi)\right)$ is the gauge link in the adjoint representation, $A^\mu = -if_{abc}A_c^\mu$. The off-light-cone vector v is introduced to regulate the light-cone singularity associated with the TMD distributions, $\zeta^2 = (2v \cdot P)^2/v^2$. An evolution equation can be derived for the TMD distributions respect to ζ,

$$\frac{\partial}{\partial \ln \zeta} xg(x, b_\perp, \zeta) = (K(\mu, b_\perp) + G(\zeta, \mu)) \times xg(x, b_\perp, \zeta) , \tag{6}$$

where K and G are the evolution kernel. Similarly, we will introduce the TMD parton distribution from incoming hadron B, which includes another light-cone singularity regulator $\bar{\zeta}^2 = (2\bar{v} \cdot \bar{P})^2/\bar{v}^2$. After resummation, the dependence on v and $\bar{v}$ will cancel out between TMD distributions and the soft factors, which we will introduce in the following.

Soft Factor. The collinear gluon radiations parallel to the final state two jets are factorized into the jet functions, and do not contribute to the low transverse momentum imbalance $q_\perp$. However, the soft gluon radiations will contribute. In this section, we will evaluate the soft factor contribution.

For the color neutral particle production in hadronic collisions, the soft factor can be easily constructed. For colored final state, factorizing out the soft gluon radiation is much more complicated, although the basic idea is the same, i.e., applying the Eikonal approximation. So, for each incoming and outgoing partons, the soft gluon radiation is factorized into an associated gauge link moving along the parton momentum direction. Because of color entanglement among the four partons in the $2 \to 2$ process, gauge invariant construction of the soft factor depends on the overall color configurations. In our calculations, we follow the procedure of Ref. [16], where the soft gluon radiations are evaluated on the orthogonal color basis. For example, in the partonic process $gg \to q\bar{q}$, there are three color bases,

$$C_1 = \delta^{a_1 a_2}\delta_{a_3 a_4} , \qquad C_2 = if^{a_1 a_2 c} T_{a_3 a_4}^c , \qquad C_3 = d^{a_1 a_2 c} T_{a_3 a_4}^c . \tag{7}$$

where $a_{3,4}$ are the color indices for the finial state quark and antiquark, and $a_{1,2}$ for the initial state gluons. For $qg \to qg$ channel, similar color bases will be chosen; whereas there are 8 independent color structures for $gg \to gg$ channel. The soft factor is constructed as matrix elements on the above bases, and

$$S_{IJ} = \int_0^\pi \frac{d\phi_0}{\pi} C_{Iii'}^{bb'} C_{Jll'}^{aa'} \langle 0|\mathcal{L}_{vcb'}^\dagger(b_\perp)\mathcal{L}_{vbc}(b_\perp)\mathcal{L}_{\bar{v}ca'}^\dagger(0)\mathcal{L}_{\bar{v}ac}(0)\mathcal{L}_{nji}^\dagger(b_\perp)\mathcal{L}_{\bar{n}i'k}(b_\perp)\mathcal{L}_{\bar{n}kl}^\dagger(0)\mathcal{L}_{nl'j}(0)|0\rangle , \tag{8}$$

where we have integrated out the azimuthal angle ϕ_1 of the leading jet and traded the relative azimuthal angle ϕ for $q_\perp$. In the above equation, I, J represent the color basis index, n and $\bar{n}$ represent final state quark and antiquark momentum directions (for the above mentioned $gg \to q\bar{q}$ channel), and v and $\bar{v}$ for the initial state two momentum directions. The soft factor is expressed by a 3×3 matrix. Accordingly, the hard factor can also be calculated on the same color bases and expressed as a 3×3 matrix.

Following the TMD formalism, we need to choose the off-light-cone gauge links for the two incoming partons in Eq. (8). As mentioned in the Introduction, not all the soft gluon radiations can contribute to the imbalance $q_\perp$ between the two jets. Those gluon radiation inside the jet cone will be part of the jet and will not contribute to the soft factor. In order to exclude these contributions, we can impose a kinematic constraint on the phase space integral for the radiated gluon. Equivalently, we find that it is much simpler to require an off-shellness of n and $\bar{n}$ in Eq. (8) so that $n^2 = P_J^2 R_1^2/Q^2$ and $\bar{n}^2 = P_J^2 R_2^2/Q^2$, where $R_{1,2}$ are the cone sizes for the two jets [5, 16]. By doing so, we obtain the exact same leading logarithms of $R_{1,2}$ for the soft factor as compared to the kinematic constraint calculation in the small cone approximation.

The soft factor satisfies the renormalization group equation [16]:

$$\frac{d}{d \ln \mu} S_{IJ}(\mu) = -\Gamma_{IJ'}^{s\dagger} S_{J'J}(\mu) - S_{IJ'}(\mu)\Gamma_{J'J}^s , \tag{9}$$

where Γ is the relevant anomalous dimension. By solving the renormalization group equation, we resum the large logarithms associated with the soft factor. More specifically, we include the cone size dependent term in Eq. (4) and keep the rest in the γ^s term in Eq. (3).

Factorization implies that the differential cross section contributions from the partonic processes can be written as

$$W_{ab \to cd}(x_i, b) = x_1 f_a(x_1, b, \zeta^2, \mu^2, \rho) x_2 f_b(x_2, b, \bar{\zeta}^2, \mu^2, \rho) \text{Tr}\left[\mathbf{H}_{ab \to cd}(Q^2, \mu^2, \rho)\mathbf{S}_{ab \to cd}(b, \mu^2, \rho)\right] ,$$

where the dependence on $\rho = (2v \cdot \bar{v})^2/v^2\bar{v}^2$ and the factorization scale μ cancel out among different factors. To derive the final resummation results, we have to solve the evolution equation for the parton distributions and the renormalization group equation for the soft factor. In particular, we will choose the factorization scale $\mu = Q$, and
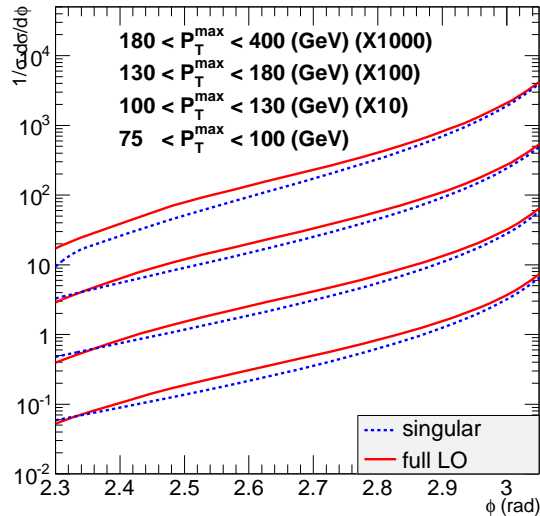
FIG. 1: Comparison between the α_s expansion of the resummation calculation (dashed curve) and the full leading order (LO) calculation (solid curve) in the region around $\phi = \pi$ for the kinematics specified by the D0 Collaboration [7]. The LO curves come from NLOJET++ code [7, 28].

evolve the parton distribution and soft factor from the scale $1/b$ to Q. After this, we arrive at the resummation results, as shown in Eqs. (2) and (3). The detailed expressions for $\mathbf{H}$, $\mathbf{S}$ and γ^s will be given in a separate publication. Two important cross checks are found in the final results: (1) ρ dependence cancels out; (2) cone size R dependence is diagonal in the color basis matrix and can be removed from the soft factor, which leads to a universal resummation factor as shown in Eqs. (3) and (4).

Compare to the Experimental Data. Before we apply our resummation formula to compare with experimental data, we would like to demonstrate the consistency between the perturbative expansion of our resummation result and the fixed order calculation around $\phi = \pi$. In Fig. 1, we plot the normalized distribution of the dijet azimuthal angle ϕ for the kinematics specified by the D0 collaboration [7]. The solid curves are from a full leading order (LO) calculation (with three partons in the final state) [7, 28] and the dashed curves are from the expansion of the $W_{ab \to cd}$ term to the same order in strong coupling constant α_s, labelled as "singular" piece. In the numeric calculations, we used the CT10 next-to-leading order (NLO) parton distribution functions [29], and the jet cone size $R = 0.7$ [7]. From this figure, we clearly see that the leading order expansion of the resummation results agrees well with the fixed order calculation in the limit $\phi \to \pi$. This demonstrates that the resummation result captures the most important contribution from the fixed order calculation.

In Fig. 2, we compare our resummation results to the experimental data from D0 Collaboration at the Tevatron [7]. We have included $A^{(1,2)}$, $B^{(1)}$ and $D_{1,2}^{(1)}$ in the Sudakov form factor. It is worthwhile to mention that including $A^{(2)}$ improves the agreements with the data, which may support the factorization we argue in this paper. B and D coefficients are generally process-dependent. Here, we only include their one-loop results. It is desirable to obtain higher order results and study their effects, which is however beyond the scope of this paper.

When Fourier transforming the $b_\perp$-expression to obtain the transverse momentum distribution, we follow the b_* prescription of CSS resummation [10], *i.e.*, replacing $b_\perp$ by $b_* = b/\sqrt{1 + b^2/b_{max}^2}$ in the calculation. By doing so, we will also introduce the non-perturbative form factors for the quarks and gluons from the initial states. In our calculations, we have used $b_{max} = 0.5 \, \text{GeV}^{-1}$, and the non-perturbative form factors follow the parameterizations in Refs. [30]. However, we would like to emphasize that because the jet energy is so large that our final results are not sensitive to the non-perturbative form factors at all.

From Fig. 2, we see that our resummation results agree well with the experimental data, from ϕ near to π down to much smaller values. For smaller value of ϕ (away from the back-to-back configuration), the resummation calculations match to the fixed order results at NLO [28], which has also been separately shown in Fig. 2. We note that a full NLO calculation cannot describe experimental data for $\phi \sim \pi$ [7], where the fixed order calculation becomes divergent. Our resummation calculation, after being matched with the NLO result.clearly improves the theory prediction and can describe the experimental data in a wider kinematic region. This demonstrates the importance of all order resummation in perturbative calculations for these type of hard QCD processes.
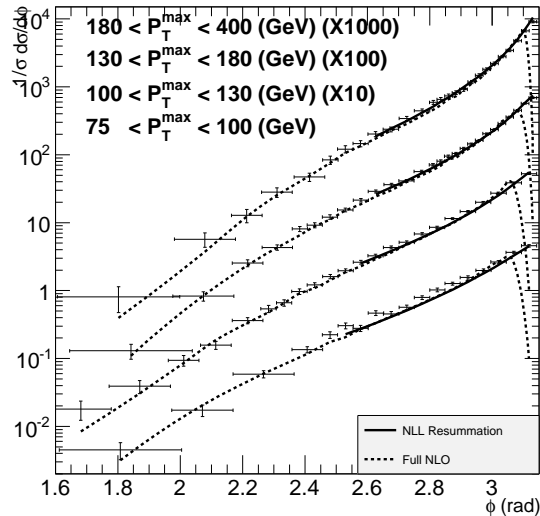
FIG. 2: Resummation results on dijet azimuthal correlations at the Tevatron, compared to the experimental data from D0 Collaboration [7], where both jets are produced in central-rapidity region with $|y| < 0.5$. Away from $\phi = \pi$, the resummation results (solid curves) are matched to a full NLO calculation (dashed curves) [7, 28].

Conclusions. In this paper, we have performed the QCD resummation calculation for dijet azimuthal angular correlation in hadron collisions, at the next-to-leading logarithmic level. The relevant resummation coefficients are computed at one-loop order. We show that the perturbative expansion of our resummation calculation agree well with the fixed order calculation in the back-to-back correlation limit (ϕ around π). After being matched with the full NLO calculation at a smaller ϕ value, our resummation results can describe a much wider range of the experimental data, particulary for ϕ around π where event rate dominates. The agreements between the resummation results and the experimental data encourage further developments along this direction. We will present the detailed derivations and more phenomenological studies in a future publication. We will also discuss jet algorithm dependence of our results and the issues concerning the "non-global" logarithms arising from α_s^2 calculations [11, 13].

Extension to other jets production processes will be interesting to carry out, in particular, for those that involve Higgs boson in the final state, such as Higgs plus one jet or two jets production at the LHC. Applications to the dijet production in heavy ion collisions and proton-nucleus collisions [31] are interesting to follow as well. We also notice that recently, the CMS collaboration at the LHC has measured the dijet azimuthal correlation with large rapidity separation between the jets, which has been interpreted as the BFKL resummation effects [15]. It will be worthwhile to compare these theoretical calculations with our results to pin down the underlying physics mechanism for the observed phenomena.

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